EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 12

Mon 4/28/2008 and Wed 4/30/2008

1. (Equalization cont.) Consider the following communication channel with intersymbol interference (ISI)

$$y[1] = 2x[1] + w[1]$$
$$y[2] = x[1] + 2x[2] + w[2]$$

where $x[1]$ and $x[2]$ are two data symbols and $w[1]$ and $w[2]$ are i.i.d. Gaussian noise random variables with mean zero and variance σ^2.

(h) Write down an expression for the matched filter for estimating $x[1]$ as a function of $y[1]$ and $y[2]$. If $x[1]$ and $x[2]$ are independent 2-PAM symbols each taking on values $+\sqrt{E}$ and $-\sqrt{E}$ with equal probability, draw a constellation diagram for the receive space and illustrate the filtering operation as a projection.

(i) Assume now that $x[1]$ is 2-PAM but $x[2]$ is a Gaussian random variable with zero mean and variance E. Compute the probability of decoding symbol $x[1]$ incorrectly. What happens when SNR is large? Why?

(j) Now compute the actual error probability when $x[2]$ is also equiprobable 2-PAM. What happens when SNR is large? Why? Why might the error probability expression derived in (i) be a more meaningful measure of typical matched filter performance at high SNR? *Hint: think about what happens when we use M-PAM.*

(k) Suppose $x[1]$ and $x[2]$ are independent equiprobable 2-PAM symbols. We wish to jointly decode $x[1]$ and $x[2]$ based on receiving $y[1]$ and $y[2]$. Find an expression for the ML decoder and draw a constellation diagram illustrating the four decoding regions.

(l) Find an approximate expression for the error probability of the ML decoder for the symbol $x[2]$, when SNR is large. Compare this expression to your answer to (i) and explain.

(j) Write down an expression for the zero-forcing equalizer for $x[1]$. Compute the error probability of it and compare your answer to the ML decoder.

2. (Introduction to the Viterbi algorithm) Consider the ISI channel

$$y[n] = 2x[n] + x[n-1] + w[n]$$

where the $w[n]$ are i.i.d. Gaussian noise random variables with mean zero and variance 1. A sequence $x[1], x[2], \ldots$ is sent. Each $x[n]$ is a 2-PAM data symbol taking on values 1 and -1 with equal probability.

(a) Denote the state by $s[n] = [x[n]\ x[n-1]]$. How many possible states are there? Draw a state diagram illustrating the possible state transitions.

(b) Suppose we receive the sequence $y[1] = 1$, $y[2] = -2$, $y[3] = -2$. Compute the likelihood of each of the eight possible transmit sequences $x[1], x[2], x[3]$ being sent, and based on this make an ML decoding decision. How does the complexity of this brute force ML decoding procedure scale with the length of the transmit sequence n?