EE123
Digital Signal Processing

Lecture 14

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Announcements

- How was midterm?
- New homework
- New Lab

- Last time:
  - Sampling of C.T. signals
- Today:
  - D.T processing of C.T signals
  - C.T processing of D.T signals (ha????)

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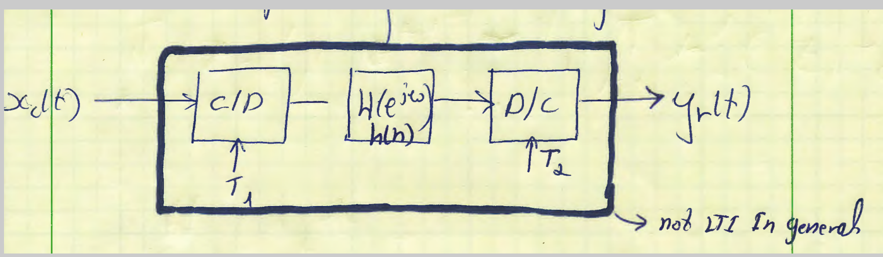
Discrete-Time Processing of C-T signals



- Q: If *h[n]* is LTI $\rightarrow H(e^{jw})$ exists,
  Is the whole system LTI?

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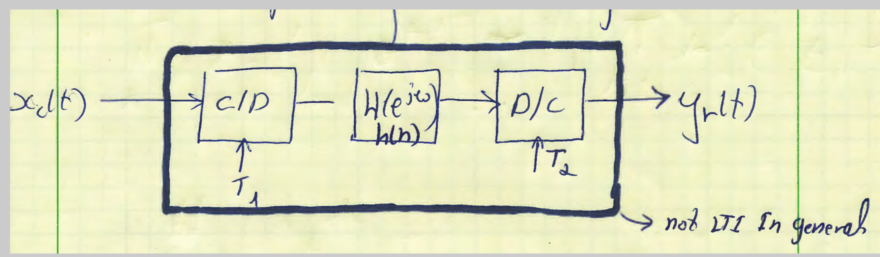
Discrete-Time Processing of C-T signals



- Q: If *h[n]* is LTI $\rightarrow H(e^{jw})$ exists,
  Is the whole system LTI?
- A: If xc(t) is bandlimited by $\frac{\Omega_s}{2} = \frac{\pi}{T}$ then,

$$\frac{Y_r(j\Omega)}{X_c(j\Omega)} = H_{\text{eff}}(j\Omega) = \begin{cases} H(e^{j\omega})\big|_{\omega=\Omega T} & |\Omega| < \Omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

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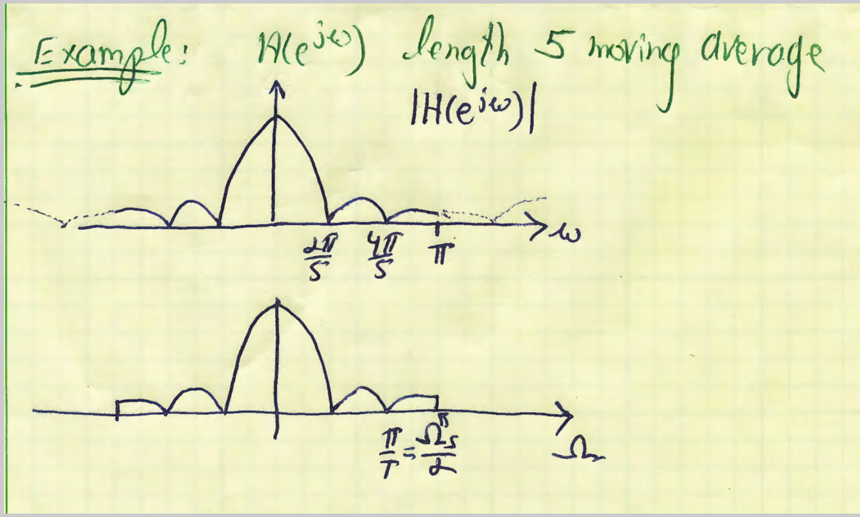
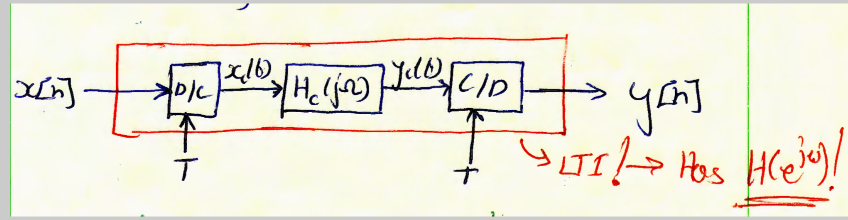
Example:



C.T Processing of D.T. signals



- Useful to interpret D.T. systems with no simple interpretation in discrete domain.
- Tool: recall: $x_c(t) = \sum_{n=-\infty}^{\infty} x[n]\mathrm{sinc}\left(\frac{t-nT}{T}\right)$

Derivation

$$X_c(j\Omega) = \begin{cases} T X(e^{j\omega})\Big|_{\omega=\Omega T} & |\Omega| \le \Omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

$$Y_c(j\Omega) = H_c(j\Omega) X_c(j\Omega)$$ ⇒ also bandlimited

so,

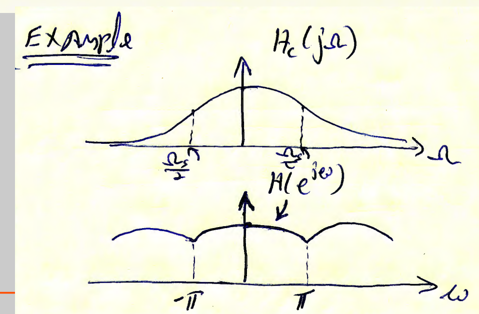
no aliasing!

$$Y(e^{j\omega}) = \frac{1}{T}\sum_k Y_c(j(\Omega - k\Omega_s))\Big|_{\Omega=\frac{\omega}{T}} = \frac{1}{T} Y_c(j\Omega)\Big|_{\Omega=\frac{\omega}{T}}$$

Example

combining the above,

$$Y(e^{j\omega}) = \underbrace{H_c(j\Omega)\Big|_{\Omega=\frac{\omega}{T}}}_{H(e^{j\omega})} \cdot X(e^{j\omega}) \quad |\omega| < \pi$$



Example:

Non-Integer delay: $H(e^{j\omega}) = e^{-j\omega\Delta}$

- What is the time-domain operation when Δ is not an integer (Δ=1/2)?

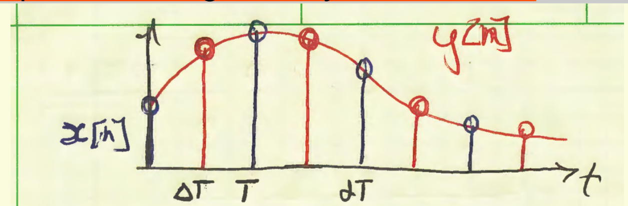
Block diagram gives $H(e^{j\omega}) = e^{-j\omega\Delta}$

LET: $H_c(j\Omega) = e^{-j\underbrace{\Omega T}_{\omega}\Delta}$ delay ΔT in time

$$x[n] \to \boxed{D/C}_{\uparrow T} \to \boxed{e^{-j(\Delta T)\Omega}} \to \boxed{C/D}_{\uparrow T} \to y[n]$$

C.T Recon delay ΔT sampling

Example: Non Integer Delay



The block diagram is only for interpretation.

$$y_c(t) = x_c(t-\Delta) \Rightarrow y[n] = y_c(nT) = x_c(nT - T\Delta)$$

$$= \sum_k x[k]\,\mathrm{sinc}\left(\frac{t-kT-T\Delta}{T}\right)\Big|_{t=nT}$$

T's cancel

$$= \sum_k x[k]\,\mathrm{sinc}(n-k-\Delta)$$

Example: Non Integer Delay

which results in:

$$h[n] = \mathrm{sinc}(n-\Delta)$$

Example: a discrete delta is a representation of a sampled sinc.

shifting by partial samples results in many coefficients!

Changing Sampling-rate via D.T Processing

Down Sampling: $x[n] \to \boxed{\downarrow M} \to x_d[n] = x[nM]$

$= x_c(nT)$ $= x_c(n\underbrace{MT}_{T'})$

The DTFT:

$$X(e^{j\omega}) = \frac{1}{T}\sum_k X_c\left(j\left(\underbrace{\frac{\omega}{T}}_{\Omega} - \underbrace{\frac{2\pi}{T}}_{\Omega_s}k\right)\right)$$

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_k X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}r\right)\right)$$

Changing Sampling-rate via D.T Processing

The DTFT:

$$X(e^{j\omega}) = \frac{1}{T}\sum_k X_c\left(j\left(\underbrace{\frac{\omega}{T}}_{\Omega} - \underbrace{\frac{2\pi}{T}}_{\Omega_s}k\right)\right)$$

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_k X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}r\right)\right)$$

we would like to bypass X_c and go from $X(e^{j\omega}) \to X_d(e^{j\omega})$

substitute $r = kM + i$ $\quad i = 0,1,\ldots,M-1$

two counters $\quad k = -\infty,\ldots,+\infty$

e.g. k: hour $\quad$ i: minutes

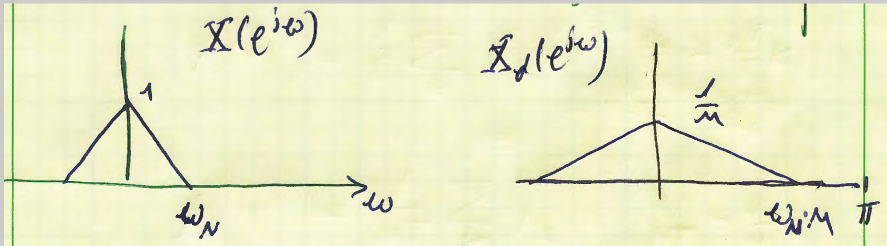
Changing Sampling-rate via D.T Processing

$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}i - \frac{2\pi}{T}k\right)\right)}_{X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi}{M}i\right)}\right)}$$

$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi}{M}i\right)}\right)$$

strech by M $\qquad$ aliasing.

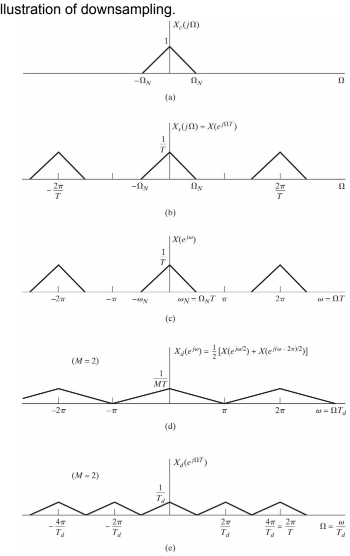
Example:



aliasing avoided if

$\omega_N \cdot M \leq \pi$

Figure 4.20 Frequency-domain illustration of downsampling.

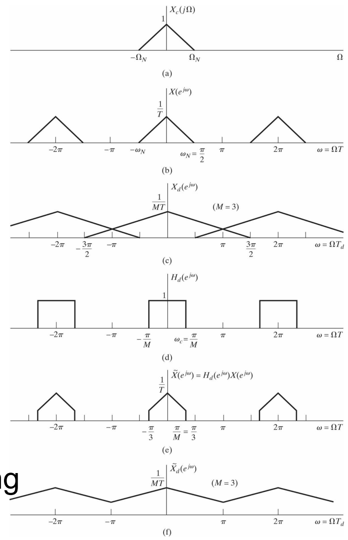


$$\left.\begin{array}{l} M = 2 \\ w_N = \pi/2 \end{array}\right\} w_N \cdot M = \pi$$

no aliasing

Discrete-Time Signal Processing, Third Edition
Alan V. Oppenheim • Ronald W. Schafer

Figure 4.21 (a)–(c) Downsampling with aliasing. (d)–(f) Downsampling with prefiltering to avoid aliasing.



$$\left.\begin{array}{l} M = 3 \\ w_N = \pi/2 \end{array}\right\} w_N \cdot M = 3\pi/2 > \pi$$

aliasing!

can use anti-aliasing filter.

$$x[n] \longrightarrow \boxed{\begin{array}{c}LPF\\ \pi/M\end{array}} \xrightarrow{\tilde{x}[n]} \boxed{\downarrow M} \longrightarrow \tilde{x}_d[n] = \tilde{x}[nM]$$

Up-sampling

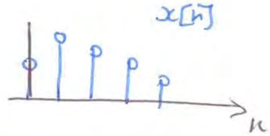
$x[n] = x_c(nT)$

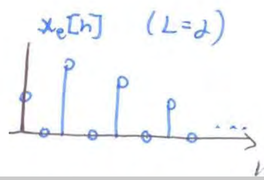
$x_i[n] = x_c(nT')$ where $T' = \frac{T}{L}$, L integer

obtain $x_i[n]$ from $x[n]$ in two steps:

(1) Generate $x_e = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2L, \ldots \\ 0 & \text{otherwise} \end{cases}$

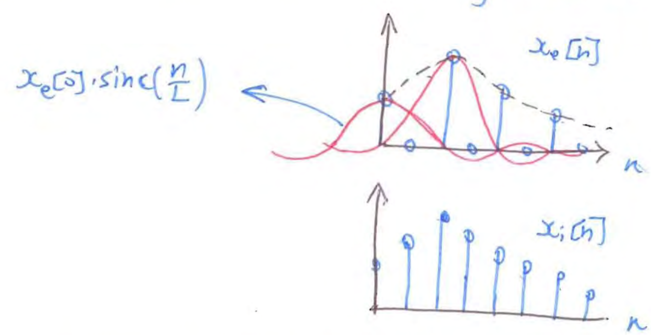
e ↓ expansion





Up-Sampling

(2) Obtain $x_i[n]$ from $x_e[n]$ by bandlimited interpolation.



$x_i[n] = x_e[n] * \text{sinc}(\frac{n}{L})$

Up-Sampling

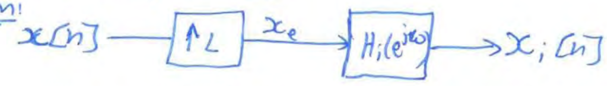
since $x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL]$

$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k]\,\mathrm{sinc}\left(\frac{n-kL}{L}\right)$$

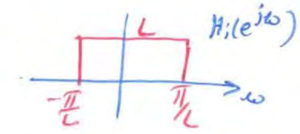
Frequency Domain Interpretation

Freq. domain Interpretation:

system:



$h_i = \mathrm{sinc}\left(\frac{n}{L}\right) \xRightarrow{DTFT}$



Frequency Domain Interpretation

$$X_e(e^{j\omega}) = \sum_n \underbrace{x_e[n]}_{\neq 0 \text{ only when } n=mL,\ m\ \text{integer}} e^{-j\omega n} = \sum_m x_e[mL]e^{-j\omega Lm} = X(e^{j\omega L})$$

compress $X(e^{j\omega})$ by factor L!