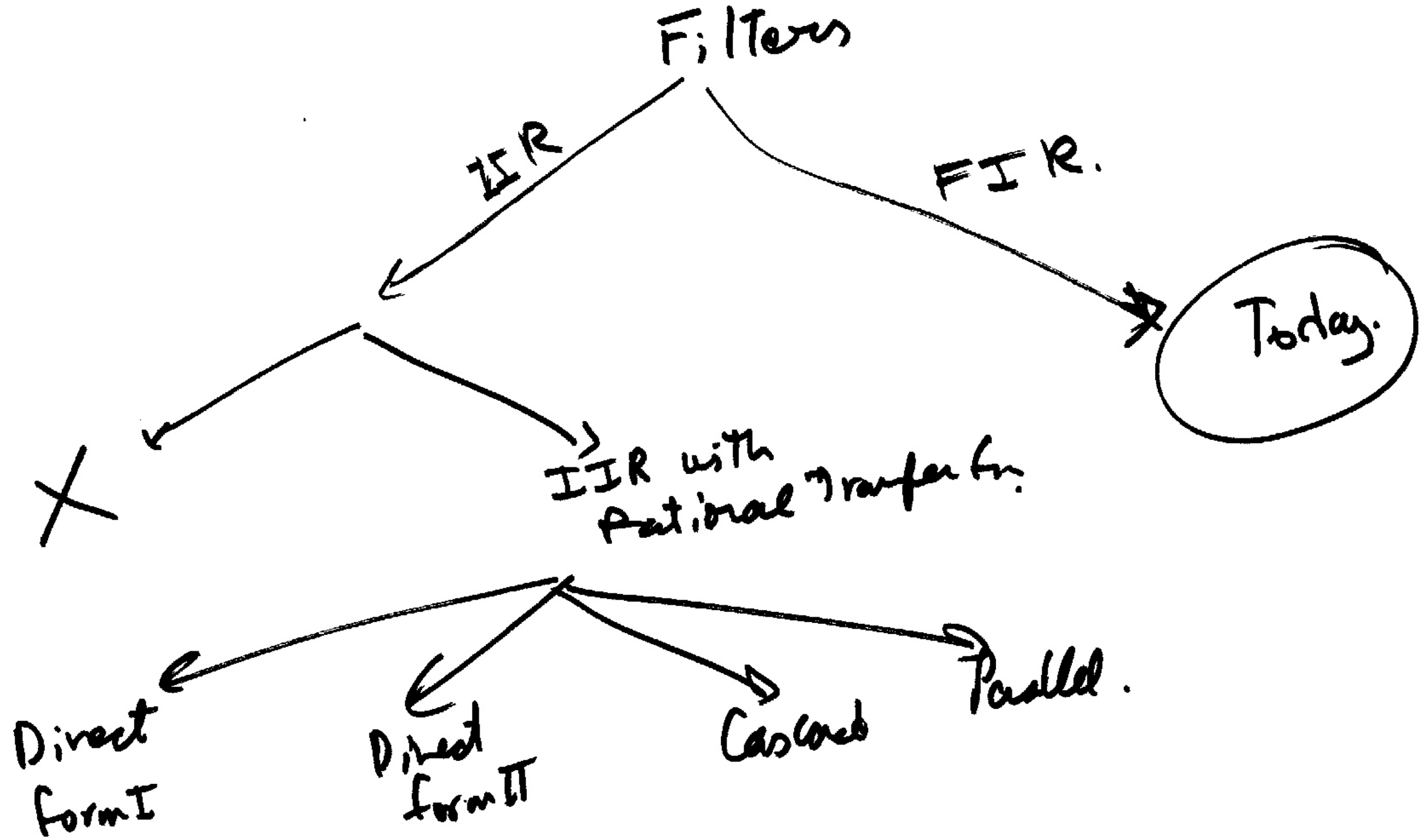
OCT 12, 05

Realization of FIR filters



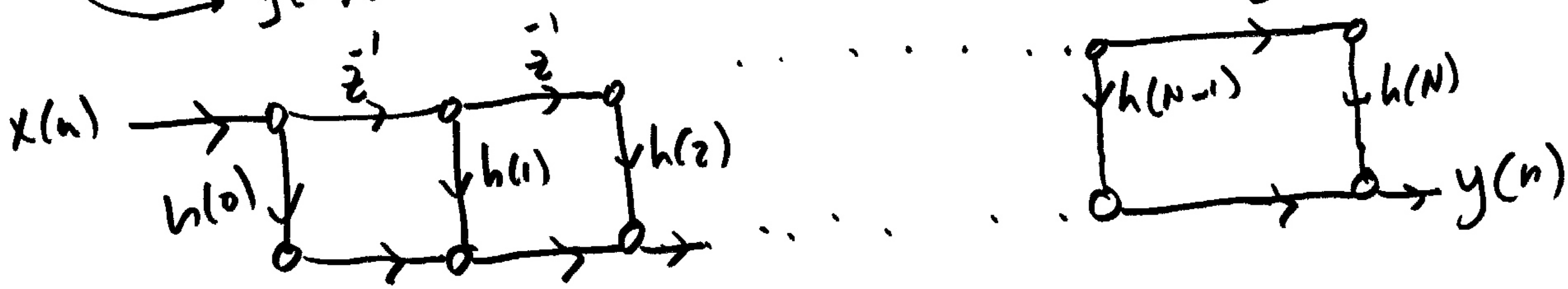
$$x \longrightarrow \boxed{h(n)} \longrightarrow y$$

h(n) has N+1 taps.

$$y(n) = \sum_{k=0}^{N} h(k)\, x(n-k)$$

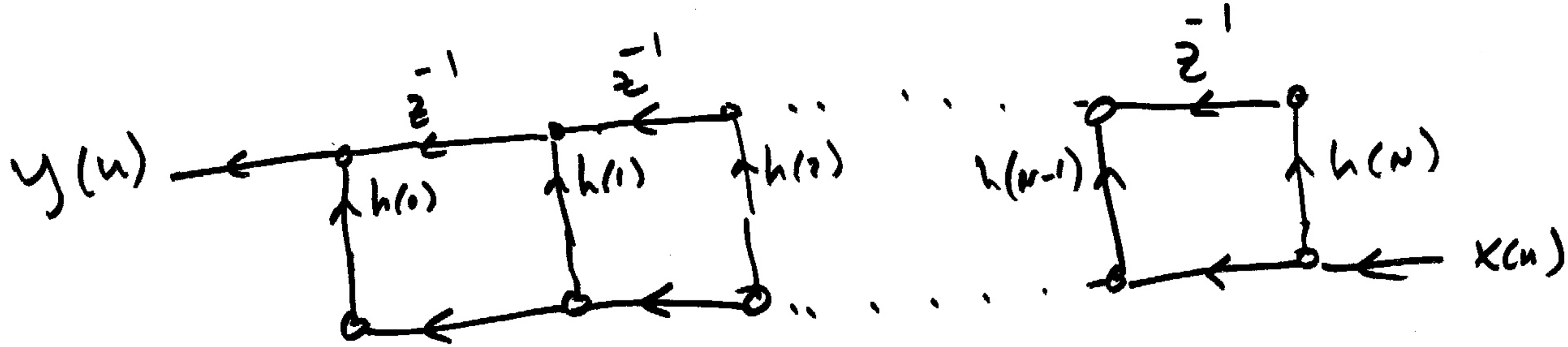
$$H(z) = \sum_{k=0}^{N} h(k)\, z^{-k}$$

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + \cdots + h(N)\,x(n-N)$$



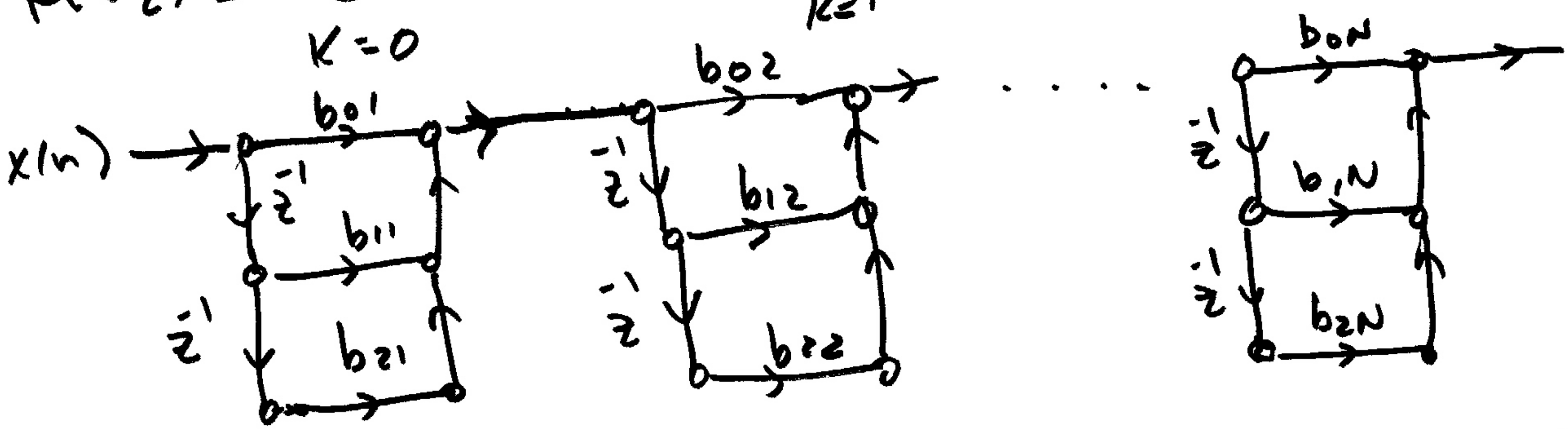
Direct form I, II.

Transposed version of)



Cascade.

$$H(z) = \sum_{k=0}^{N} h(k) z^{-k} = \prod_{k=1}^{N} \left(b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2} \right)$$



show Fig 6.39, 6.40, 6.41 O&S.

Binary Representation of #s.

Many Formats for representation of binary #s.

- one's complement
- sign & mag.
- two's complement. ← most comonly used.

- Real # in two's complement with ∞ precision.

$$X = X_m \left(-b_0 + \sum_{i=1}^{\infty} b_i \, 2^{-i} \right)$$

X_m = arbitrary scale factor $\quad |X| < X_m$

b_i = either zero or 1

b_0 = sign bit $\longrightarrow \begin{cases} b_0 = 0 & 0 \le x \le X_m \\ b_0 = 1 & -X_m \le x < 0 \end{cases}$

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with finite # of bits $(B+1)$ we get representation:

$$\hat{X}_B = Q_B[x] = X_m\left(-b_0 + \sum_{i=1}^{B} b_i 2^{-i}\right)$$

$\hat{x}$: is quantized version of x.

smallest difference between any 2 #s in quantized domain $\Delta = X_m 2^{-B}$

5.87923 , only use 2 bits 5.87924

- quantized #s are in the rang.

$$-X_m \leq \hat{x} < X_m$$

$$\hat{X}_B = b_0 \,\square\, b_1\, b_2\, b_3 \ldots\ldots b_B$$

binary point.

$$5.3924 \longrightarrow \begin{cases} 5.4 & \text{rounding} \\ 5.3 & \text{Truncation} \end{cases}$$

Start a real number x
to get $\hat{x}_B$, one can either
rounding or Truncation:

Show Fig 6.37 (a), 6.37(b) in O&S.

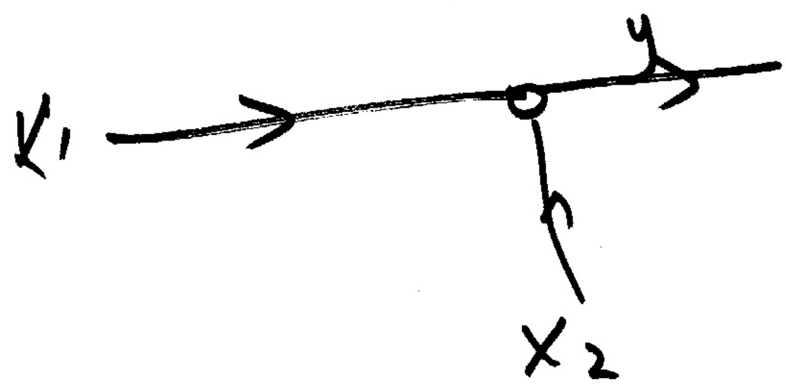
Quantization error

$$e = Q_B[x] - x$$
$$= \hat{x}_B - x$$

2's complement:

rounding error $-\Delta/2 < e < +\Delta/2$

Truncation error $0 \leq e \leq \Delta$



$|x_1 + x_2| < 10{,}000$

$y = x_1 + x_2$

$|x_1| < 5000$

$|x_2| < 5000$

overflow.

Saturate

natural overflow

6.38

O&S

- Interesting property of Two's Complement + natural overflow:

  Add few #s, if The final sum doesn't overflow, Then result is Correct even though The intermediate results overflow.

- Tradeoffs between overflow & rounding error.

  $X_m \uparrow \longrightarrow$ overflow is less likely but $\Delta \uparrow$, $e \uparrow$

$X_m \downarrow$ $\longrightarrow$ overflow is more likely.

but $\Delta\downarrow$ $e\downarrow$

But B can come to rescue.

- Keep X_m large to minimize chances of overflow

- But keep B large to keep Δ, e small.

Multiplication also introduces
- overflow
- vanishing curve