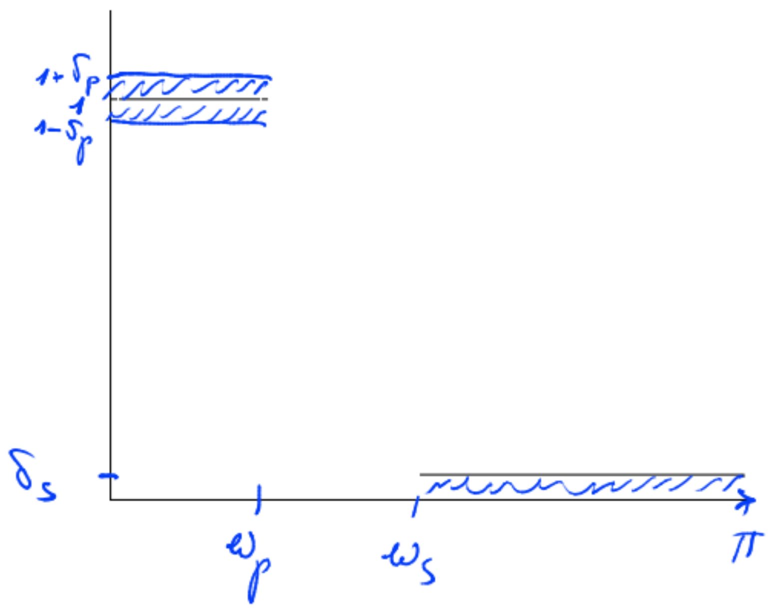
Lecture 11/09/11

Optimal min-max design through optimization:

SPECS:



(•) Maximum pass-band ripple:

$$1-\delta_p \leq |H(e^{j\omega})| \leq 1+\delta_p, \quad 0 \leq \omega \leq \omega_p$$

(•) Minimum stop-band attenuation

$$|H(\omega)| \leq \delta_s \qquad \omega_s \leq \omega \leq \pi$$

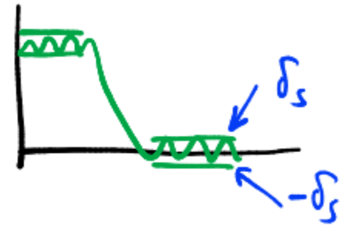
maximum stop-band attenuation (linear phase) 2

minimize δ_s Recall $\tilde{H}(e^{j\omega})$ is Re

Subject to: $1-\delta_p \le \tilde{H}(e^{j\omega_k}) \le 1+\delta_p, \quad 0 \le \omega_k \le \omega_p$

$-\delta_s \le \tilde{H}(e^{j\omega_k}) \le \delta_s, \quad \omega_s \le \omega_k \le \pi$

$\delta_s > 0$



$$\tilde{H}(e^{j\omega_k})\Big|_{0\le\omega_k\le\omega_p} = \begin{bmatrix} 1 & 2\cos(\omega_k) & \cdots & 2\cos(\omega_k \frac{M}{2}) \\ \vdots & & & \\ 1 & 2\cos(\omega_p) & \cdots & 2\cos(\omega_p \frac{M}{2}) \end{bmatrix}$$

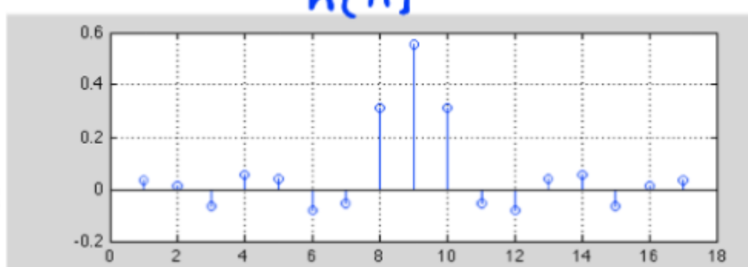
- Similarly for $\omega_s \le \omega_k \le \pi$

- $\omega_p, \omega_s, M, \delta_p$ given
- solution is a linear program in $\tilde{h}, \delta_2$
- know & used since 60's
- can add constraints e.g. $|h_i| \le \alpha$

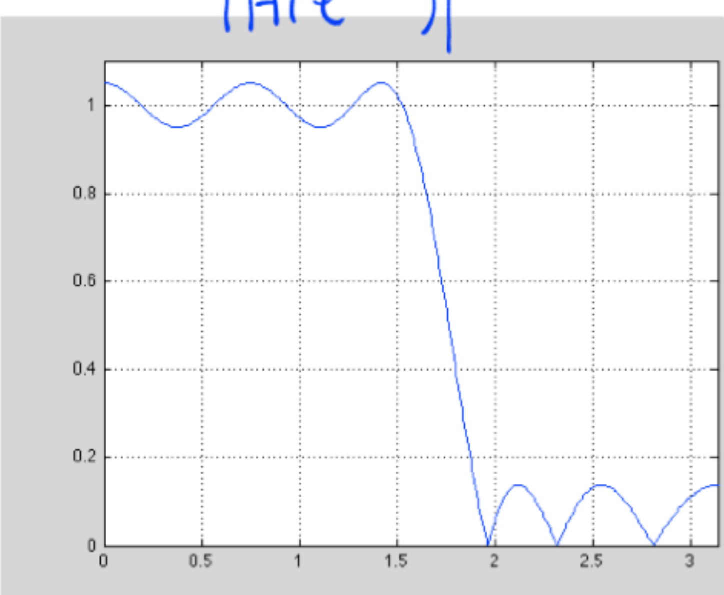
* Based on notes by Steaphen boyd

```
M = 16;
wp = 0.5*pi;
ws = 0.6*pi;
dp = 0.05;

MM = M*15;
w = linspace(0,pi,MM);

idxp = find(w <=wp);
idxs = find(w >=ws);

Ap = [ones(length(idxp),1) 2*cos(kron(w(idxp)',[1:M/2]))];
As = [ones(length(idxs),1) 2*cos(kron(w(idxs)',[1:M/2]))];

% optimization

cvx_begin
    variable hh(M/2+1,1);
    variable ds(1,1);

    minimize(ds)
    subject to
        Ap*hh <=1+dp;
        Ap*hh >=1-dp;
        As*hh < ds;
        As*hh > -ds;
        ds>0;
cvx_end

h = [hh(end:-1:1) ; hh(2:end)];
```

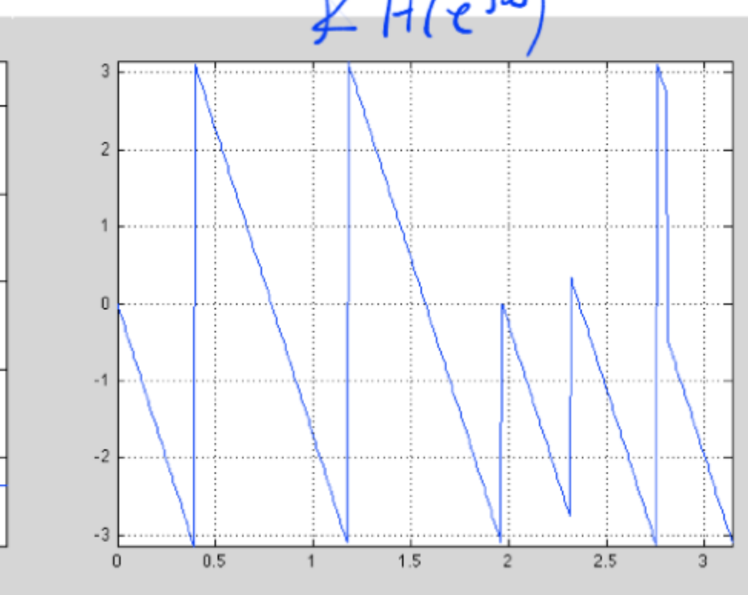
$h[n]$



$|H(e^{j\omega})|$



$\measuredangle H(e^{j\omega})$



Variations

- fix δ_s, minimize δ_p

- fix δ_s and δ_p, M min. $\omega_s - \omega_p = \Delta\omega$

  use bisection: - fix $\Delta\omega$, δ_p, M, solve $\tilde{h}$, δ_s

  - if feasible decreas $\Delta\omega$ if not increase

- fix δ_p, δ_s, ω_p, ω_s minimze M

  bisection!

IIR Design

Historically → Continuouse IIR design was highly advanced use results from C.T to D.T

- C.T IIR designs have closed form → easy to Use.

- With these designs, easy to control amp. response but not phase response.

Common types:

(1) Butterworth → monotonic, no ripple

(2) Chebyshev → type I pass-band ripple
type II stop band ripple

(3) Elliptic → Ripples in both s.b and P.b
Has zeroes.

<u>Design of DT IIR Filters from Analog.</u> ⑦

Discretize by using one of many techniques.

$$H_c(s) \longrightarrow H(z)$$

<u>must satisfy:</u>

(1) Imaginary axis $\rightarrow$ mapped to unit circle

$$s = j\Omega \Rightarrow z = e^{j\omega}$$

(2) stability of $H_c(s)$ should result in stable $H(z)$

<u>Two methods:</u>

(1) Impulse invariance $\rightarrow$ match impulse

$$h[n] = T h_c(nT)$$

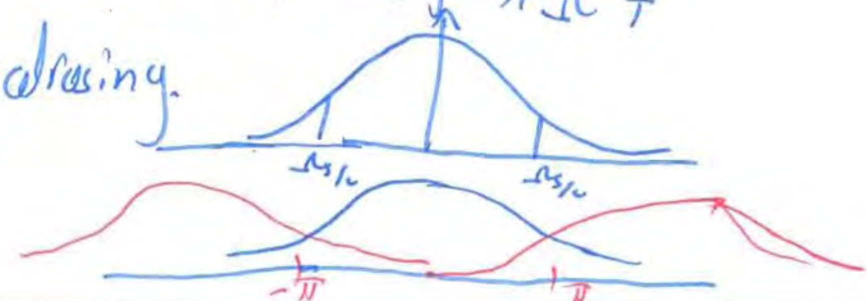
$$\Downarrow$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c(j\Omega + j\Omega_s k)\Big|_{\Omega = \frac{\omega}{T}}$$

if. $H_c(j\Omega)$ band limited $|\Omega| \leq \Omega_s/2$ then:

$$H(e^{j\omega}) = H_c(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

otherwise $\rightarrow$ aliasing.



(–) Bilinear transform → basic idea.

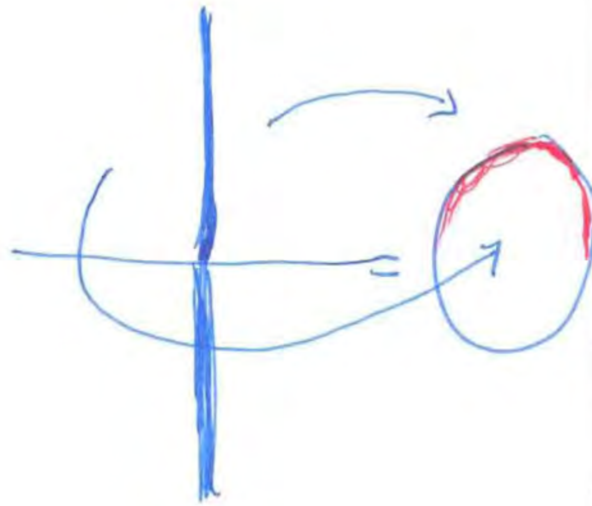
(•) Distorts $H_c(j\Omega)$ such that it is bandlimited.

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$$

for $s=0 \Rightarrow z=1$

$s=\infty \Rightarrow z=-1$



Matlab: butter, cheby1, cheby2, ellip

Next time FIR ...