EECS 121: Introduction to Digital Communication Systems

Problem Set 4
Due Tuesday, Feb 26, 2008

1. Ex. 2.36 (a) and (b) in Gallager's book.

2. Consider an i.i.d. source $U_1, U_2, \ldots$, where U_i has pdf $f_U(u) = c(1-|u|)$ for $u \in [-1, 1]$ and 0 otherwise.

(a) What is the constant c?

(b) Suppose each U_i is quantized by a uniform quantizer with M values, where M is even. The output of the quantizer is represented by a fixed length code. What is the number of bits per symbol required?

(c) Suppose now instead of representing each output of the quantizer by a fixed length code, the sequence of outputs of the quantizer is coded using a variable length block Huffman code, with the block size chosen to be arbitrarily large. Give an expression for the minimum achievable average number of bits per source symbol required.

(d) Compute the gap in the performance between the scheme in (b) and (c) as $M \to \infty$. Is this gap bounded as $M \to \infty$? Does this gap converge to some limit? Can you compute this limit?

3. Consider communication over an erasure channel where each packet transmission is erased with probability p independent of other transmissions. Reliable communication is assured using NACK (negative acknowledgements) and retransmissions.

(a) First assume NACKs are instantaneous so that retransmission can be done in the next time slot in case a packet gets erased in the current time slot. What is the pmf of the number of transmissions of a packet until it successfully gets through? What is the expected number of transmissions?

(b) Suppose packets arrive at an average rate of λ packets per time slot. Packets wait in a queue and take turn in being transmitted. What is the largest arrival rate for which the queue can be made stable? (No rigorous proof needed, just an intuitive justification of your answer.)

(c) Do you think a higher arrival load can be sustained than in (b) if forward error correction coding over the packets is done instead of using feedback to ensure reliability? Explain.

(d) Now suppose NACKs take 10 time slots to reach the sender after erasures occur. Qualitatively, explain (i) how this affects the delay in transmitting a packet; (ii) how the answer in (b) is changed. (You are free to schedule when to transmit fresh packets in the queue and retransmit previously erased packets.)

4. Consider the use of a repetition code where each binary information symbol is repeated

n times.

(a) Suppose the channel is an erasure channel with erasure probability $p < 1/2$ (independent erasures). Compute the probability of the receiver unable to decode an information symbol.

(b) Suppose the channel is a binary symmetric channel with crossover probability p. Give an expression for the probability of the receiver making a mistake in decoding an information symbol. (Use an appropriate detection rule.)

(c) Find a good bound or estimate of the error probability in (b) and use that to compare it with the error probability in (a). In what sense is an erasure channel "easier" to communicate over than a binary symmetric channel?

5. A $(7, 4)$ Hamming code is used for an erasure channel with erasure probability p (independent erasures). Find an expression of the probability that a transmitted codeword cannot be unambiguously decoded. Does this probability depend on the transmitted codeword? Explain. Give a good simple approximation to this probability when p is small.