EE 121: Introduction to Digital Communication Systems

Problem Set 3

Due: February 14 in class

1. Let X_1, X_2, X_3 be three discrete-valued random variables. Verify that (a) $H(X_1, X_2) = H(X_1) + H(X_2|X_1)$; (b) $H(X_1, X_2, X_3) = H(X_1) + H(X_2|X_1) + H(X_3|X_2, X_1)$. Show that if X_1 and X_3 are independent conditional on X_2, then $H(X_3|X_2, X_1) = H(X_3|X_2)$.

2. Consider the symmetric "Mickey mouse" Markov chain, with parameter $\alpha \in [0, 1]$.

(a) Define what it means for a distribution to be a stationary distribution of a Markov chain. What is the stationary distribution of the Mickey mouse chain?

(b) Starting from an initial distribution $P(X_1 = 0) = p$, compute the distribution of X_n. Is it true that for every $\alpha \in [0, 1]$, the distribution of X_n converges to the stationary distribution?

(c) Using (b) or otherwise, compute the entropy rate of the Markov chain:

$$H = \lim_{n \to \infty} \frac{H(X_1, \ldots, X_n)}{n}$$

Does the limit exist for all $\alpha \in [0, 1]$? Does it depend on the initial distribution?

(d) Now suppose the Markov chain is asymmetric with self transition probabilities α_0 and α_1 in state 0 and 1 respectively. What is its stationary distribution? Does the entropy rate exist for all $\alpha_0, \alpha_1 \in [0, 1]$? Does the entropy rate depends on the initial distribution?

3. Problem 2.21 in Gallager.

4. Problem 2.23 in Gallager.

5. Problem 2.35 in Gallager.