EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 11

Mon 4/21/2008 and Wed 4/23/2008

1. (Introduction to Equalization) Consider the following communication channel with intersymbol interference (ISI)

$$y[1] = 2x[1] + w[1]$$
$$y[2] = x[1] + 2x[2] + w[2]$$

where $x[1]$ and $x[2]$ are two data symbols and $w[1]$ and $w[2]$ are i.i.d. Gaussian noise random variables with mean zero and variance σ^2.

(a) Describe a simple way of estimating the value of the symbol $x[2]$? Write your estimation rule as a linear function of $y[1]$ and $y[2]$. This estimator is called the *zero forcing (ZF) equalizer*.

(b) Describe the relationship between the ZF equalizer and the matrix

$$\mathbf{H} = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}.$$

Draw and label a picture describing the geometry of the ZF equalizer in $\mathbb{R}^2$.

(c) Assuming $x[1]$ and $x[2]$ are i.i.d. Gaussian random variables with zero mean and variance E, compute the MSE for symbol $x[2]$ for the ZF equalizer, as function of $\mathsf{SNR} \triangleq E/\sigma^2$. The MSE for symbol $x[2]$ is given by $\mathbb{E}(\hat{x}[2] - x[2])^2$, where $\hat{x}[2]$ is the decoder's estimate of $x[2]$.

(d) Assuming $x[2]$ is a BPSK symbol taking value $+\sqrt{E}$ or $-\sqrt{E}$ with equal probability, describe how you would use the output of the ZF equalizer to decode $x[2]$. Compute $P_{ZF}(\mathcal{E})$, the probability of decoding error for $x[2]$ as a function of SNR.

(e) Suppose you use the following estimate of $x[2]$,

$$\hat{x}[2] = y[2].$$

This estimation rule is called the *matched filter (MF) equalizer*. Assuming that $x[1]$ is a Gaussian random variable with zero mean and variance E, but $x[2]$ is an independent BPSK symbol taking values $+\sqrt{E}$ or $-\sqrt{E}$ with equal probability, compute $P_{MF}(\mathcal{E})$, the probability of decoding error for $x[2]$ as a function of SNR, treating $x[1]$ as independent Gaussian noise with zero mean and variance E.

(f) Describe the relationship between the matched filter equalizer and the matrix $\mathbf{H}$ from part (b). Draw and label a picture describing the geometry of the MF equalizer in $\mathbb{R}^2$.

(g) Comment on the relative error performance of the two equalizers at different SNR levels.