EECS 121: Introduction to Digital Communication Systems

Problem Set 8

Due Tues, April 29.

1.(a) Consider the use of 2-PAM modulation on the AWGN channel. Using MATLB, plot the error probability as a function of the signal-to-noise ratio $\mathsf{SNR} := \mathsf{E}/\sigma^2$ on a log-log plot, for the range of SNR from 0 dB to 40 dB.

(b) The Shannon capacity formula $C = \frac{1}{2}\log_2(1 + \mathsf{SNR})$ gives the maximum reliable rate of *coded* communication over the AWGN channel at a given SNR. Inverting the formula also gives the minimum signal-to-noise ratio to achieve a given desired data rate using *any* communication scheme. Using the plot in (a), eyeball how much more SNR does 2-PAM need beyond the minimum SNR at error probability 10^{-4}? How about at error probability 10^{-5}? As the target error probability decreases, what happens to this gap to the minimum SNR? Is this consistent to what you learnt in class about the value of coding?

(c) Repeat parts (a) and (b) for M-PAM, for $M = 4, 8, 16$. You may put all these plots on the same figure.

(d) Repeat (a) for M-PPM (pulse position modulation), $M = 2, 4, 8, 16$. You can either use the exact error probability expression derived in class or the union bound. Choose an appropriate range of SNR's to do the plot. at error probability 10^{-4}, how does the performance of PPM compared to the minimum SNR predicted by Shannon's capacity formula for various M? Does the gap decrease or increase as M becomes large?

2. You are given a bandwidth [-1 MHz, 1 MHz] and an AWGN channel with noise power spectral density $N_0/2$. You want to design a communication link that delivers *at least* R bits/s with an error probability of *no more* than 10^{-4}. You can use M-ary PAM symbols, $M = 2, 4, 8$ or 16, in conjunction with a repetition code of any length k. You can also use a M-ary orthogonal code, $M = 2, 4, 8, 16, 32, 64$. You can also use either of these codes in conjunction with a repetition code, i.e. repeating each of the symbols k times for some k of your choosing. For the A-to-D and D-to-A, you can assume the use of ideal sinc pulses.

Pick a good scheme that uses small energy per bit for

(a) $R = 4$ Mbits/s.

(b) $R = 150$ kilobits/s

Be sure to explain clearly how you arrive at your choice and how other choices are ruled out. Also, calculate the E_b/N_0 for the scheme you have chosen in each of the two cases. (You may find the plots you generated in Q. 1 useful for this question.)

3. (a)Consider a communication system for the AWGN channel using ideal sinc pulses at

rate $\frac{1}{T} = 2W$. Let $\{w(t)\}$ be the additive noise before receive filtering, $\{\tilde{w}(t)\}$ be the noise after the receive filtering and let $\{w[m]\}$ be the additive noise after sampling. Sketch a system diagram with the relevant signals labelled.

(b) In class we sketch out an argument why:

$$\lim_{n\to\infty} \frac{1}{nT} \sum_{m=-n/2}^{m=+n/2} E[w[m]^2] = \lim_{\tau\to\infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} E[\tilde{w}(t)^2]dt \tag{1}$$

Repeat this argument carefully here.

(c) Show that eq. (1) remains valid if instead of the sinc pulses, another transmit pulse $g(t)$ at rate $1/T$ is used as long as the time-shifted versions of the pulse are orthonormal, i.e.

$$\int_{-\infty}^{+\infty} g(t-nT)g(t-mT)dt = \delta_{mn}$$

and $g(-t)$ is used as the receive filter in the A-to-D.

4. Consider communication over a casual LTI channel with impulse response:

$$h(t) = \frac{1}{D}e^{-t/D}, \qquad t \geq 0.$$

(a) If t is in seconds, what unit should D be in? Explain the physical significance of the variable t and the parameter D.

(b) In class, we define the *delay spread* of a channel response to be the range of t for which the impulse response is non-zero. Explain why it doesn't make sense for this channel. Propose an alternative definition which makes more sense here, and use your definition to calculate the delay spread in terms of the parameter D.

(c) Using MATLAB or otherwise, compute and plot the discrete-time channel response assuming ideal sinc pulse D-to-A and A-to-D. You can assume the symbol time is 10^{-6} seconds and do the plot for $D = 10^{-8}, 10^{-7}, D = 10^{-6}$ and 10^{-5} seconds. For which values of D is the discrete-time channel effectively memoryless? with memory? Is this consistent to what you learnt in class?