Oct 26, 05

Filter Design

LTI

FIR: Determining coeffs of $h(n)$

IIR

Rational Transfer fn $H(z) = \frac{P(z)}{Q(z)}$

Determining coeffs $Q(z)$ and $P(z)$

implement using D.E.

not a Rational Transfer fn.

1

3 steps of building Filters

1. Specification $\longrightarrow$ Application dependent
2. Design $\longrightarrow$ Determing coeff.
3. Realization $\longrightarrow$ Directform 1, 2, Cascade, Parallel.
4. Implementation
   - C program on PC.
   - matlab prog.
   - Program DSP chip.
   - Xilix FPGA.
   - Programmable ASIC
   - ASIC

FIR

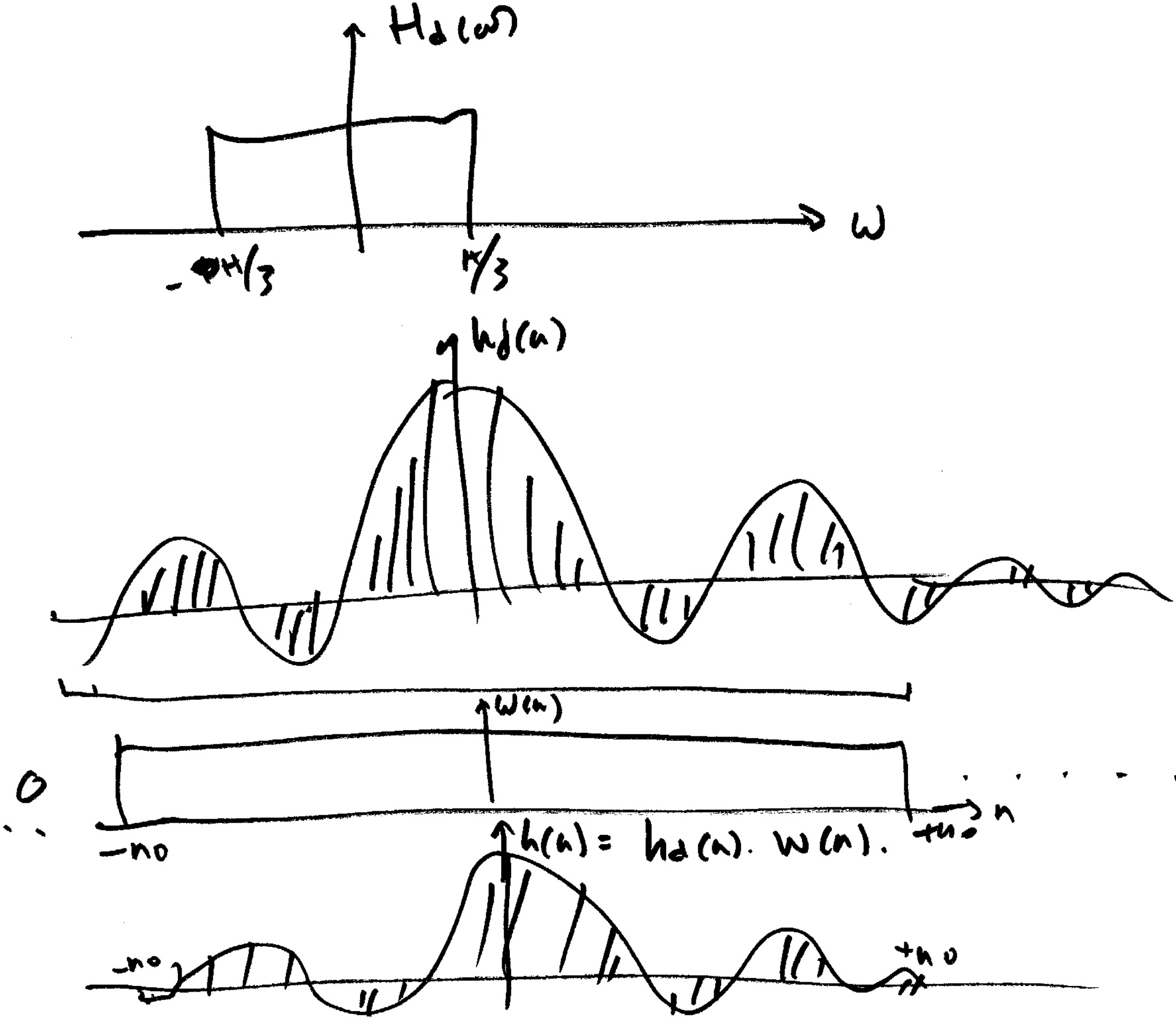
Windows.　　　　Optimum.

FIR Filter Design Using Windows.

1. Start with desired freq. Response $H_d(\omega)$

2. Compute IDTFT $\{H_d(\omega)\} = h_d(n) =$
   = Desired impulse response.

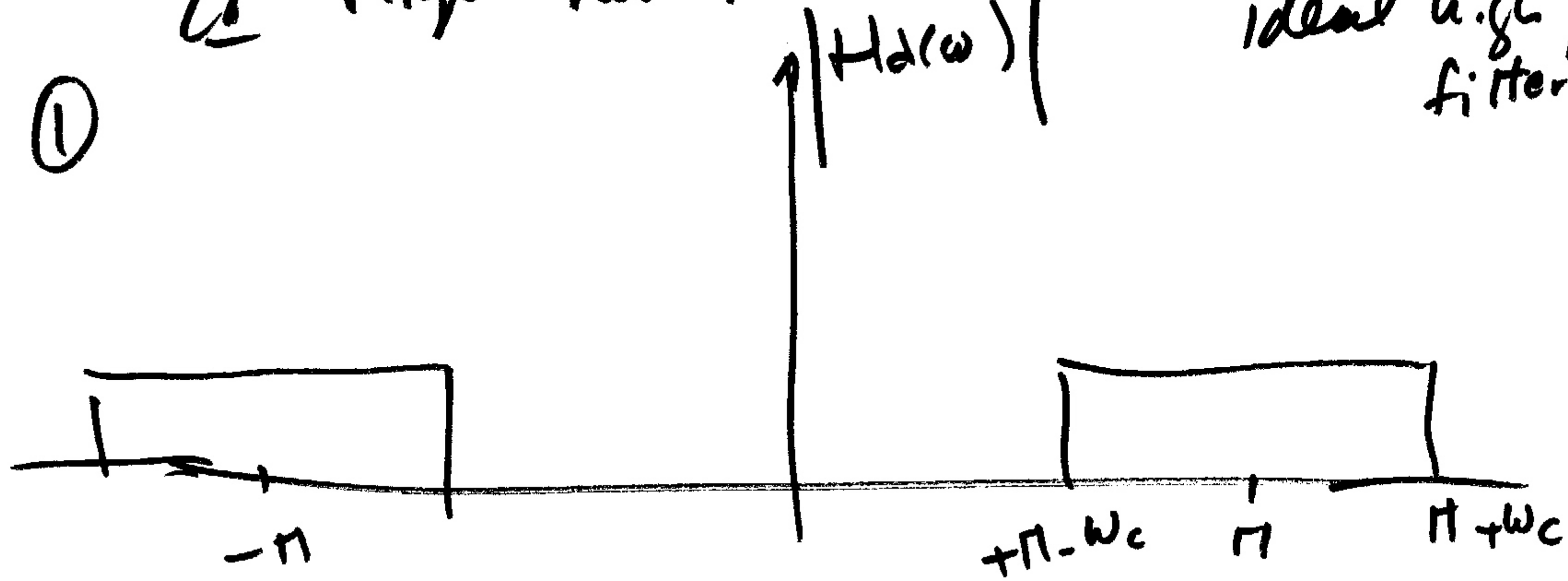
3. $h(n) = h_d(n)\, w(n)$ — finite length window function.


$H_d(\omega)$
ω
$-\pi/3$
$\pi/3$
$h_d(n)$
$w(n)$
n
$-n_0$
$+n_0$
$h(n) = h_d(n) \cdot w(n)$.
$-n_0$
$+n_0$

Ex High Pass Filter

ideal high pass filter.

①



② Compute I.D.T.F.T. $\{H_d(\omega)\}$

Assume linear Phase filter. (desired/ideal, final FIR)

Assume Type I.

$$\left.\begin{aligned} &\beta = 0 \\ &\alpha = \frac{N-1}{2} \\ &\#\text{ of taps is odd} \end{aligned}\right\} \rightarrow H_d(\omega) = \underbrace{H_r(\omega)}_{\text{Real}} e^{-j\alpha\omega}$$

$$|H_d(\omega)| = \begin{cases} 1 & \pi - \omega_c < \omega < \pi + \omega_c \\ 0 & \text{otherwise.} \end{cases}$$

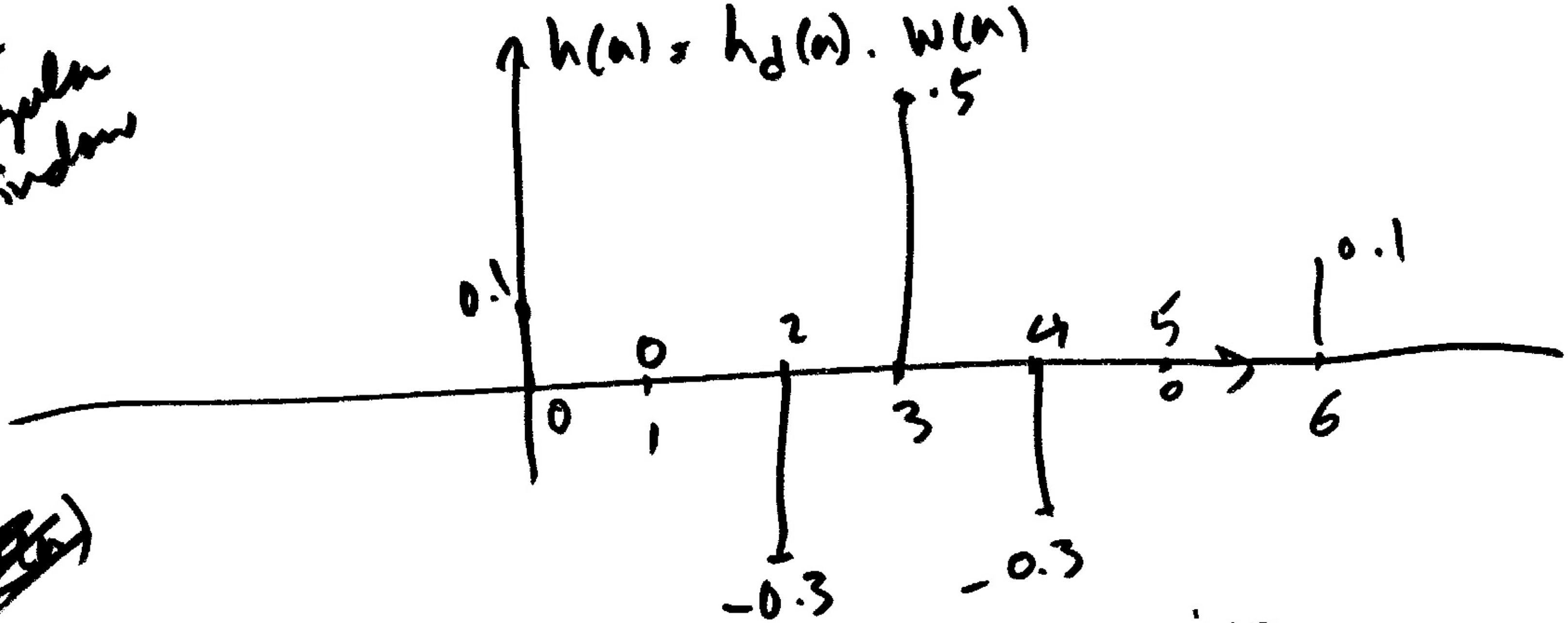
$$H_d(\omega) = \begin{cases} e^{-j\alpha\omega} & \pi - \omega_c \leq \omega < \pi \\ 0 & \text{otherwise.} \end{cases}$$

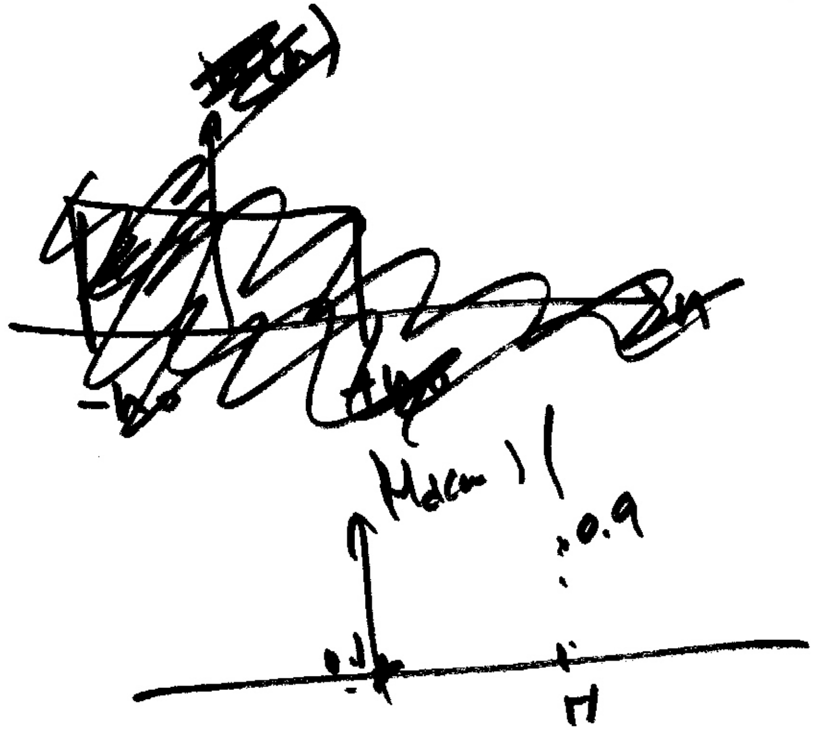
$$\text{I.DT FT}\{H_d(\omega)\} = \frac{1}{2\pi}\int_{\pi-\omega_c}^{\pi+\omega_c} e^{-j\alpha\omega} e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{(-1)^{n-\alpha}}{\pi(n-\alpha)} \sin\left(\omega_c (n-\alpha)\right)$$

(3) Multiply $h_d(n)$ by a finite length window to get FIR filter

$N = 7$
Rectangular Window





$$H(w) = \sum h(n) e^{-jwn}$$

$$\left[H(w)\right]_{w=0} = \sum h(n) = 0.1$$

$\Rightarrow$ D.C. value of $H(w)$ in $\sum h(n)$

$$\left[H(w)\right]_{w=\pi} = \sum h(n) e^{-j\pi n}$$

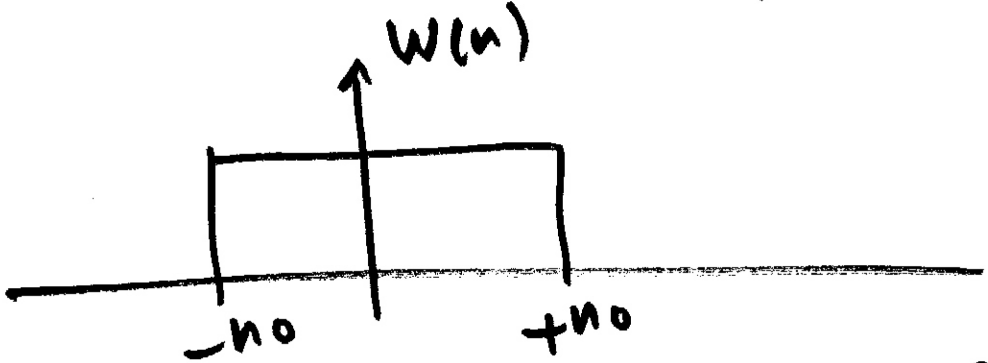
$$\sum_{n=0}^{N-1} h(n)(-1)^n$$

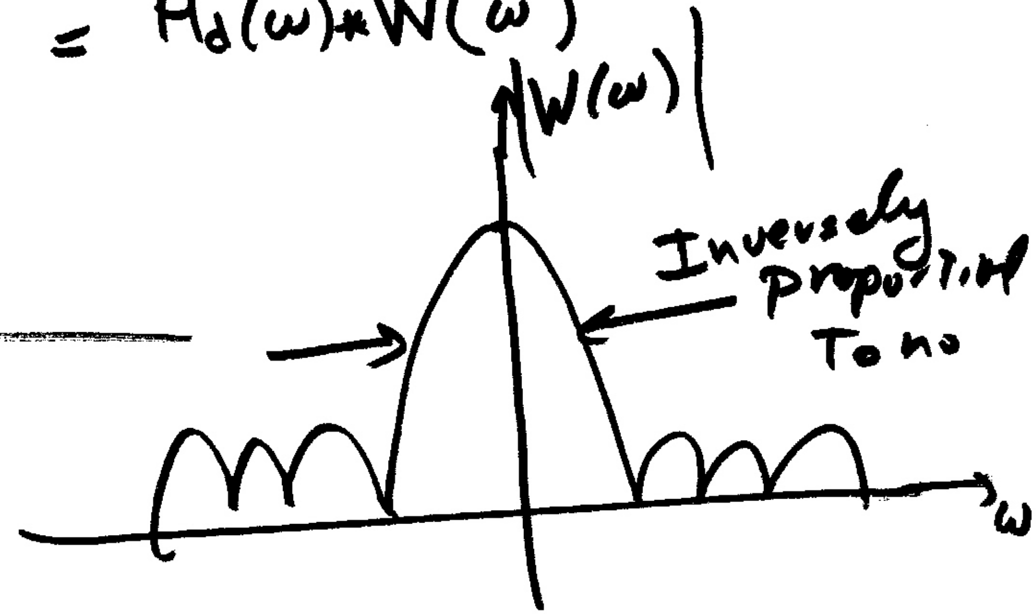
$$= 0.1 - 0.3 - 0.5 - 0.3 + 0.1$$

$$= -0.9$$

$$h(n) = h_d(n)\, w(n) \leftarrow \text{window}$$

$$H(w) = H_d(\omega) * W(\omega)$$

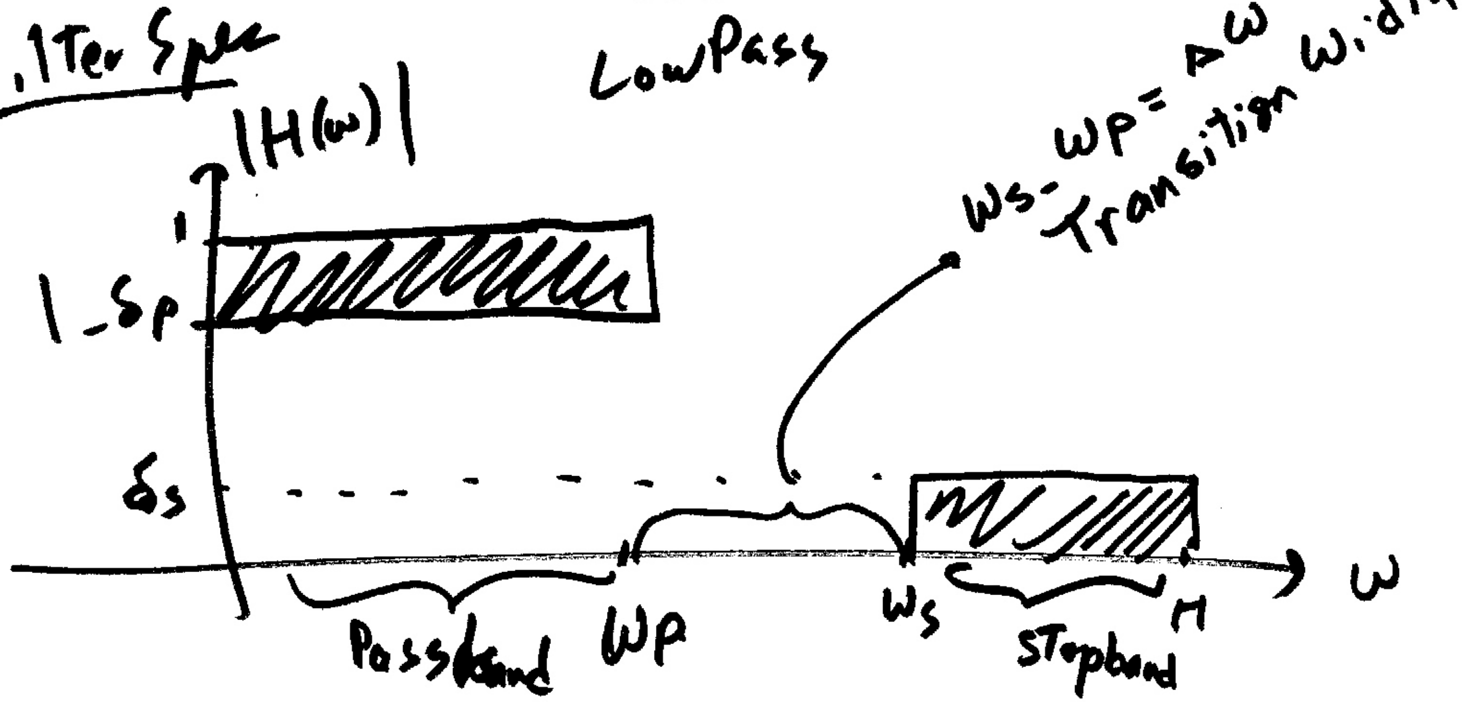




Terminology

Filter Spec

LowPass



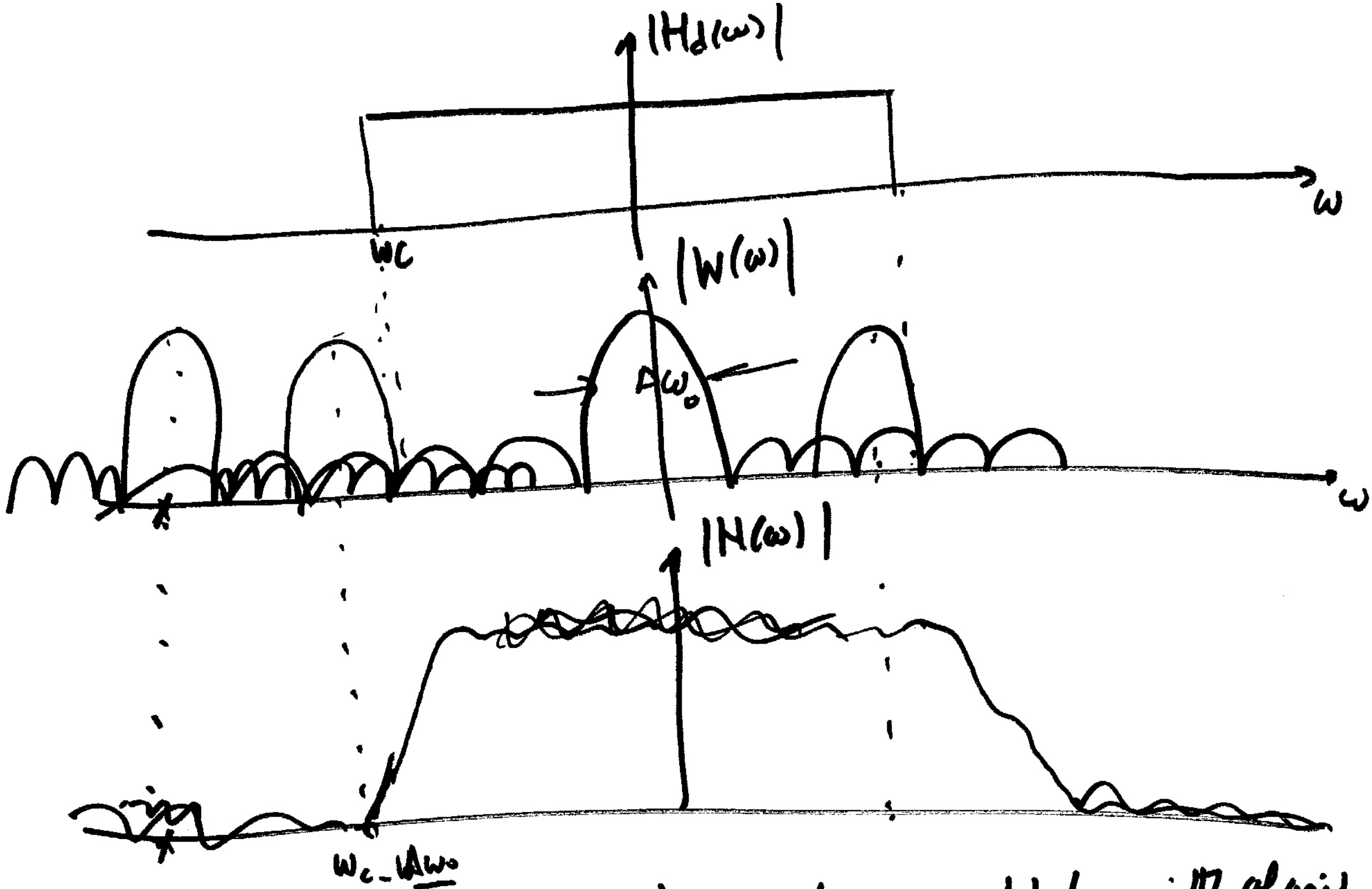
$0 \le \omega \le \omega_p$ passband.

$\omega_s \le \omega \le \pi$ stopband.

$\Delta\omega = \omega_s - \omega_p$ = Transition width

δ_p = passband ripple

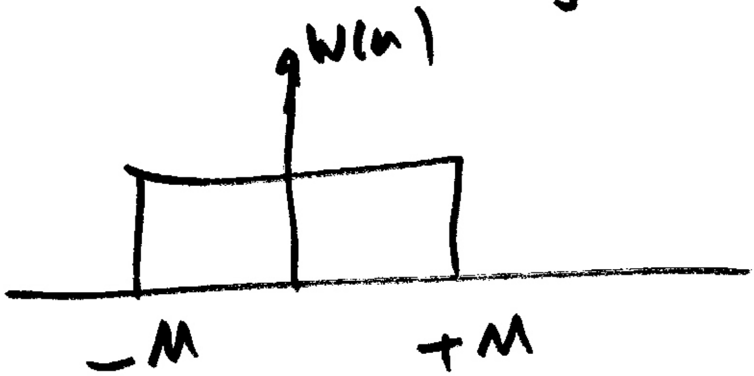
δ_s = stopband ripple.

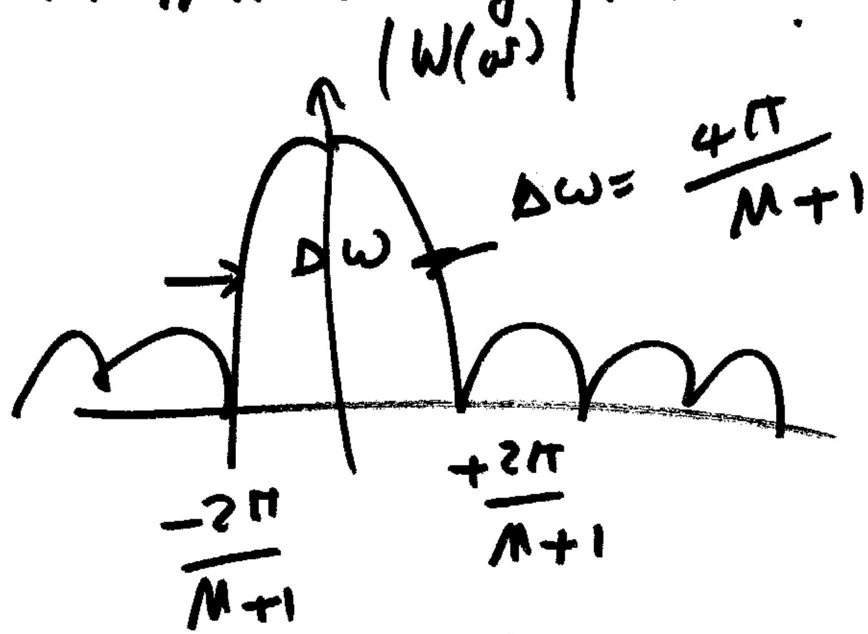


① Transition width of $H(\omega)$ depends on mainlobe width of window.
② Ripple in $H(\omega)$ depends on ripple of window fn.

① How To design window $w(n)$ To get good mainlobe/sidelobe behavior $H(\omega)$?
δ_p small δ_s small $\omega_s - \omega_p$ small

Q1 How To control Transition Width of $H(\omega)$ i.e. Mainlobe of $W(\omega)$?





ⓐ * Longer Windows in Time domain have narrower mainlobe width in freq Domain $\Longrightarrow$ small Transition width in final FIR filter.

(b) Shape of Window:

Fixed size (duration) window.
but different shape. have different.
main lobe width

Fig 7.22 & 25

Rectangular $\longrightarrow$ smallest mainlobe width.

Blackman $\longrightarrow$ highest mainlobe width.

Q2 How to design $w(n)$ to get good side lobe behavior for $w(n)$ i.e.. a good ripple behavior for my final FIR filter.

- Shape of $w(n)$ controls sidelobe behavior.
- But size of $w(n)$ (duration) does not significantly affect sidelobe behavior

shape → sidelobe and mainlobe $w(n)$.

size $\xrightarrow{\text{only}}$ mainlobe of $w(n)$

Strategy

1. use shape to control sidelobe behaviour of $W(n)$
2. use size to control mainlobe behaviour