EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 6

Wed 3/5/2008 and Mon 3/10/2008

1. Consider the reception of a QAM data symbol in additive white Gaussian noise

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}.$$

$\mathbf{X} = [X_1\ X_2]^T$ takes on one of four values $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$ with equal probability. The average power constraint is $E[X_1^2] + E[X_2^2] \leq 2E$. Both W_1 and W_2 have variance σ^2.

(a) What values should be chosen for $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$ in order to minimize the probability of error for the optimal detector?

(b) What are the decision regions for the optimal detector, given your choice of $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$?

(c) Calculate the probability of error for the optimal detector. Give an approximate expression for the probability of error when the ratio E/σ^2 is large.

(d) Suppose $P(\mathbf{X} = \mathbf{x}_1) = 0$ and $P(\mathbf{X} = \mathbf{x}_i) = 1/3$ for $i = 2, 3, 4$. Repeat parts (a)-(c).