Passband Wireless Communication

Introduction

Beginning with this lecture, we will study wireless communication. The focus of these lectures will be on point to point communication. Communication on a wireless channel is inherently different from that on a wireline channel. The main difference is that unlike wireline channel, wireless is a shared medium. The medium is considered as a federal resource and is federally regulated. The entire spectrum is split into many licensed and unlicensed bands. An example of the the point to point communication in the licensed band is the cellular phone communication, whereas wi-fi, cordless phones and blue tooth are some of the examples of communication in the unlicensed band.

The transmission over a wireless channel is restricted to a range of frequencies $(f_c - \frac{W}{2}, f_c + \frac{W}{2})$ around the central carrier frequency f_c. The wire is a low pass filter and hence the carrier frequency for the wireline channel is $f_c = 0$.

This restriction immediately poses some questions about the design of the wireless communication systems. The foremost question being how is reliable communication related to the carrier frequency? Is the communication strategy and hence the transmitter-receiver design particular to the specific carrier frequency? Do we have to design the system based on f_c?

It turns out that we can always work in with the baseband signal (i.e., the signal with $f_c = 0$) even for the wireless communication and then convert the baseband signal to the passband signal (a signal that is centered around some nonzero carrier fraquency) with the desired carrier frequency. This makes the design of the transmitter and receiver transparent to the carrier frequency. Thus, only the front end of of the sytem needs to be changed if we change f_c. Also since the bandwidth of the signal W (typically in KHz) is much smaller than the carrier frequency f_c (typically in MHz), the design of DAC and ADC becomes much easier and modular.

The focus of this lecture will be on the conversion of the baseband signal to the passband signal and vice-versa. Also, the actual wireless channel affects the passband signal. How do these effects translate in the baseband domain, i.e., is there a baseband equivalent of the wireless channel? We will also address this question.

Baseband Representation of the Passband Signals

As mentioned before, most of the processing such as coding/decoding, modulation/ demodulation etc. is done at the baseband. At the transmitter, the last stage of the operation is to "up-convert" or "mix" the signal with the carrier frequency and transmit it via the antenna. Similarly, the first step at the receiver is to "down-convert" the RF signal to the

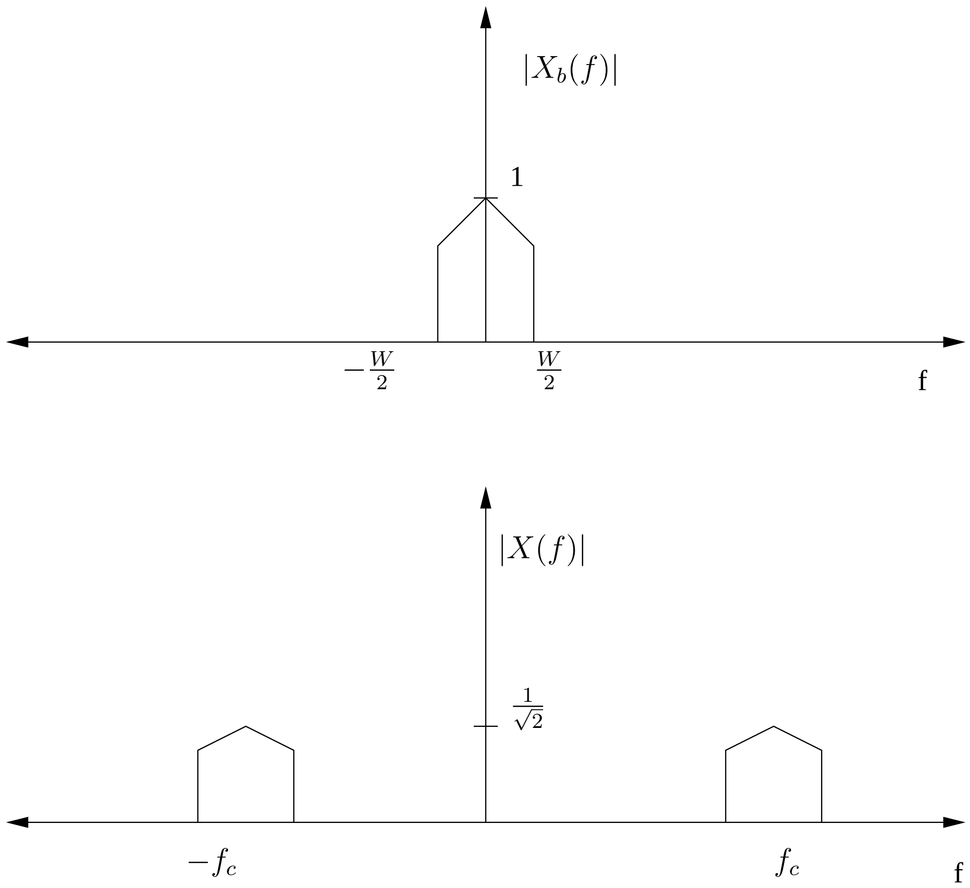

Figure 1: Magnitude spectrum of the real baseband signal and its passband signal

baseband before processing. Therefore it is most important to have a baseband equivalent representation of signals.

Let's begin with the real baseband signal $x_b(t)$ (of double sided bandwidth W) that we want to transmit over the wireless channel in a band centered around f_c. In wireline channel, $x_b(t)$ would be the signal at the output of the DAC.

We know that we can up-convert this signal by multiplying it by $\cos 2\pi f_c t$.

$$x(t) = x_b(t) \cdot \sqrt{2} \cos 2\pi f_c t \tag{1}$$

The resulting signal $x(t)$ has spectrum centered around f_c and $-f_c$. Figure 1 shows this transformation diagramatically. We scale the carrier by $\sqrt{2}$ as $\cos 2\pi f_c t$ has power $\frac{1}{2}$. Thus, by scaling, we are keeping the power in $x_b(t)$ and $x(t)$ same. Note that since $x_b(t)$ is real, the magnitude of its Fourier transform, $X_b(f)$ is symmetric in f and hence the magnitude of the spectrum of the RF signal, $X(f)$ is symmetric around f_c and $-f_c$. We note that to get real $x(t)$, we need not have $X(f)$ symmetric around f_c and $-f_c$. This is a consequence of $x_b(t)$ being real.

To get back the baseband signal, we multiply $x(t)$ again by $\sqrt{2} \cos 2\pi f_c t$ and then pass

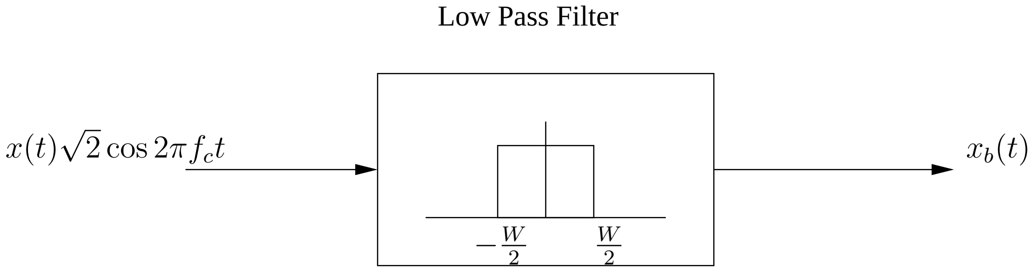

Figure 2: Down-conversion at the receiver

the signal through a low pass filter with bandwidth W.

$$\begin{aligned} x(t)\sqrt{2}\cos 2\pi f_c t &= 2\cos^2(2\pi f_c t).x_b(t) & (2) \\ &= (1+\cos 4\pi f_c t)x_b(t) & (3) \end{aligned}$$

The low pass filter will discard the signal $x_b(t)\cos 4\pi f_c t$ as it is the bandpass signal centered around $2f_c$. Figure 2 shows this transformation diagramatically.

One can see that if we multiply $x(t)$ by $\sqrt{2}\sin 2\pi f_c t$ instead of $\sqrt{2}\cos 2\pi f_c t$, we get $x_b(t)\sin 4\pi f_c t$ and low pass filter will discard this signal completely. There will be a similar outcome had we modulated the baseband signal on $\sqrt{2}\sin 2\pi f_c t$ and try to recover it by using $\sqrt{2}\cos 2\pi f_c t$. Thus,

1. Since the only difference in $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ is the phase lag of $\frac{\pi}{2}$, synchronization of carrier phase is crucial in up-conversion and down-conversion.

2. We also note that the signals modulated on $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ never get mixed up in the process of down-conversion. Though both the signals share same frequency band, they are orthogonal to each other. Thus, we could have transmitted two real baseband signals in the same frequency band and doubled the data rate. This is possible as now we are using total double sided bandwidth of $2W$ instead of W as in wireline channel. The resulting RF signal is still real. However, the magnitude of the spectrum of the RF signal need not be symmetric around f_c and $-f_c$.

Thus, we can now have the RF signal $x(t)$ which is

$$x(t) = x_{b_1}(t)\sqrt{2}\cos 2\pi f_c t - x_{b_2}(t)\sqrt{2}\sin 2\pi f_c t \tag{4}$$

The baseband signals $x_{b_1}(t)$ and $x_{b_2}(t)$ are obtained at the receiver by multiplying $x(t)$ by $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ separately and then passing both the outputs through the low pass filters. Here we are modulating the amplitude of the carrier by the baseband data. Such a scheme is called amplitude modulation. When we modulate both sin and cos parts of the carrier by two undependent baseband signals, the scheme is called Quadrature Amplitude Modulation (QAM). [1]

[1] Can we up-convert one more baseband signal and still be able to recover it at the receiver? The answer is negative. This is because, we can express $\cos(2\pi f_c t+\theta)$ as $\cos\theta\cos 2\pi f_c t - \sin\theta\sin 2\pi f_c t$. Thus any phase change θ is uniquely determined by the amplitudes of $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$.

The baseband signal $x_b(t)$ is now defined in terms of the pair $(x_{b_1}(t), x_{b_2}(t))$. In literature, this pair is denoted as $(x_b^I(t), x_b^Q(t))$, where I stands for "in phase" signal and Q stands for "quadrature phase" signal. To make the notation compact we can think of $x_b(t)$ as a complex signal defined as follows:

$$x_b(t) \stackrel{\text{def}}{=} x_b^I(t) + j x_b^Q(t) \tag{5}$$

We will follow this notation hereafter.

If the wireless channel is just the AWGN channel, then we know how to recover the baseband signal from the RF signal at the receiver and we are done. However, wireless channel is not AWGN channel. If $h(t)$ denote the impulse response of the (time-invarient) wireless channel, the received RF signal is

$$y(t) = h(t) * x(t) + w(t) \tag{6}$$

where $w(t)$ is the RF noise. We will ignore the noise for the time being. Then, $y(t) = h(t) * x(t)$. $x(t)$ is obtained by up-converting the baseband signal $x_b(t)$. We obtain the baseband signal $y_b(t)$ at the receiver by down-converting the received RF signal $y(t)$.

The question we want to address now is

> How does the channel impulse response manifests itself in baseband? How are the the baseband signals $y_b(t)$ and $x_b(t)$ related?

It turns out that there is an baseband equivalent filter $h_b(t)$ of the channel filter $h(t)$. The transmitted baseband signal $x_b(t)$ is filtered through the baseband channel filter $h_b(t)$ to give the received baseband signal $y_b(t)$. [2]

$$y_b(t) = h_b(t) * x_b(t) \tag{7}$$

To understand the relation between $h(t)$ and $h_b(t)$, let's consider a few examples.

1. Let's take the simple case when $h(t) = \delta(t)$. Then, $y(t) = x(t)$ and hence $y_b(t) = x_b(t)$. Hence, $h_b(t) = \delta(t)$.

2. Let's consider $h(t) = \delta(t - t_0)$. In this case,

$$y(t) = x(t - t_0) = x_b^I(t - t_0)\sqrt{2}\cos 2\pi f_c(t - t_0) - x_b^Q(t - t_0)\sqrt{2}\sin 2\pi f_c(t - t_0) \tag{8}$$

[2]The intuition behind this relation can be obtained by representing the signals in frequency domain. We know that in frequency domain, $Y(f) = H(f)X(f)$. We have also seen from Figure 1 that the $X(f)$ and $X_b(f)$ are related by translation in frequency domain and so are $Y(f)$ and $Y_b(f)$. Thus, if we translate $H(f)$ appropriately, we will have $H_b(f)$ so that $Y_b(f) = H_b(f)X_b(f)$, i.e., in time domain, $y_b(t) = h_b(t) * x_b(t)$.

We obtain the baseband signal $y_b(t)$ as

$$\begin{aligned}
y_b^I(t) &= \text{LPF}\left(y(t)\sqrt{2}\cos 2\pi f_c t\right) & (9)\\
&= \text{LPF}\big((2\cos 2\pi f_c(t-t_0)\cos 2\pi f_c t)x_b^I(t-t_0)\\
&\quad -(2\sin 2\pi f_c(t-t_0)\cos 2\pi f_c t)x_b^Q(t-t_0)\big) & (10)\\
&= \text{LPF}\big((\cos 2\pi f_c(2t-t_0)+\cos 2\pi f_c t_0)x_b^I(t-t_0)\\
&\quad -(\sin 2\pi f_c(2t-t_0)-\sin 2\pi f_c t_0)x_b^Q(t-t_0)\big) & (11)\\
&= x_b^I(t-t_0)\cos 2\pi f_c t_0 + x_b^Q(t-t_0)\sin 2\pi f_c t_0 & (12)\\
&= \mathcal{R}\left\{x_b(t-t_0)e^{-j2\pi f_c t_0}\right\} & (13)
\end{aligned}$$

Similarly, we obtain $y_b^Q(t)$ as

$$\begin{aligned}
y_b^Q(t) &= \text{LPF}\left(-y_b(t)\sqrt{2}\sin 2\pi f_c t\right) & (14)\\
&= \mathcal{I}\left\{x_b(t-t_0)e^{-j2\pi f_c t_0}\right\} & (15)
\end{aligned}$$

Thus,

$$y_b(t) = x_b(t-t_0)e^{-j2\pi f_c t_0} \quad (16)$$

Hence,

$$h_b(t) = e^{-j2\pi f_c t_0}\delta(t-t_0) \quad (17)$$

Thus, the baseband signal also gets delayed by the same amount as the passband signal. However, its phase also changes. This phase lag depends on the delay t_0 as well as on the carrier frequency f_c.

We can generalize the second example to obtain the baseband equivalent representation of a generalized channel. Suppose the wireless channel is given by

$$h(t) = \sum_{l=0}^{L-1} a_l \delta(t-t_l) \quad (18)$$

Then, the baseband equivalent of the channel will be

$$h_b(t) = \sum_{l=0}^{L-1} a_l e^{-j2\pi f_c t_l}\delta(t-t_l) \quad (19)$$

We will see in the next lecture that the wireless channel can actually be modeled as Equation 18.

Looking Ahead

In this lecture, we have seen the baseband representation of the RF signal. Writing baseband signal as a complex number simplifies the notation a lot. In next few lectures, we will see that this notation will enable us to use many of the results from the wireline channel.

We have also obtained the baseband equivalent of the channel given by Equation 18. In the next lecture we will see that this is a reasonable model for the wireless channel. We will also see that how the wireless cannels are different from the wireline channels.