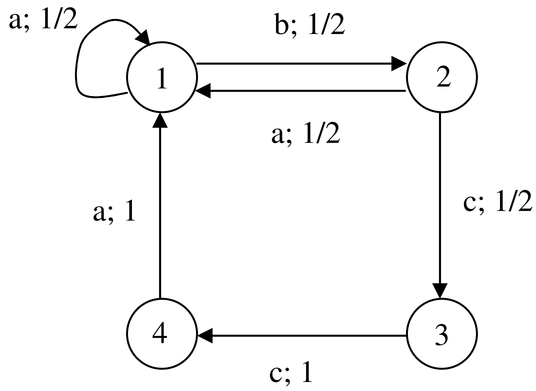
EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 3

Mon 2/11/2007 and Wed 2/13/2007

1. The Markov chain $S_0, S_1, \ldots$ below starts in steady state at time 0 and has 4 states, $\mathcal{S} = \{1, 2, 3, 4\}$. The corresponding Markov source $X_1, X_2, \ldots$ has a source alphabet $\mathcal{X} = \{a, b, c\}$ of size 3.



(a) Find the steady-state probabilities $\{q(s)\}$ of the Markov chain.

(b) Find $H(X_1)$.

(c) Find $H(X_1|S_0)$.

(d) Describe a uniquely decodable encoder for which $\overline{L} = H(X_1|S_0)$. Assume that the initial state is known to the decoder. Explain why the decoder can track the state after time 0.

(e) Suppose you observe the source output without knowing the state. What is the maximum number of source symbols you must observe before knowing the state?

2. (a) Find the entropy rate of the two-state Markov chain with transition matrix

$$\mathbf{P} = \begin{pmatrix} \alpha_0 & 1-\alpha_1 \\ 1-\alpha_0 & \alpha_1 \end{pmatrix}.$$

(b) What values of α_0, α_1 maximize the rate of part (a)?

(c) Find the entropy rate of the two-state Markov chain with transition matrix

$$\mathbf{P} = \begin{pmatrix} p & 1 \\ 1-p & 0 \end{pmatrix}.$$

(d) Find the maximum value of the entropy rate of the Markov chain of part (c). We expect that the maximizing value of p should be greater than $1/2$, since the 0 state permits more information to be generated than the 1 state.

(e) Let $N(n)$ be the number of allowable state sequences of length n for the Markov chain of part (c). Find $N(n)$ and calculate

$$H_0 = \lim_{n\to\infty} \frac{\log N(n)}{n}.$$

Explain why H_0 is an upper bound on the entropy rate of the Markov chain? Compare H_0 with the maximum entropy found in part (d) and comment.