EE 121: Introduction to Digital Communication Systems

Problem Set 1

Due: Jan. 31 in class

1. Ex. 2.2 in Gallager's notes.

2. Ex. 2.5 in Gallager's book.

3. A *binary symmetric channel* (BSC)is one in which the both the input symbol and the output symbol are binary and the input is flipped with probability ϵ to get the output.

(a) We transmit a bit of information which is 0 with probability p and 1 with $1-p$. It passes through a BSC with cross-over probability ϵ. Suppose we observe a 1 at the output. Find the conditional probability p_1 that the transmitted bit is a 1.

(b) The same bit is transmitted again through the BSC and you observe another 1. Find a formula to update p_1 to get p_2, the conditional probability that the transmitted bit is a 1. You may find following fact useful: if A, B, C are three events, then

$$P(A|B,C) = \frac{P(C|A,B)P(A|B)}{P(C|B)}.$$

(c) Using (b) or otherwise, calculate p_n, the probability that the transmitted bit is a 1 given that you have observed n 1's at the BSC output. Plot p_n as a function of n. What happens as $n \to \infty$?

4. (a) Let $X_1, \ldots, X_n, \ldots$ be an i.i.d. sequence of Bernouilli random variables each with probability p being 1. Let $Y_n := \frac{1}{n}\sum_{t=1}^{n} X_t$. For your favorite value of p, plot the pmf of Y_n for several different values of n and explain as carefully as you can how the plots validate the *law of large numbers*.

(b) Suppose you are designing a symbol-by-symbol variable length source coder for which you need the marginal pmf of each symbol. Explain precisely how you would estimate it from a given training text (i.e. a sequence of symbols from the same source). Which theorem from probability justifies the validity of your procedure and explain clearly how.

5. Ex. 2.6 in Gallager's book.