EE123 Digital Signal Processing

Lecture 15

Announcements

- Last Time
  - – Today:
  - – Resampling
  - – Interchanging operations
    - multi-rate processing
- Today
  - –Multi-Rate
  - –Polyphase
  - –Practical A/D
    - Quantization noise

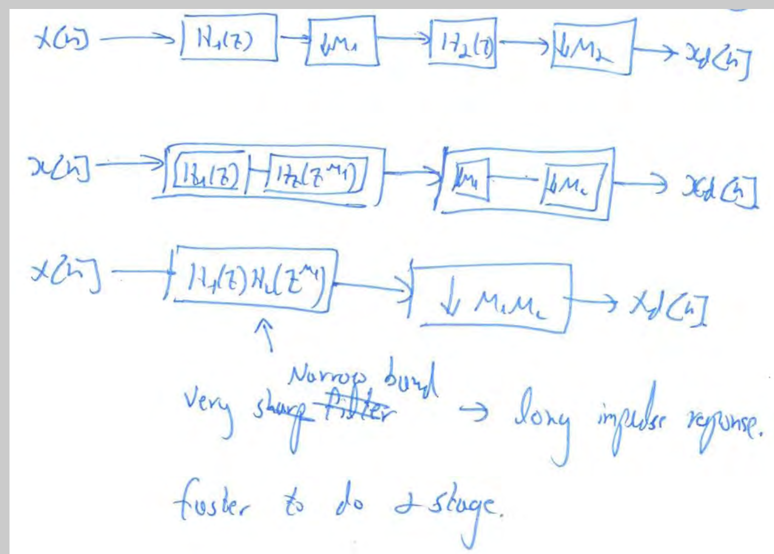
Interchanging Operations

- Last time, showed:

$$x[n] \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow L} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z^L)} \rightarrow y[n]$$

$$x[n] \rightarrow \boxed{\downarrow M} \rightarrow \boxed{H(z)} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{H(z^M)} \rightarrow \boxed{\downarrow M} \rightarrow y[n]$$

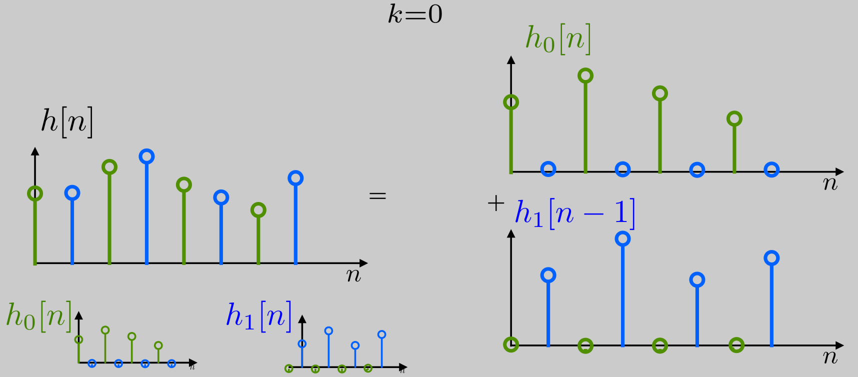
Multi-Rate Filtering



Polyphase Decomposition

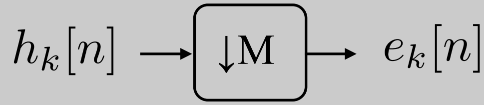
- We can decomposed an impulse response to:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

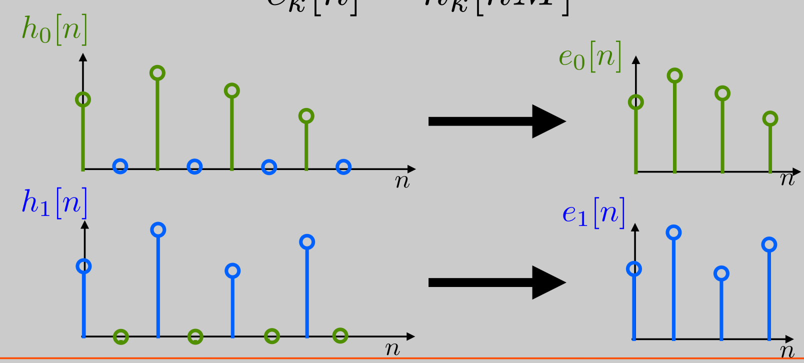


Polyphase Decomposition

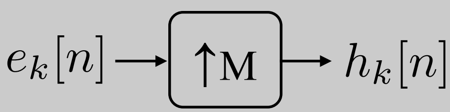
- Define:



$$e_k[n] = h_k[nM]$$



Polyphase Decomposition



recall upsampling ⇒ scaling

$$H_k[z] = E_k[z^M]$$

Also, recall:

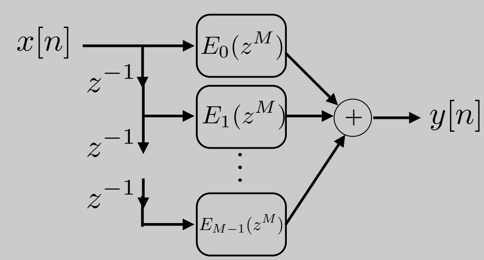
$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

So,

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$

Polyphase Decomposition

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$



Why should you care?

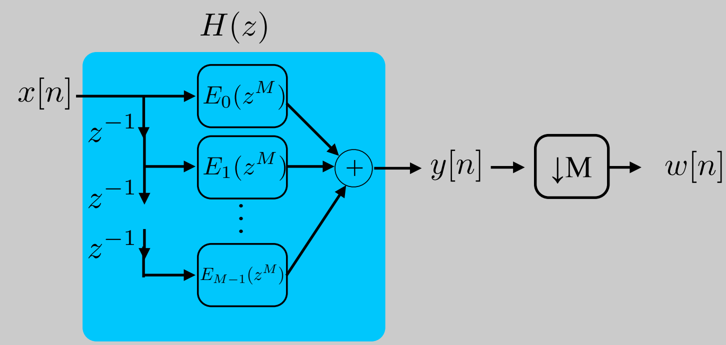
Polyphase Implementation of Decimation

$$x[n] \rightarrow \boxed{H(z)} \rightarrow y[n] \rightarrow \boxed{\downarrow M} \rightarrow w[n] = y[nM]$$

- Problem:
  - –Compute all y[n] and then throw away -- wasted computation!
    - For FIR length N ⇒ N mults/unit time
  - –Can interchange Filter with compressor?
    - Not in general!

Polyphase Implementation of Decimation

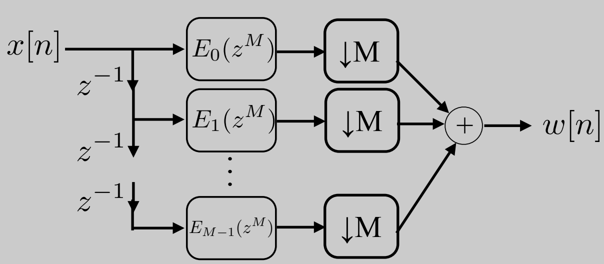
$$x[n] \rightarrow \boxed{H(z)} \rightarrow y[n] \rightarrow \boxed{\downarrow M} \rightarrow w[n] = y[nM]$$



Polyphase Implementation of Decimation

$$x[n] \rightarrow \boxed{H(z)} \rightarrow y[n] \rightarrow \boxed{\downarrow M} \rightarrow w[n] = y[nM]$$

Interchange sum with decimation

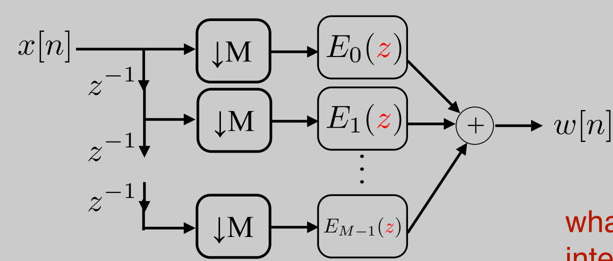


now, what can we do?

Polyphase Implementation of Decimation

$$x[n] \rightarrow \boxed{H(z)} \rightarrow y[n] \rightarrow \boxed{\downarrow M} \rightarrow w[n] = y[nM]$$

Interchange filter with decimation



what about interpolation?

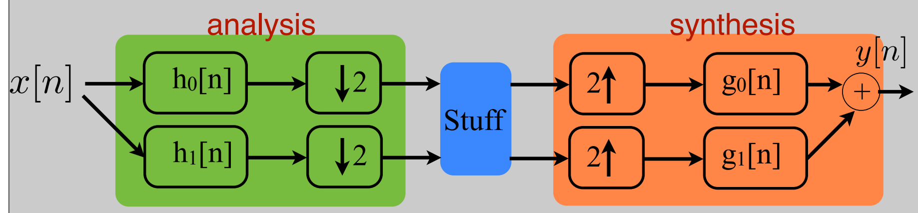
Computation:
Each Filter: N/M *(1/M) mult/unit time
Total: N/M mult/unit time

Multirate Filter Bank

- For many types of applications we would like to process bands in frequency separately
  - Analysis/synthesis of speech, Audio, more
- Idea:
  - Split frequency
  - Decimate to reduce computation
  - (Much like wavelets!)

Multirate FilterBank

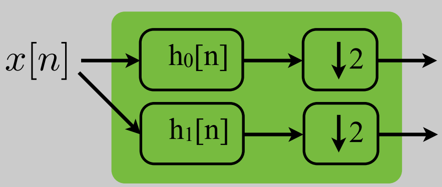
- $h_0[n]$ is low-pass, $h_1[n]$ is high-pass
- Often $h_1[n] = e^{j\omega n} h_0[n]$ or $H_1(e^{j\omega}) = H_0(e^{j(w-\pi)})$

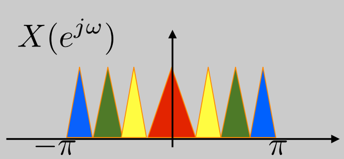


Note: notation different than in lecture 9,10 for Haar

Subtleties in Time-Freq Tiling

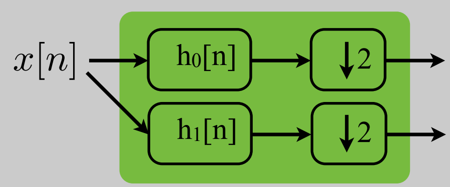
- Assume h_0, h_1 are ideal low,high pass filters

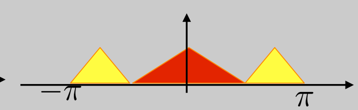


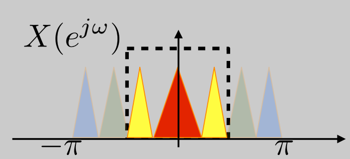


Subtleties in Time-Freq Tiling

- Assume h_0, h_1 are ideal low,high pass filters

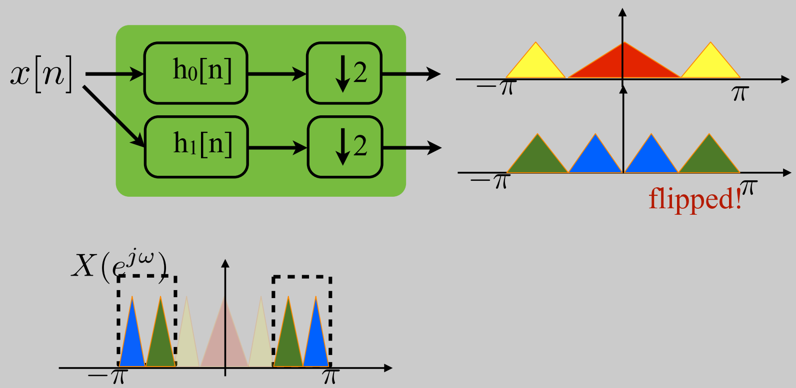






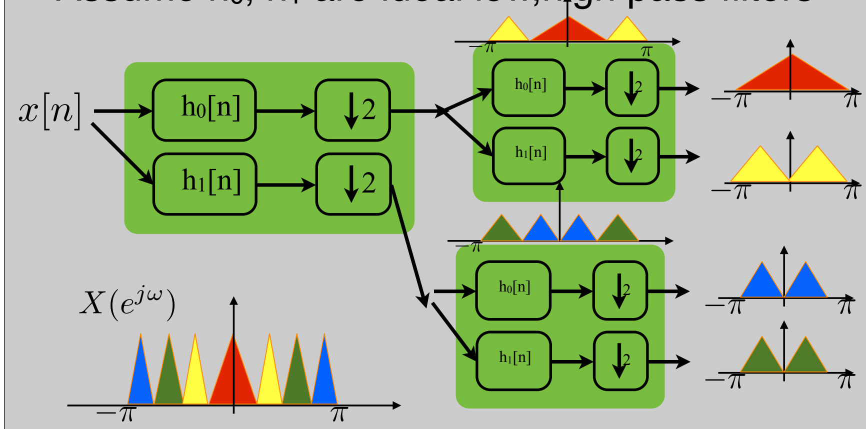
Subtleties in Time-Freq Tiling

- Assume h_0, h_1 are ideal low,high pass filters

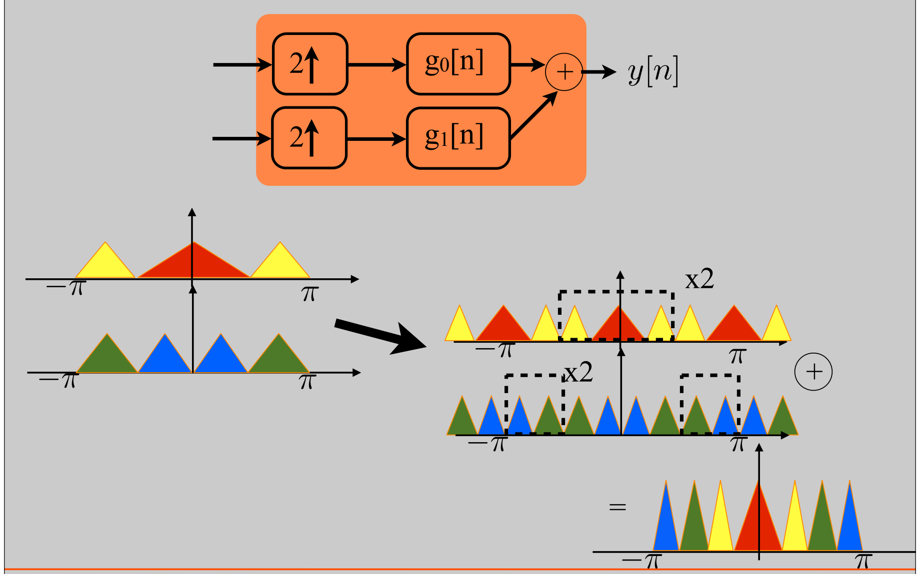


Subtleties in Time-Freq Tiling

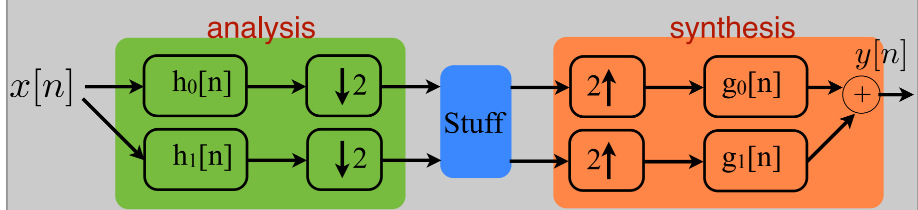
- Assume h_0, h_1 are ideal low,high pass filters



Perfect Reconstruction Ideal Filters



Perfect Reconstruction non-Ideal Filters

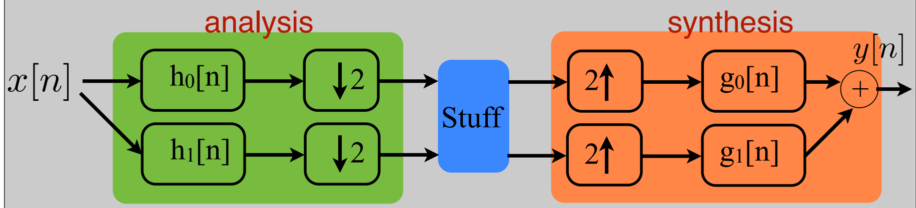


$$Y(e^{j\omega}) = \frac{1}{2}\left[G_0(e^{j\omega})H_0(e^{j\omega}) + G_1(e^{j\omega})H_1(e^{j\omega})\right] X(e^{j\omega})$$
$$+\frac{1}{2}\left[G_0(e^{j\omega})H_0(e^{j(\omega-\pi)}) + G_1(e^{j\omega})H_1(e^{j(\omega-\pi)})\right] X(e^{j(\omega-\pi)})$$

need to cancel!

aliasing

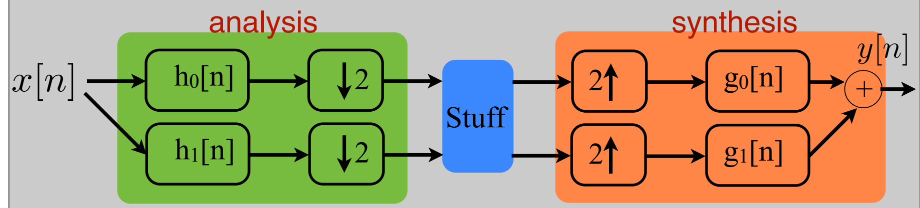
Quadrature Mirror Filters - perfect recon



QMF - mirror around pi/2

$$
\begin{aligned}
H_1(e^{j\omega}) &= H_0(e^{j(\omega-\pi)}) \\
G_0(e^{j\omega}) &= 2H_0(e^{j\omega}) \\
G_1(e^{j\omega}) &= -2H_1(e^{j\omega})
\end{aligned}
$$

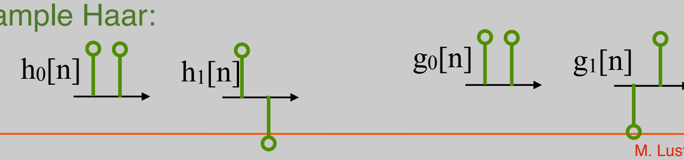
Quadrature Mirror Filters - perfect recon



QMF - mirror around pi/2

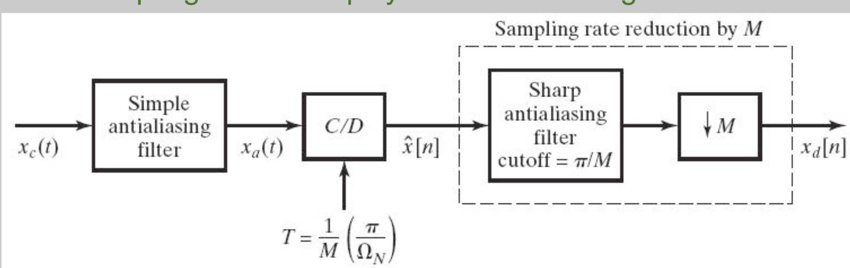
$$
\begin{aligned}
H_1(e^{j\omega}) &= H_0(e^{j(\omega-\pi)}) \\
G_0(e^{j\omega}) &= 2H_0(e^{j\omega}) \\
G_1(e^{j\omega}) &= -2H_1(e^{j\omega})
\end{aligned}
$$

Example Haar:

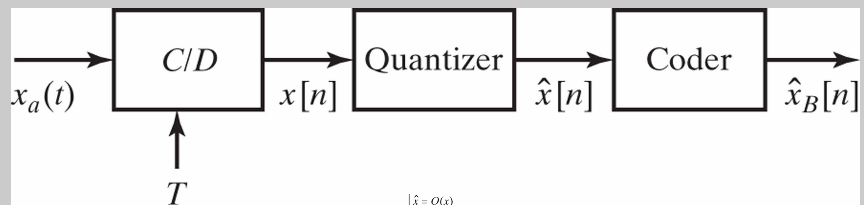


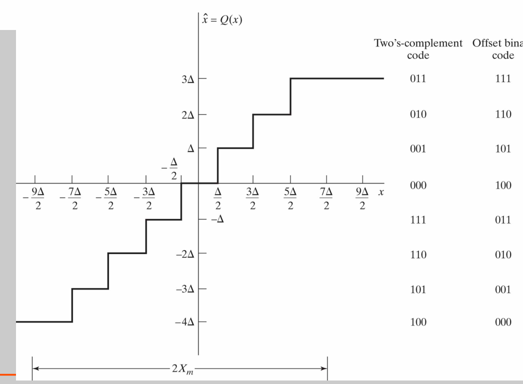
A Few Notes on Practical A/D

Oversampling A/D to simplify C.T. Anti-Aliasing Filter



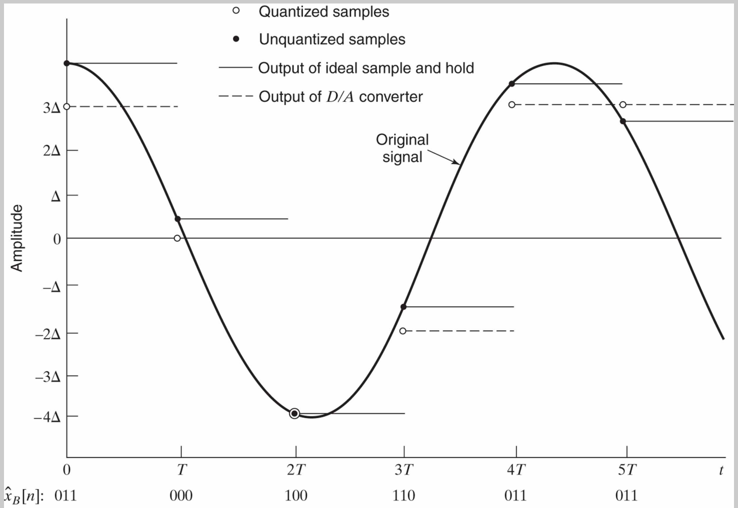
Sampling and Quantization



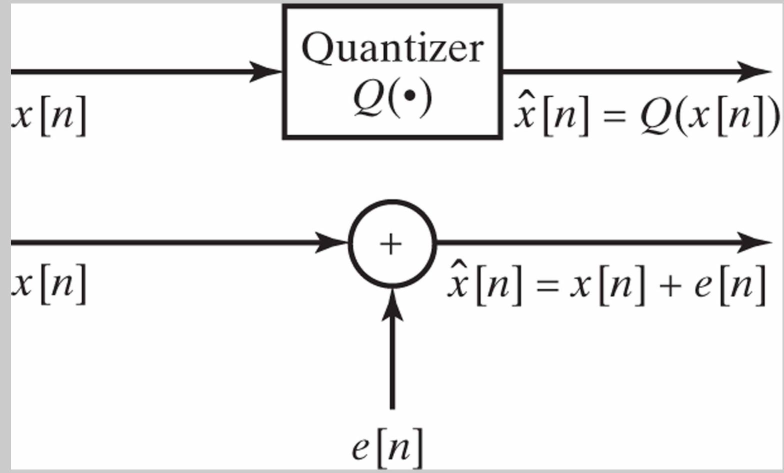


Sampling and Quantization

Figure 4.55 Sampling, quantization, coding, and D/A conversion with a 3-bit quantizer.

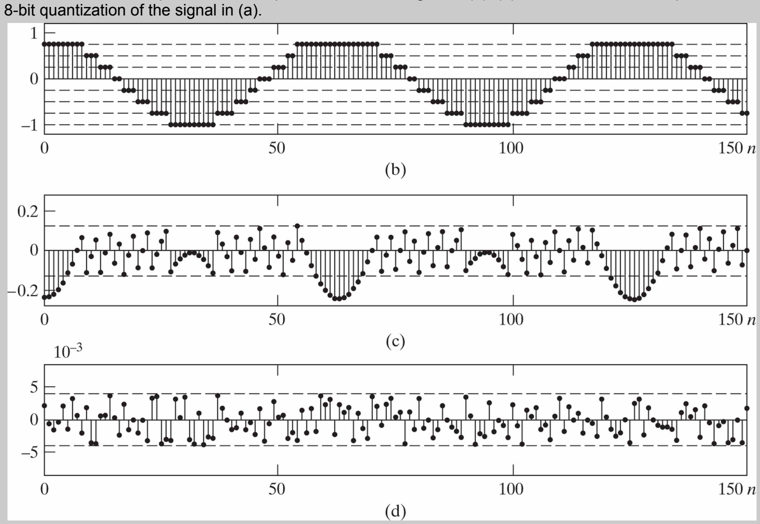


Noise Model For Quantization

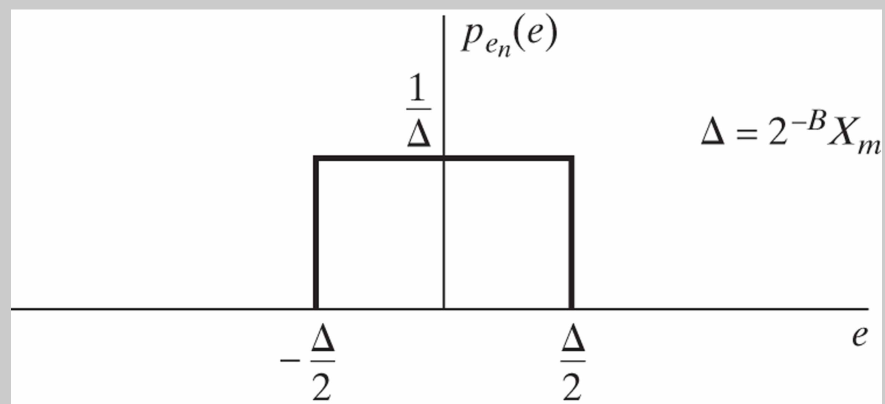


Quantization Noise

Figure 4.57 (*continued*) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a). (d) Quantization error sequence for 8-bit quantization of the signal in (a).



PDF of Quantization Noise



$$\text{SNR}_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

Signal amp

rms of amp