

- $x[n] = x^*[n-n]$ iff $X[k]$ real

Suppose $X[k]$ is real. Then $X[k] = X^*[k]$ (a nb is real iff it is equal to its conjugate)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi kn}{N}} = X^*[k] = \sum_{n=0}^{N-1} x^*[n] e^{+j \frac{2\pi kn}{N}}$$

$$\begin{aligned} \sum_{n=0}^{N-1} x^*[n] e^{-j \frac{2\pi kn}{N}} &= x^*[0] + \sum_{n=1}^{N-1} x^*[n] e^{+j \frac{2\pi kn}{N}} \\ &= x^*[0] + \sum_{m'=1}^{N-1} x^*[N-m'] \underbrace{e^{+j \frac{2\pi k(N-m')}{N}}}_{\text{4) change of variable } m' = N-m} \\ &= \underbrace{e^{+j \frac{2\pi k 0}{N}}}_{1} \cdot \underbrace{e^{-j \frac{2\pi km'}{N}}}_{1} \end{aligned}$$

$$\text{Thus } X^*[k] = x^*[0] + \sum_{m'=1}^{N-1} x^*[N-m'] e^{-j \frac{2\pi km'}{N}} = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi kn}{N}}$$

$$\begin{cases} x[0] = x^*[0] = x^*[(N-0)]_N \\ x[n] = x^*[N-n] \text{ for } 1 \leq n \leq N-1 \end{cases}$$

The problem says $x_1[n] = x_1[N-n]$. It should be $x_1[n] = x_1[(N-n)]_N$

There is an "abuse of notation", but it is not too confusing!

If you suppose $x[n] = x^*[N-n]$, you can prove similarly that $X[k] = X^*[k]$.

This will complete the proof that $x[n] = x^*[N-n]$ iff $X[k]$ real

• Decompose a finite sequence into real and odd part

let $x[n]$ finite length sequence : $x[n] = 0$ if $n < 0$ or $n > N-1$

$$\text{Define } x_{\text{ep}}[n] = \frac{1}{2} (x[n] + x^*[N-n]) \quad 1 \leq n \leq N-1$$

$$\left\{ \begin{array}{l} x_{\text{ep}}[0] = \text{Re}(x[0]) \end{array} \right.$$

$$\left\{ \begin{array}{l} x_{\text{op}}[n] = \frac{1}{2} (x[n] - x^*[N-n]) \end{array} \right.$$

$$\left\{ \begin{array}{l} x_{\text{op}}[0] = j \text{Im}(x[0]) \end{array} \right.$$

$$\text{Equivalently, we could define: } x_{\text{ep}}[n] = \frac{1}{2} (x[n] + x^*[(N-n)_N])$$

$$\text{for } n=0, x_{\text{ep}}[n] = \frac{1}{2} (x[0] + x^*[(N)_N]) = \frac{1}{2} (x[0] + x^*[0]) = \text{Re}(x[0])$$

$$\begin{aligned} x_{\text{ep}}[N-n]^* &= \left(\frac{1}{2} (x[N-n] + x^*[N-(N-n)]) \right)^* \quad \text{for } 1 \leq n \leq N-1 \\ &= \frac{1}{2} (x[n] + x^*[N-n]) \quad \text{for } 1 \leq n \leq N-1 \end{aligned}$$

$$x_{\text{ep}}[0]^* = \text{Re}(x[0])^* = \text{Re}(x[0])$$

thus, we have $x_{\text{ep}}[n] = x_{\text{op}}^*[(N-n)_N] \Rightarrow x_{\text{ep}}[k]$ is real

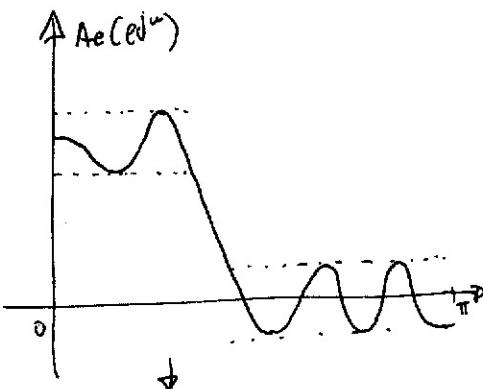
We can show similarly that $x_{\text{op}}[k]$ is imaginary

$$\text{Since } x[n] = \text{Re}(x[n]) + j \text{Im}(x[n]) = x_{\text{ep}}[n] + x_{\text{op}}[n]$$

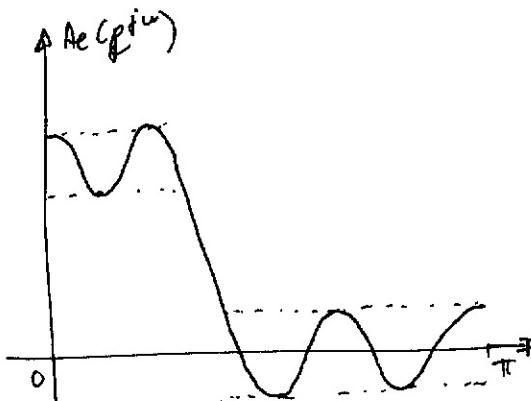
$$\Rightarrow X[k] = \underbrace{x_{\text{ep}}[k]}_{\text{real}} + \underbrace{x_{\text{op}}[k]}_{\text{imaginary}} \quad \left\{ \Rightarrow \begin{cases} x_{\text{ep}}[k] = \text{Re } X[k] \\ x_{\text{op}}[k] = j \text{Im } X[k] \end{cases} \right.$$

$$X[k] = \underbrace{\text{Re } X[k]}_{\text{real}} + \underbrace{j \text{Im } X[k]}_{\text{imaginary}}$$

• Nb of alternations - Parks McClellan algorithm



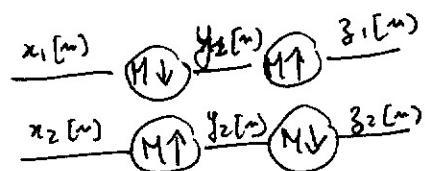
no alternations at 0 \Rightarrow L+2 case



Alternations at 0 and π . It can be L+2 or L+3 (extrapolate case). You cannot find out without further information. In an exam problem it would be specified whether it is L+2 or L+3

• Upsampling-Downsampling

Are these 2 systems equivalent:



$$* Y_1(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M-1} X_1(e^{j\frac{\omega - 2\pi i}{M}})$$

$$z_1(e^{j\omega}) = Y_1(e^{jM\omega}) = \frac{1}{M} \sum_{i=0}^{M-1} X_1(e^{j\frac{M\omega - 2\pi i}{M}}) = \frac{1}{M} \sum_{i=0}^{M-1} X_1(e^{j\omega - \frac{2\pi i}{M}})$$

$$* Y_2(e^{j\omega}) = X_2(e^{jM\omega})$$

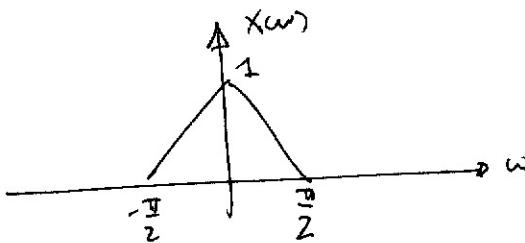
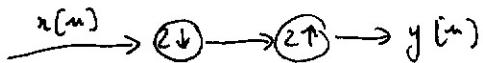
$$z_2(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M-1} Y_2(e^{j\frac{\omega - 2\pi i}{M}}) = \frac{1}{M} \sum_{i=0}^{M-1} X_2(e^{j\frac{M\omega - 2\pi i}{M}})$$

$$= \frac{1}{M} \sum_{i=0}^{M-1} X_2(e^{j(\omega - 2\pi i)}) = \frac{1}{M} \sum_{i=0}^{M-1} X_2(e^{j\omega}) = X_2(e^{j\omega})$$

2π -periodic

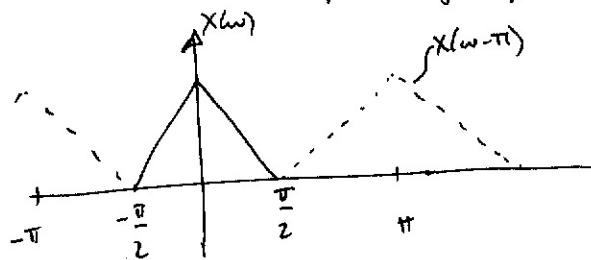
The 2 systems are not equivalent!

One can prove $-\circlearrowleft M \uparrow \circlearrowright N \downarrow \circlearrowright - \stackrel{(M,N) \text{ coprime}}{=} -\circlearrowleft N \downarrow \circlearrowright M \uparrow \circlearrowright -$

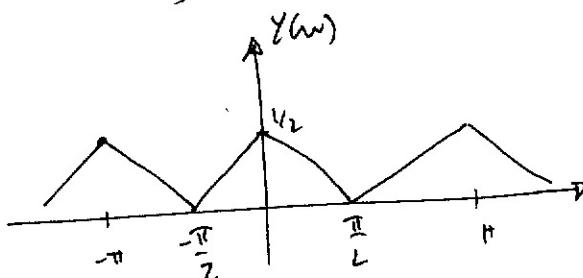


Sketch Y(ω)

From what we saw previously: $Y(\omega) = \frac{1}{2} (X(\omega) + X(\omega - \pi))$



=>



- Under what conditions on $x[n]$ do we have $x[n] = y[n]$?

You need $X(\omega) = X(\omega - \pi)$

$$\Rightarrow \sum_n x[n] e^{-j\omega n} = \sum_n x[n] e^{-j(\omega - \pi)n}$$

$$= \sum_n (-1)^n x[n] e^{-j\omega n}$$

$$\Rightarrow \forall n, x[n] = (-1)^n x[n]$$

This is true if $x[n] = 0$ when n is odd!

- Problem 8.50a from HW #9

$$x[n] = 1 + \cos \frac{\pi n}{4} - \frac{1}{2} \cos \frac{3\pi n}{4} \quad \text{for } 0 \leq n \leq 7.$$

$$X_8[k] = \sum_{n=0}^{8-1} x[n] W_8^{nk} = \sum_{n=0}^{8-1} x[n] e^{-j \frac{2\pi nk}{8}} = x[0] + x[1] e^{-j \frac{\pi}{4}} + x[2] e^{-j \frac{3\pi}{4}} + x[3] e^{-j \frac{5\pi}{4}} + \dots + x[7] e^{-j \frac{7\pi}{4}}$$

Also, we have:

$$x[n] = \frac{1}{8} \sum_{k=0}^7 X_8[k] e^{j \frac{2\pi nk}{8}} = \frac{1}{8} \sum_{k=0}^7 X_8[k] e^{j \frac{\pi k n}{4}}$$

$$= \frac{1}{8} X_8[0] + \frac{1}{8} X_8[1] e^{j \frac{\pi n}{4}} + \frac{1}{8} X_8[2] e^{j \frac{3\pi n}{4}} + \frac{1}{8} X_8[3] e^{j \frac{5\pi n}{4}} + \dots + \dots + \frac{1}{8} X_8[7] e^{j \frac{7\pi n}{4}}$$

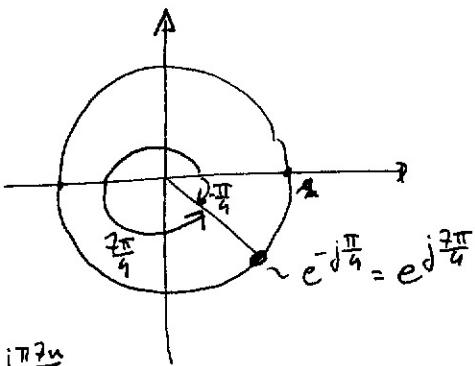
We will identify this expression to $x[n]$ to find the $X_8[k]$ coefficients.

$$x[n] = \frac{1}{2} + \frac{1}{2} \left(e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}} \right) - \frac{1}{2} \cdot \frac{1}{2} \left(e^{j\frac{3\pi n}{4}} + e^{-j\frac{3\pi n}{4}} \right)$$

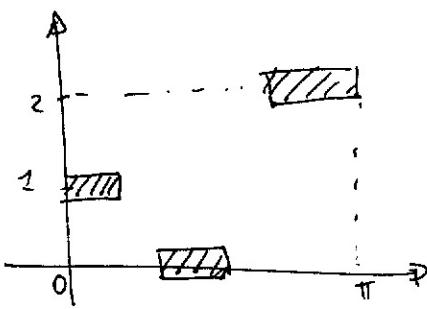
$$e^{-j\frac{\pi n}{4}} = \left(e^{-j\frac{\pi}{4}} \right)^n = \left(e^{j\frac{3\pi}{4}} \right)^n = e^{j\frac{\pi n}{4}}$$

Similarly, ~~$x[n]$~~ = $e^{-j\frac{3\pi}{4}} = e^{j\frac{5\pi}{4}}$

$$\Rightarrow x[n] = \underbrace{\frac{1}{2}}_{\frac{1}{8}X_8[0]} + \underbrace{\frac{1}{2}e^{j\frac{\pi n}{4}}}_{\frac{1}{8}X_8[1]} - \underbrace{\frac{1}{4}e^{j\frac{3\pi n}{4}}}_{\frac{1}{8}X_8[3]} - \underbrace{\frac{1}{4}e^{j\frac{5\pi n}{4}}}_{\frac{1}{8}X_8[5]} + \underbrace{\frac{1}{2}e^{j\frac{7\pi n}{4}}}_{\frac{1}{8}X_8[7]}$$



Kaiser Window



these are your specs. You design your filter using a Kaiser window with parameter δ .

This will produce ripples of maximum amplitude 3δ . Thus, you have to make sure that 3δ is less than what your specs allow.