EECS 121: Introduction to Digital Communication Systems

Problem Set 9

Due 5pm, Monday, May 12, 2008

1. Consider a two-tap baseband ISI channel:

$$y[m] = h_0 x[m] + h_1 x[m-1] + w[m]$$

where $\{w[m]\}$ is white Gaussian noise with variance σ^2 and $x[0] = +\sqrt{E}$ or $-\sqrt{E}$ with equal probability and independent from symbol to symbol. You can assume that transmission starts at time 0. We observe $y[0]$ and $y[1]$.

(a) Suppose first that we do not transmit anything on symbol time 1, i.e. $x[1] = 0$. Give the matched filter receiver to detect $x[0]$. Draw a picture to illustrate its operation. Give an *exact* expression for the average probability of detection error in terms of the Q-function.

(b) Suppose now we send another symbol $x[1] = +\sqrt{E}$ or $-\sqrt{E}$ with equal probability and independent of $x[0]$. We use exactly the same detector as in (a). Give an *exact* expression for the average probability of error in detecting $x[0]$ in terms of the Q-function. (Hint: Be careful. The interference from $x[1]$ is *not* Gaussian.)

(c) What happens to the performance of the matched filter receiver at high SNR?

(d) Argue that the performance of the maximum likelihood sequence detector (MLSD) based on $y[0]$ and $y[1]$ is much better than the matched filter in detecting $x[0]$ at high SNR. (Exact computation of the error probability in detecting $x[0]$ using the MLSD is unnecessary. A good upper bound is all you need.)

2. Consider a two-tap baseband ISI channel:

$$y[m] = 1.5x[m] + 0.6x[m-1] + w[m]$$

with additive white Gaussian noise, and suppose transmission starts at time 0. The symbols are BPSK $+1$ or -1. The transmitted sequence is $x[0] = 1, x[1] = 1, x[2] = 1, x[3] = -1, x[4] = 1, x[5] = -1, x[6] = -1, x[7] = 1$ and the noise sequence is $w[0] = -0.4, w[1] = 0.8, w[2] = -1.6, w[3] = -2.7, w[4] = +1.7, w[5] = -0.3, w[6] = 0.2, w[7] = 1.4$. Draw the trellis diagram and run the Viterbi algorithm to find the maximum likelihood sequence given the observation until time 7. Show in details the steps of your computation.

3. Consider the following encoding scheme for mapping a sequence of bits $\{b[m]\}$ to a transmitted sequence $\{x[m]\}$ over a baseband additive Gaussian channel with noise variance σ^2.

$b[m] = 0$ is mapped to $x[m] = 0$ and $b[m] = 1$ is mapped to either $x[m] = +\sqrt{E}$ or $-\sqrt{E}$, with the signs of the mapping of 1 alternate. This encoding scheme is commonly used to generate signals that do not have a DC. You can assume that the bits $\{b[m]\}$ are i.i.d. equally likely to be 1 or 0.

(a) Just an example to get started: what is the transmitted sequence if the information sequence is 011001010?

(b) What is the data rate (in bits per symbol time) of this scheme? What is the average energy per symbol?

(c) Consider first a symbol-by-symbol detection scheme where the channel output $y[m] = x[m] + w[m]$ at time m is used to detect $b[m]$. Find the maximum-likelihood rule and compute the error probability.

(d) Explain why symbol-by-symbol detection is not optimal. Explain in detail how you would perform maximum likelihood sequence detection. (Your explanation should be detailed enough that it can be easily translated into MATLAB.)

4. Let $\vec{h}$ and $\vec{x}$ be two N-dimensional vectors. Show that the DFT of the cyclic convolution of $\vec{h}$ and $\vec{x}$ is the product of the DFT of $\vec{x}$ and the DFT of $\vec{h}$ times $\sqrt{N}$.

5. Consider the 2-tap ISI channel:

$$y[m] = x[m] + \frac{1}{2}x[m-1] + w[m], \quad m \geq 1.$$

We wish to communicate using the OFDM approach with $N = 2$ tones.

(a) Write down explicitly the system diagram for the entire OFDM scheme at both the transmit and receive ends. Your description should be explicit enough so that it can be readily implemented on MATLAB.

(b) Write down explicitly the OFDM channel coefficients $\tilde{h}_0$ and $\tilde{h}_1$.

(c) Is an ODM system using such small number of tones efficient? Explain.

6. You are designing an outdoor OFDM wireless system. The system parameters are: carrier frequency $f_c = 5$ GHz, bandwidth $W = 5$ MHz, vehicular speed 120 km/hr, delay spread 10^{-5} seconds. A key specification of your system is that the Doppler spread should be less than 1% of the inter-carrier spacing to prevent inter-carrier interference.

(a) Design such a system, stating the values of all pertinent parameters and justifying your choices for them.

(b) Assuming uncoded QPSK symbols are used on the sub-carriers, calculate the data rate (in bits/second) your system can deliver. What is the overhead?

(c) Would this overhead be increased or decreased if you were designing an indoor wireless system instead? Explain.