EE 121: Introduction to Digital Communication Systems

Problem Set for Discussion Section 9

Wed 4/2/2008 and Mon 4/7/2008

1. (*Random block codes*) Consider a random block code represented by a matrix $\mathbf{G}$ with 2^T rows and 2^{RT} columns, where $R \leq 1$. The 2^{RT^2} entries in $\mathbf{G}$ are independent random variables, taking on values either 0 or 1 with equal probability. The minimum distance $d_{\min}$ of this block code is a random variable. We are interested in the distribution of the minimum distance in the limit of large blocklength, i.e. $T \to \infty$.

(a) Using the union bound, show that

$$\Pr(d_{\min} \leq k) \leq 2^{RT} \Pr(w_i \leq k)$$

where the random variable w_i is the weight of the ith codeword in the code, where $i > 1$, the 1st codeword being the all-zeros codeword.

(b) Find an expression for $\Pr(w_i \leq k)$ in terms of k and T.

(c) Use the bound $P(X \leq k) \leq \exp(-2(np-k)^2/n)$ for a binomial random variable $B(n,p)$ to show that if

$$R < 2(\epsilon - 1/2)^2 \log_2 e$$

the probability the minimum distance is less than ϵT goes to zero as $T \to \infty$.

(d) Based on the answer to part (c), for what rates can we say that reliable communication over a binary symmetric channel with crossover probability p is possible?