EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 2

Mon 2/4/2007 and Wed 2/6/2007

1. A computer generates a number X according to a known distribution $\Pr(X = k)$, $k \in \{1, 2, \ldots, M\}$. A player tries to guess that number by asking questions "Is X in set A?" and the computer answers "yes" or "no". What strategy should the player use to minimize the average number of questions asked?

2. A discrete memoryless source emits i.i.d. random symbols $X_1, X_2, \ldots$. Each random symbol X has the symbols $\{a, b, c\}$ with probabilities $\{0.5, 0.4, 0.1\}$, respectively.

(a) Find the average length $L_{\min}$ of the best variable-length prefix-free code for X.

(b) Find the average length $L_{\min,2}$, normalized to bits/symbol, of the best variable-length prefix-free code for X^2.

(c) Is it true that for any discrete memoryless source, $L_{\min} \geq L_{\min,2}$? Explain.

3. For a discrete memoryless source X with alphabet $\mathcal{X} = \{1, 2, \ldots, M\}$ let $L_{\min,1}, L_{\min,2}$ and $L_{\min,3}$ be the normalized average length in bits per source symbol for a Huffman code over $\mathcal{X}, \mathcal{X}^2$ and $\mathcal{X}^3$, respectively. What does it mean for a code to be over $\mathcal{X}^n$? Show that $L_{\min,3} \leq \frac{2}{3} L_{\min,2} + \frac{1}{3} L_{\min,1}$.