Lecture 11/02/11 FIR DESIGN By Windowing (1)

Given desired freq. response $H_d(e^{j\omega})$, find impulse response:

$$\boxed{h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega} \rightarrow \text{ideal}$$

obtain M^{th} order causal FIR filter by truncating:

$$h[n] = \begin{cases} h_d[n] & 0 \le n \le M \\ \phi & \text{otherwise} \end{cases}$$

Equivalently, $h[n] = h_d[n] W[n]$ where $w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$

We already saw that:

$$H(e^{j\omega}) = H_d(e^{j\omega}) \circledast W(e^{j\omega})$$

for Boxcar $\Rightarrow W(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)}$

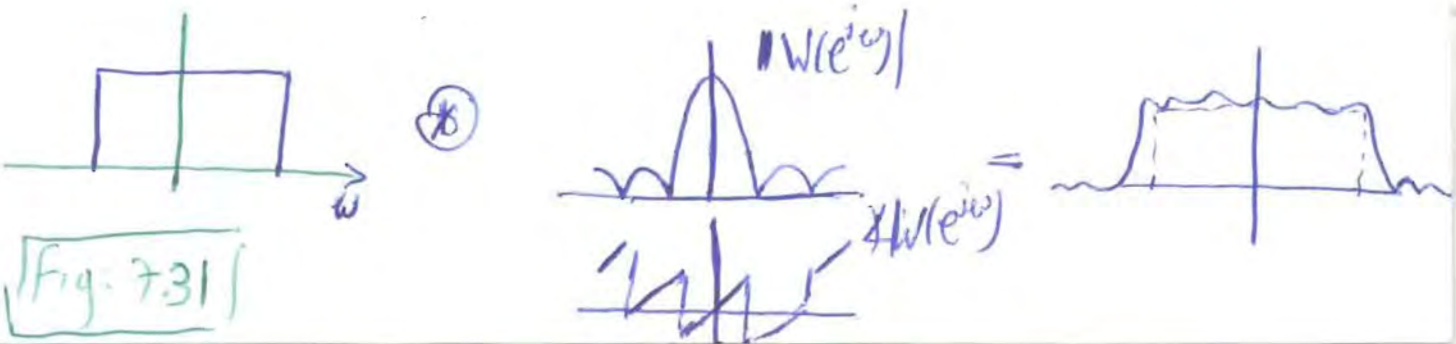


Fig. 7.31

* Boxcar window has sharpest transition, but much ripple.

"Tapered" windows:

Bartlett (triangular):

$$\Delta[n] = \begin{cases} \frac{n}{M/2} & 0 \le n \le M/2 \\ 2 - \frac{n}{M/2} & \frac{M}{2} \le n \le M \end{cases}$$

Hann

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

<Figures 7.29 7:30>

Note tradeoff between main $\underbrace{\text{lobe width}}_{\text{transition}}$ and side-lobe $\underbrace{\text{amplitude}}_{\text{ripple}}$

<7.3.1>

$\boxed{\text{Matlab command} \quad \text{fir1}}$

FIR FILTER DESIGN

Frequency response

(-) Choose a desired frequency response $H_d(e^{j\omega})$

[Non causal and infinite impulse response]

zero delay

(If is derived from CT choose T and use:

$$H_d(e^{j\omega}) = H_c\left(j\frac{\Omega}{T}\right)$$
)

window

(-) Choose window

(-) Length $M+1$ → sharpness of transition

(-) Type → transition & ripple

(-) modulat desired freq. Response to shift impulse response

$$\boxed{H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}}$$

Determin truncated impulse response $h_1[n]$

$$h_1[n] = \begin{cases} \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{-j\omega \frac{M}{2}} e^{jn\omega} & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

Apply window

$$h_\omega[n] = \omega[n] \cdot h_1[n]$$

Check

compute $H_\omega(e^{j\omega})$ if does not meet specs, increase M or use a different window.

Example FIR Low-Pass Filter Design

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases}$$

choose $M \rightarrow$ window length. $\Rightarrow H_d(e^{j\omega}) \cdot e^{-j\omega\frac{M}{2}}$

$$h_1[n] = \begin{cases} \dfrac{\sin(\omega_c(n-\frac{M}{2}))}{\pi(n-\frac{M}{2})} = \mathrm{sinc}\left(\frac{\omega_c}{\pi}(n-\frac{M}{2})\right)\cdot\frac{\omega_c}{\pi} & 0 \leq n \leq M \\ \phi & \text{otherwise} \end{cases}$$

$$\boxed{h_\omega[n] = h_1[n] \cdot \omega[n]} \Rightarrow \text{windowed sinc}$$

High-Pass: (1) design low pass $h_\omega[n]$

(2) transform $h_\omega[n] \cdot (-1)^n = e^{-j\pi n}$

general band pass

transform $h_\omega[n] \cdot \cos(\omega_0 n)$

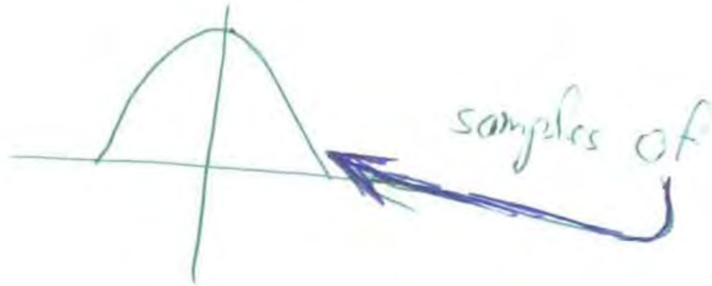
charachterization of impulse resp shape:

TIME-Bandwidth product

$$\underset{\uparrow\, M+1}{T}\cdot(\underset{\uparrow\, \frac{\omega}{2\pi}}{BW})$$

also total number of zeros of continuous sinc

$TBW = 2$



$TBW = 4$

For LP filter with desired ω_s TBW indicates sharpness of transition

Example:

$$\omega_s = \frac{\pi}{2} \Rightarrow BW = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$TBW = 2 \Rightarrow (M+1)\cdot\frac{1}{2} = 2 \Rightarrow M = 3$$

$$TBW = 8 \Rightarrow (M+1)\cdot\frac{1}{2} = 8 \Rightarrow M = 15$$