EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 10

Thu 4/17/2008 and Fri 4/18/2008

1. Consider a communication system where the information symbols $x[m]$ are mapped to a continuous waveform via

$$x(t) = \sum_{m>0} x[m]g(t - mT),$$

where $g(t)$ corresponds to the pulse shape used. The waveform is then sent over a LTI channel with impulse response $\delta(t)$. The signal is received without noise and filtered then sampled to produce the output sequence $y[n] = y(nT)$, where

$$\begin{aligned} y(t) &= x(t) * g(-t) \\ &= \int_{-\infty}^{\infty} x(\tau)g(\tau - t)d\tau \end{aligned}$$

(a) If the pulse $g(t) = \text{sinc}(t/T)$ is used, show that

$$y(t) = \sum_{m>0} x[m] \, \text{sinc}(t/T - m).$$

What is $y[n]$?

(b) Suppose that we use the same pulse $g(t) = \text{sinc}(t/T)$, but that the transmitter and receiver are not perfectly time synchronized, i.e.

$$y(t) = x(t - \Delta) * g(-t).$$

Compute $y[n]$. What is the problem?

Hint: consider the case where $x[m] = 1$ for $m = 1, 2, \ldots, \infty$.

(c) Suppose instead that we use a pulse $g(t)$ with Fourier transform

$$G(f) = \begin{cases} \sqrt{1 + 2fT}, & -1/2T < f \leq 0 \\ \sqrt{1 - 2fT}, & 0 < f < 1/2T \\ 0, & \text{otherwise.} \end{cases}$$

Compute an expression for $g(t)$. Compute $y[n]$. Does the problem from part (b) still exist?