Oct 19, 2005

What are the conditions for acheiving Linear Phase?

Generalyzed Linear Phase:

$$H(\omega) = \underbrace{H_m(\omega)}_{\substack{\text{Real} \\ \text{positive or negative.}}} e^{j(\beta - \alpha\omega)}$$

α = group delay.

$\measuredangle H(\omega) = \beta - \alpha\omega$

$- \frac{d}{d\omega} \measuredangle H(\omega) = \alpha \leftarrow$

$$H(\omega) = H_m(\omega) \cos(\beta - \alpha\omega) + j\, H_m(\omega) \sin(\beta - \alpha\omega)$$

$$\tan\left(\measuredangle H(\omega)\right) = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)} = \tan(\beta - \alpha\omega)$$

$\leftarrow$ eqn 1.

How can we deriven $\measuredangle H(\omega)$ in Terms of $h(n)$?

$$H(\omega) = \sum_n h(n)\, e^{-j\omega n}$$

$$H(\omega) = \sum_n h(n) \cos(\omega n) - j \sum_n h(n) \sin(\omega n)$$

$$\tan\left[\measuredangle H(\omega)\right] = \frac{-\sum_n h(n) \sin(\omega n)}{\sum_n h(n) \cos(\omega n)} \quad \leftarrow \text{Eqn 2.}$$

Combine Eqn 1 & 2 :

$$\boxed{\frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)} = \frac{-\sum_n h(n) \sin(\omega n)}{\sum_n h(n) \cos(\omega n)}}$$

→ necessary condition for h(n) To be G.L.P.

$$\sin(\beta - \alpha\omega) \sum_n h(n) \cos\omega n + \cos(\beta - \alpha\omega) \sum_n h(n) \sin\omega n = 0$$

$$\Rightarrow \boxed{\sum_{n=0}^{N-1} h(n) \sin\left[\omega(n-\alpha) + \beta\right] = 0}$$

→ necessary condition for $h(n)$ to be G.L.P.

Consider 2 cases:

Case ① : $\beta = 0$ or π

$$\Rightarrow \sum_n h(n) \sin(\omega(n-\alpha)) = 0$$

Can show: If $\boxed{h(n) = h(2\alpha - n)}$ then This is True.

$\boxed{N = 2\alpha + 1}$

Case ② $\beta = \pi/2$ or $3\pi/2$

$$\boxed{\sum_{n=0}^{N-1} h(n) \cos[(n-\alpha)\omega] = 0}$$

Can show: If $\boxed{h(2\alpha - n) = -h(n)}$ $\boxed{N = 2\alpha + 1}$

Then satisfy

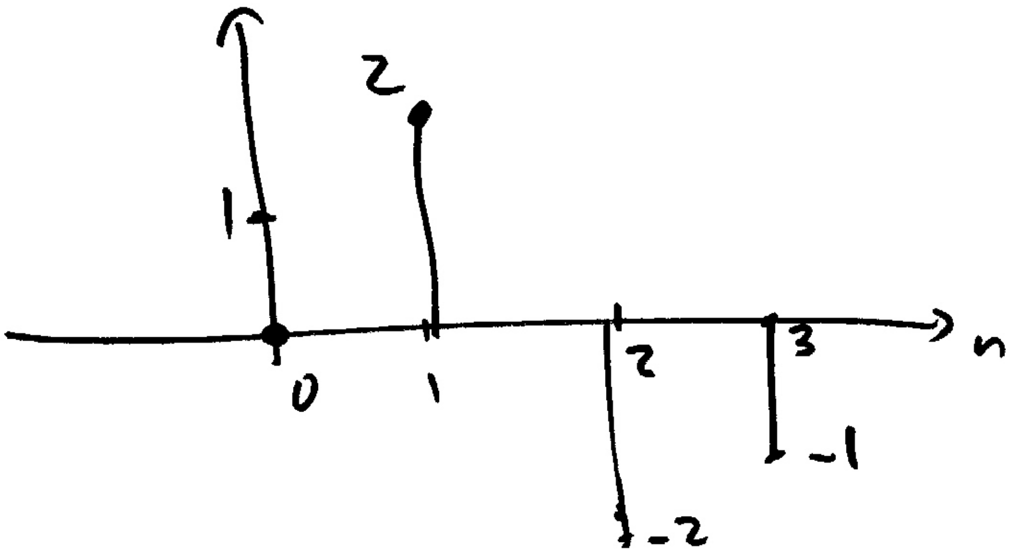
$h(N-1-n) = -h(n)$

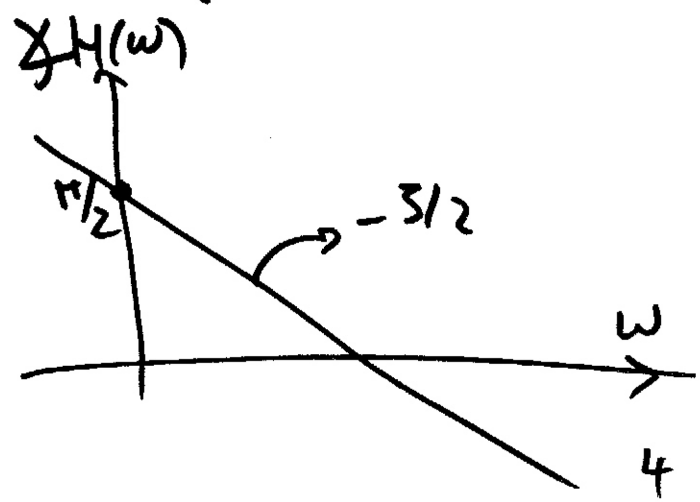
EX $N = 4 = \#$ of Taps $\qquad N = 2\alpha + 1 = \#$ of Taps.

$-h(n) = h(3-n) \qquad \beta = \pi/2 \Rightarrow \alpha = 3/2$

$$H(\omega) = H_m(\omega)\, e^{+j\left(\frac{\pi}{2} - \frac{3}{2}\omega\right)}$$

$h(3-n) = -h(n)$





Ex $h(n)$ 5 points

$N = 2\alpha + 1$

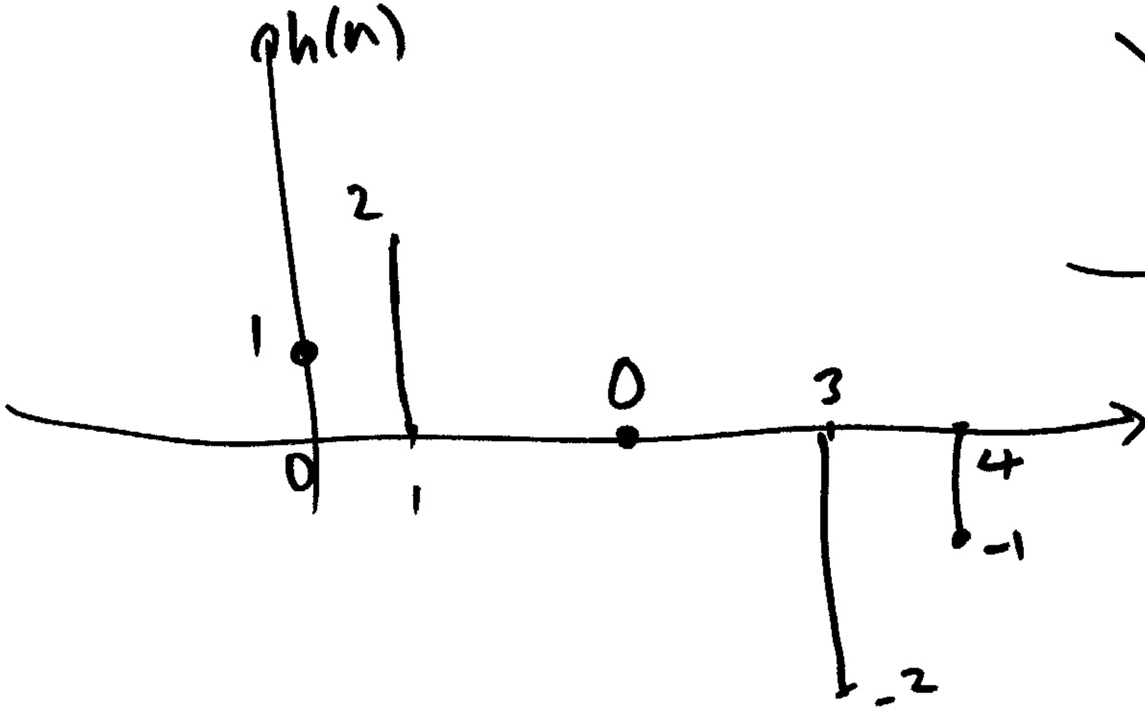
$5 = 2\alpha + 1$

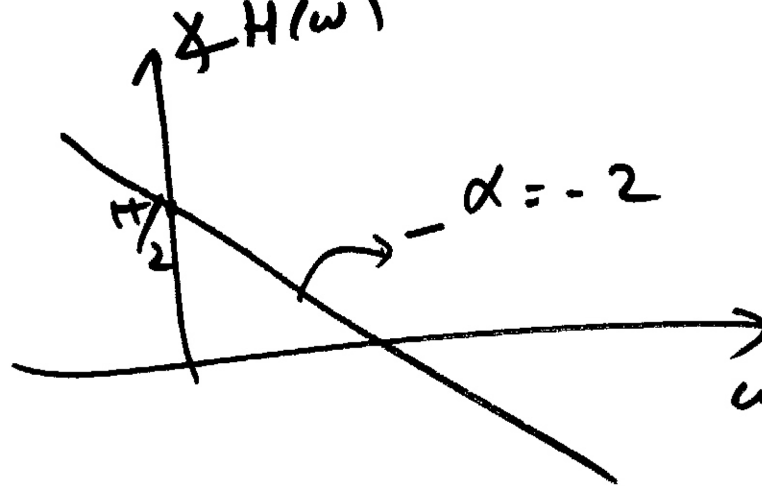
$\Longrightarrow \alpha = 2$

$h(2\alpha - n) = -h(n)$

$h(4 - n) = -h(n)$

$\beta = \pi/2$





$H(\omega) = H_m(\omega)\, e^{+j(\beta - \alpha\omega)}$

Can show 2 things.

Case 1 $\beta = 0$

$$h(n) = \begin{cases} h(N-1-n) & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$

Then

$$\boxed{H(\omega) = H_m(\omega)\, e^{-j\left(\frac{N-1}{2}\right)\omega}}$$

$\boxed{\beta = 0}$

$\boxed{\alpha = \frac{N-1}{2}}$

Case 2 $\beta = \pi/2$

$$h(n) = \begin{cases} -h(N-1-n) & 0 \le n < N \\ 0 & \end{cases}$$

$$H(\omega) = H_m(\omega)\, e^{j(\beta - \alpha\omega)}$$

with $\beta = \pi/2$, $\alpha = \frac{N-1}{2}$

$$\boxed{H(\omega) = H_m(\omega)\, e^{j\pi/2}\, e^{-j\omega\left(\frac{N-1}{2}\right)}}$$

Case ① $\beta = 0$ symetry

odd # of Taps

even

Case ② $\beta = \pi/2$ anti-symetry.

odd

even.

$\Longrightarrow$ 4 Types of GLP. FIR

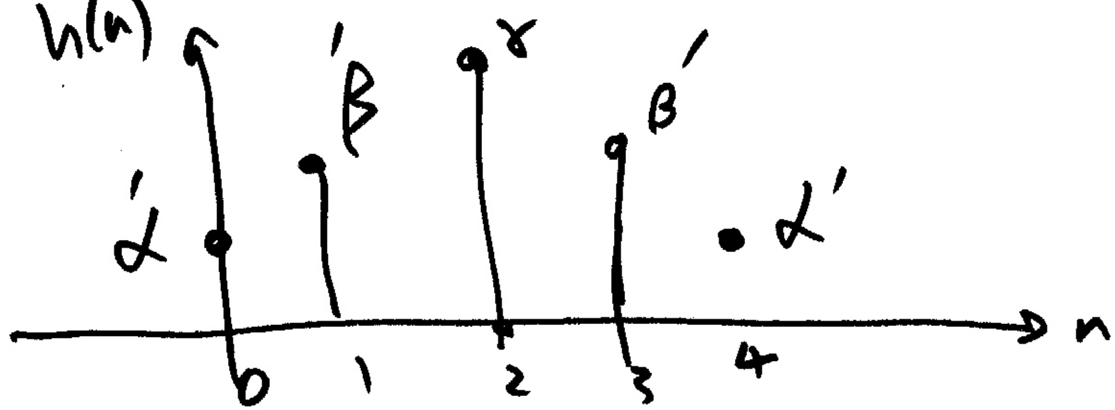
odd $\leftarrow$ Type I
even $\leftarrow$ Type II
$\Big\}\longrightarrow$ symetric

odd $\leftarrow$ Type III
even $\leftarrow$ Type IV
$\Big\}\longrightarrow$ Anti-symetric.

Type I : symmetry # of taps odd.

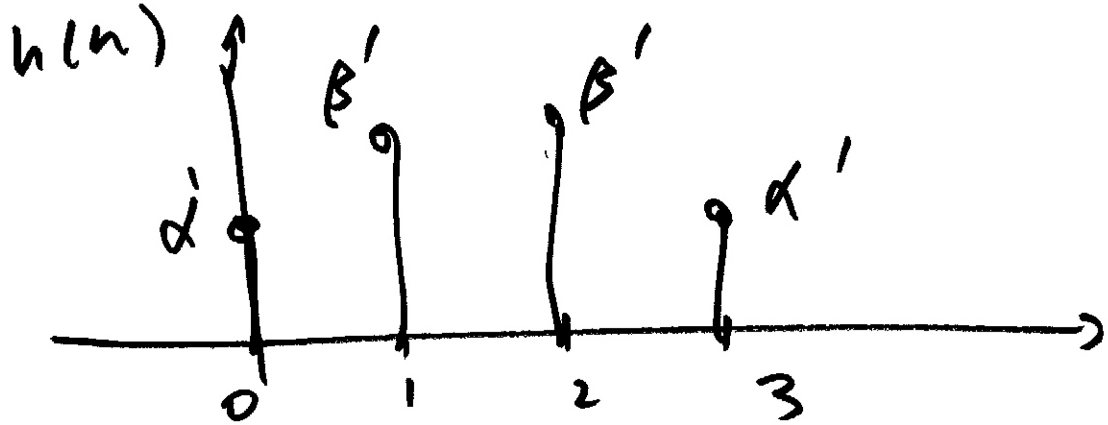
$h(n) = h(N-1-n)$

$N = 5.$



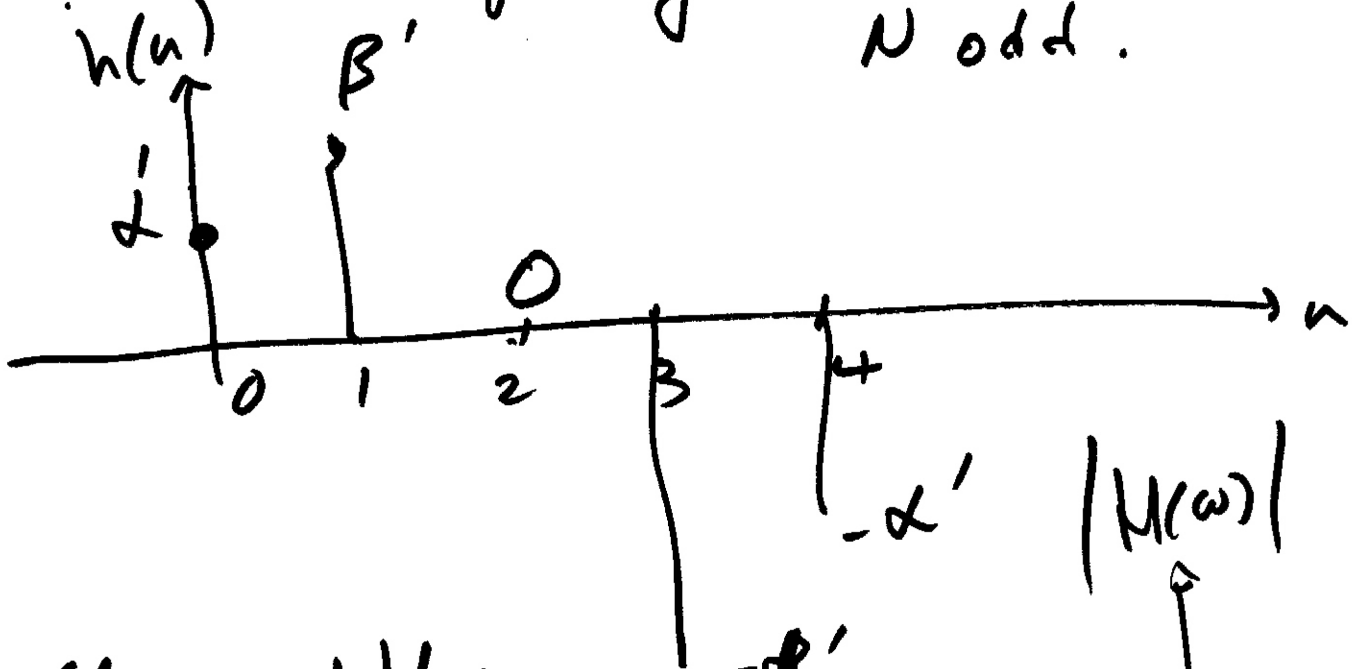
<u>Type II</u> symmetry # of taps is even.

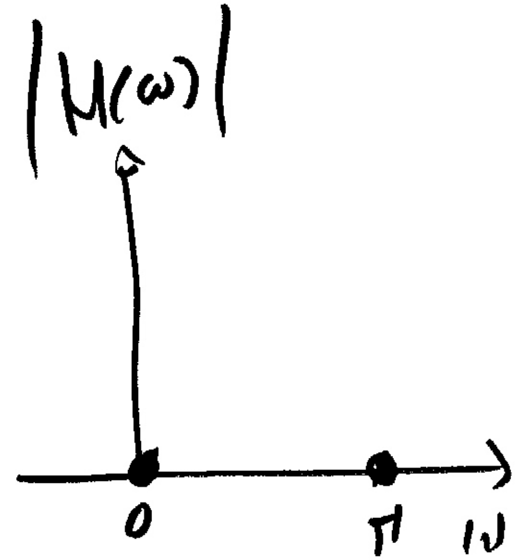
$N = 4$



Type III. Anti-Symmetry $h(n) = -h(N-1-n)$

N odd. $N=5$





Can show $H(0) = H(\pi) = 0$

Cannot be low pass

Cannot be high pass

Type IV   anti symmetry ~~8~~ $\rightarrow \beta = \pi/2$ N = even.

$N = 4$

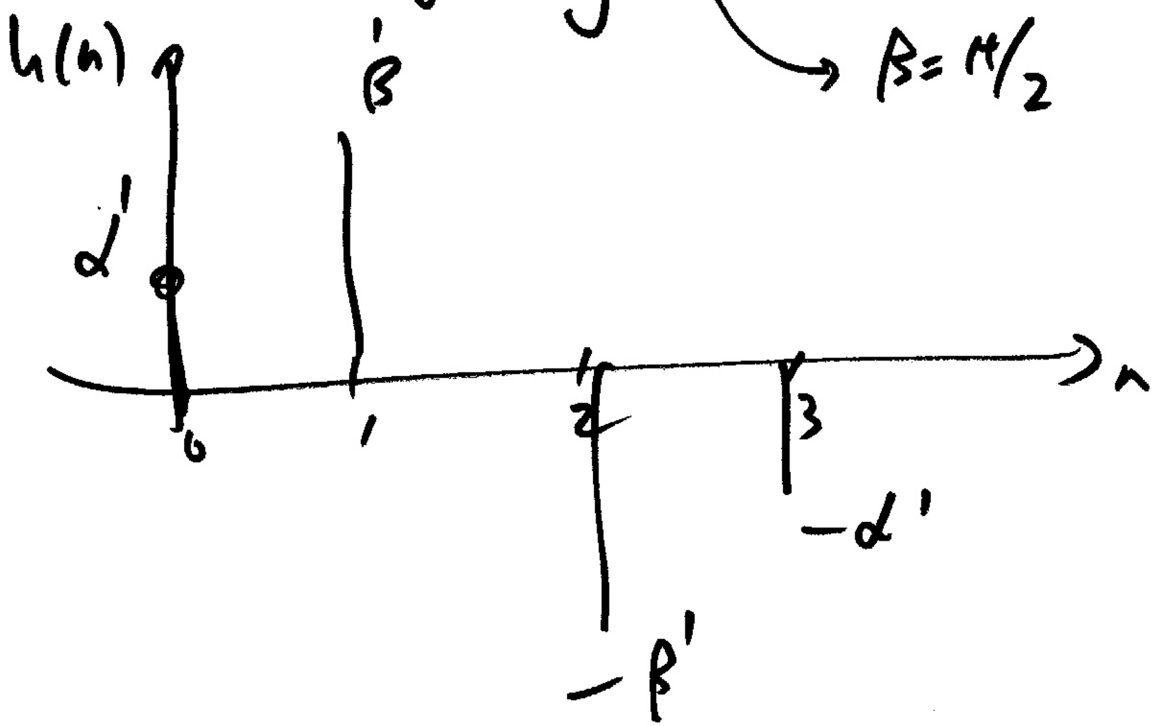
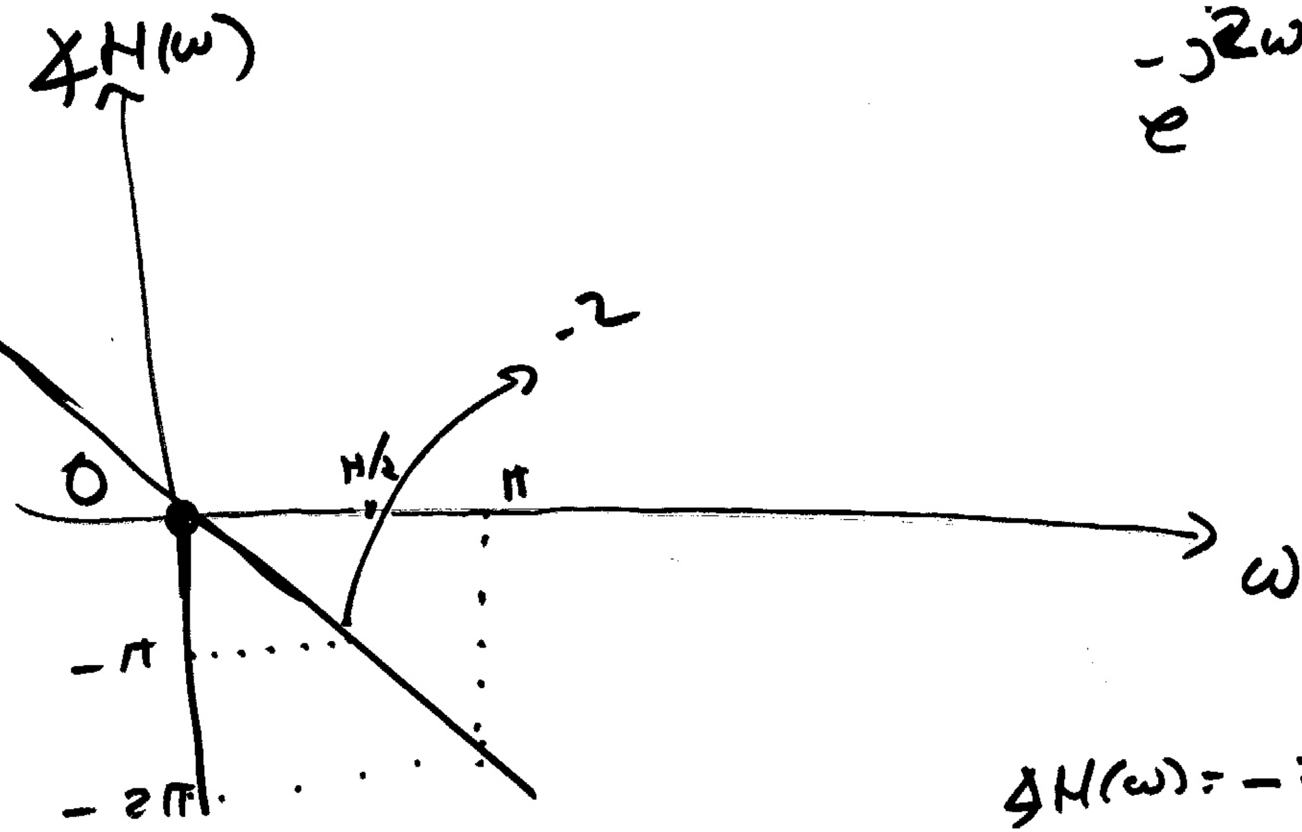


Fig 5.36, 5.37, 5.38, 5.39, 5.40

O&S

$e^{-j2\omega}$

$H_m(\omega)$



$\measuredangle H(\omega) = -2\omega$

$\omega = \pi$

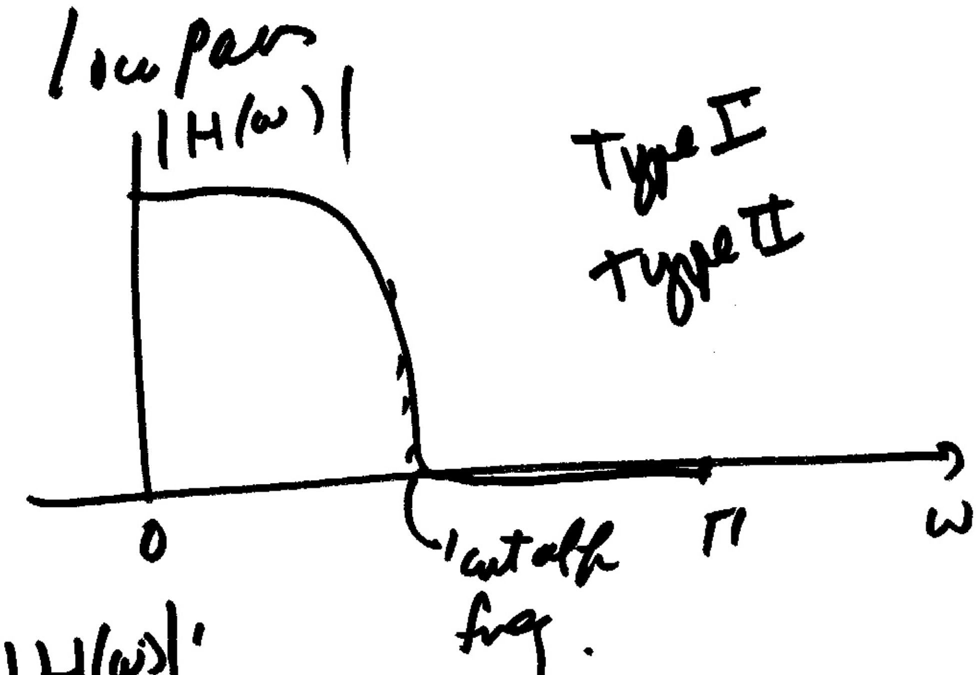
$\omega = \pi/2$

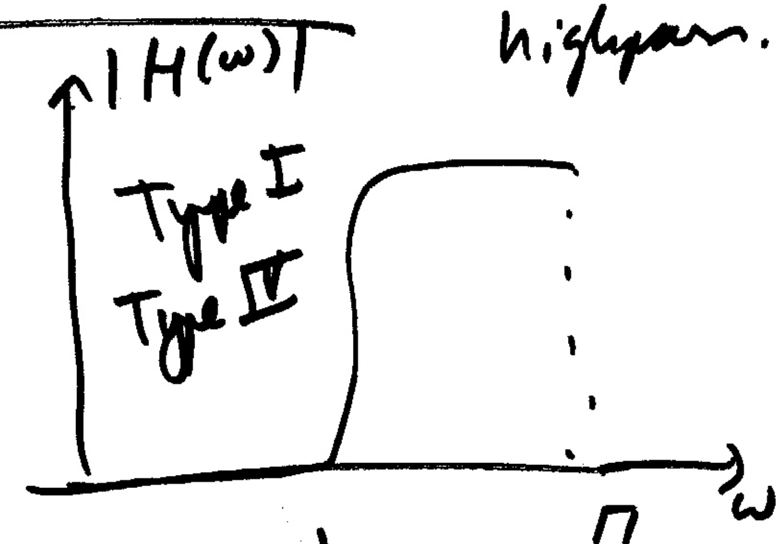
$\measuredangle H(\omega) = -\pi$

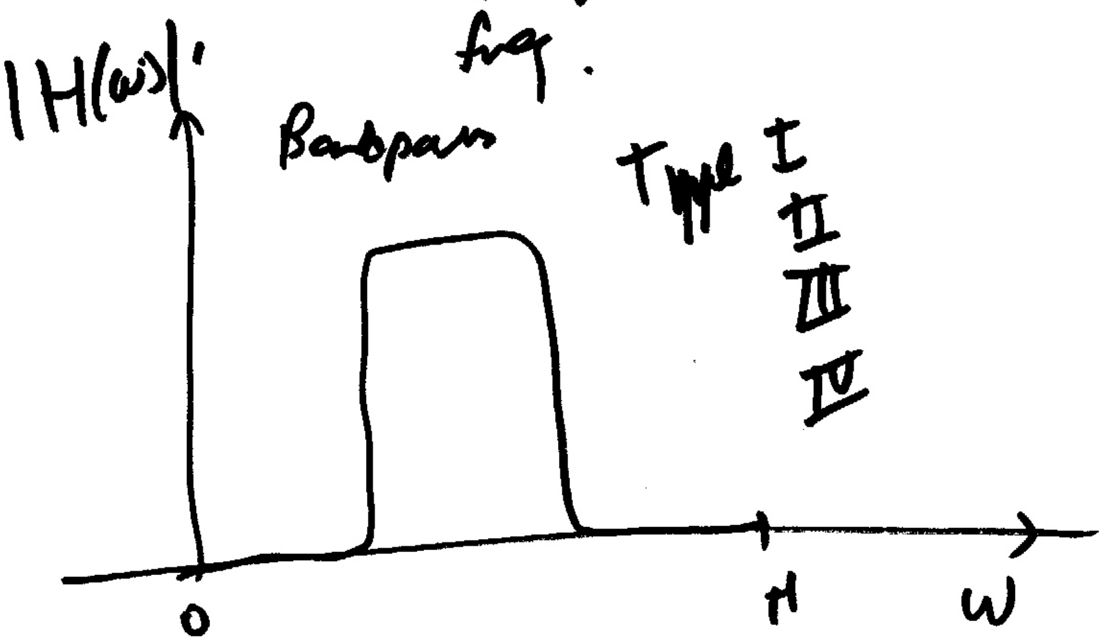
5.38

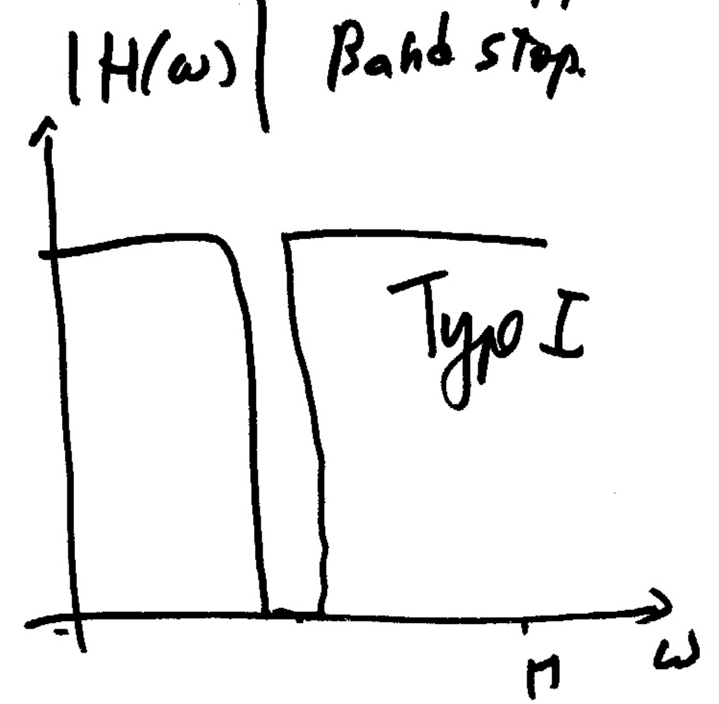
	Symmetry or antisymmetry	N even or odd	α	β	$H_r(\omega)$	Constraints $H_r(\omega)\, e^{j\beta}$
Type I	$h(n) = h(N-1-n)$	odd	$\frac{N-1}{2}$	0	$\sum_{0}^{(N-1)/2} a(n) \cos \omega n$	Real
Type II	$h(n) = h(N-1-n)$	even	$\frac{N-1}{2}$	0	$\sum_{n=1}^{N/2} b(n) \cos \omega(n-\frac{1}{2})$	Real $H(\pi)=0$
Type III	$h(n) = -h(N-1-n)$	odd	$\frac{N-1}{2}$	$\pi/2$	$\sum_{n=1}^{\frac{N-1}{2}} c(n) \sin \omega n$	purely imaginary. $H(0)=0$ $H(\pi)=0$
Type IV	$h(n) = -h(N-1-n)$	even	$\frac{N-1}{2}$	$\pi/2$	$\sum_{n=1}^{N/2} d(n) \sin \omega(n-\frac{1}{2})$	purely imaginary. $H(0)=0$

4 Classes of Filters









	Low pass	high pass	Band pass	Band stop.
I	✓	✓	✓	✓
II	✓	✗	✓	✗
III	✗	✗	✓	✗
IV	✗	✓	✓	✗