

Lecture 11/30/11

①

Generalized linear-phase systems

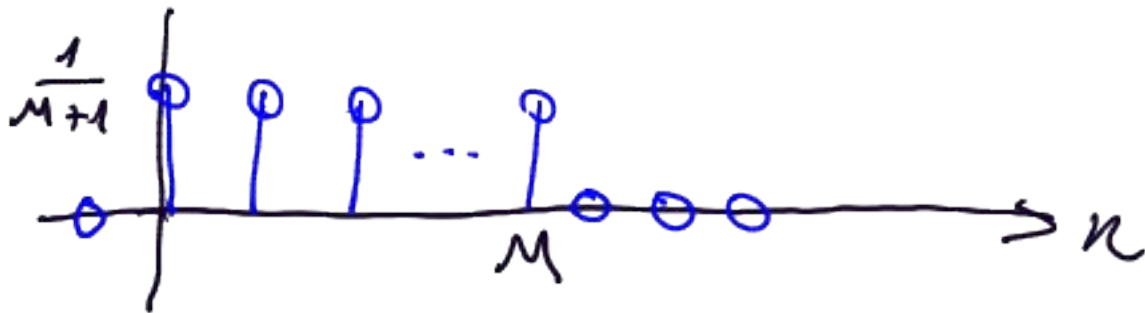
$$H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{Real, allow sign change}} e^{-j\alpha\omega + j\beta}$$

Real, allow  
sign change

$$\text{grad}[H(e^{j\omega})] = \alpha \left( \text{except when } A(e^{j\omega}) \text{ changes sign} \right)$$

Example  $(M+1)$ -point moving average

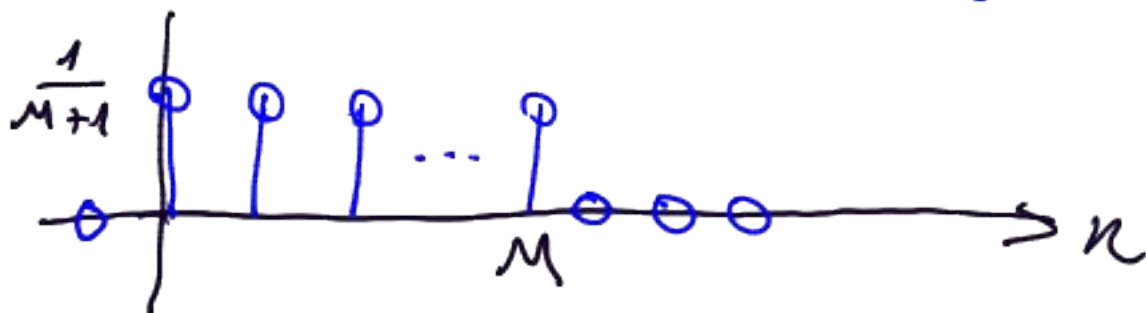
(2)



$$H(e^{j\omega}) = \underbrace{\quad}_{A(e^{j\omega})} \quad \square$$

# Example $(M+1)$ -point moving average

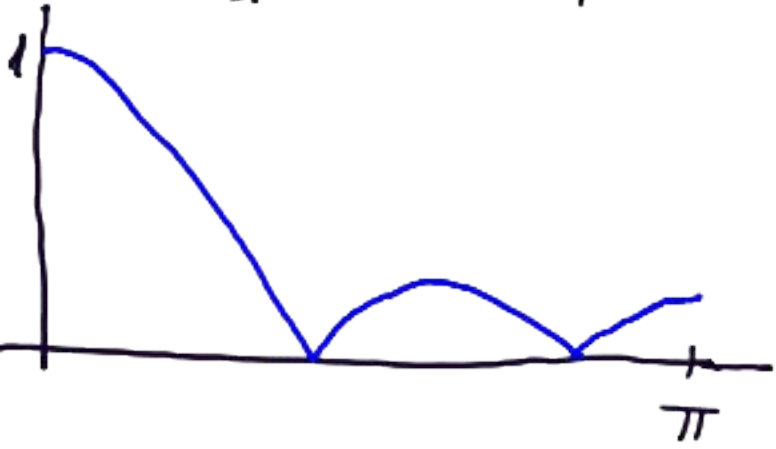
(2)



$$H(e^{j\omega}) = \underbrace{\frac{1}{M+1} \frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}}_{A(e^{j\omega})} e^{-j\omega \frac{M}{2}}$$

$M=4$

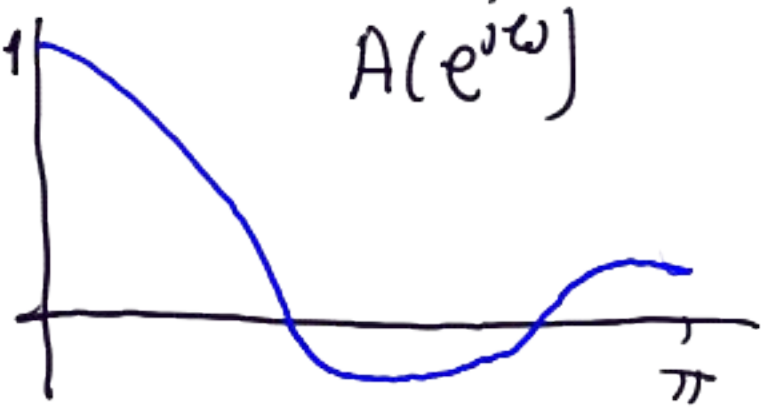
$|H(e^{j\omega})|$



$\arg[H(e^{j\omega})]$  ③



$A(e^{j\omega})$



$\text{grad}[H(e^{j\omega})]$

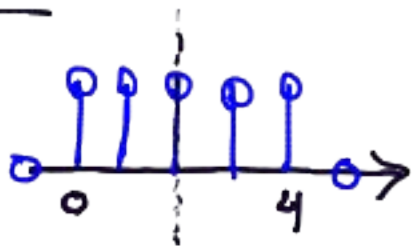


GLP for FIR  $\rightarrow$  MUST have symmetry

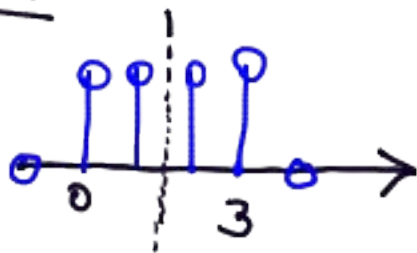
(4)

$$h[n] = h[M-n]:$$

Type I (M even)



Type II (M odd)

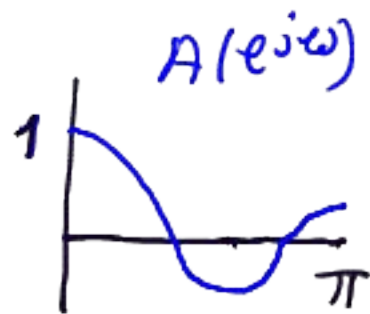
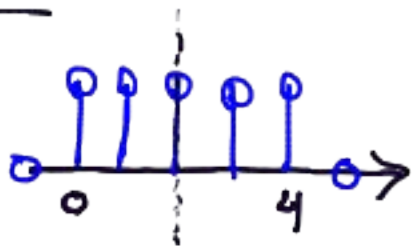


GLP for FIR  $\rightarrow$  MUST have symmetry

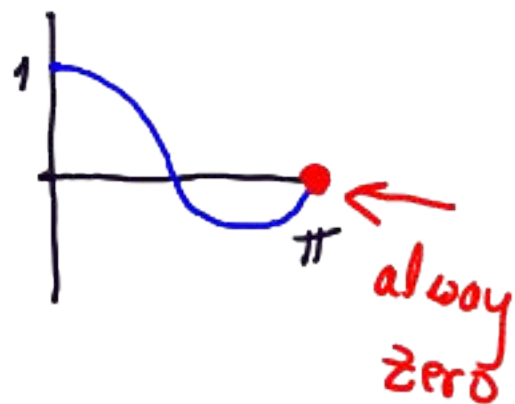
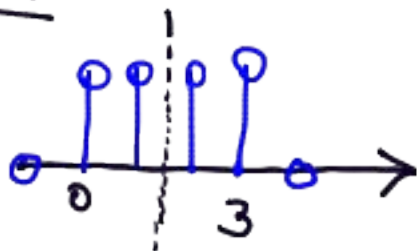
(4)

$$h[n] = h[M-n]:$$

Type I ( $M$  even)



Type II ( $M$  odd)

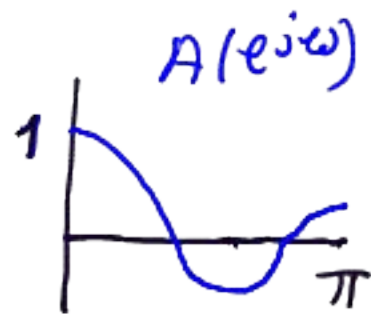
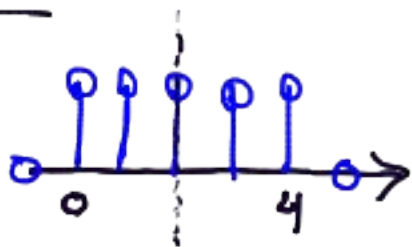


GLP for FIR  $\rightarrow$  MUST have symmetry

(4)

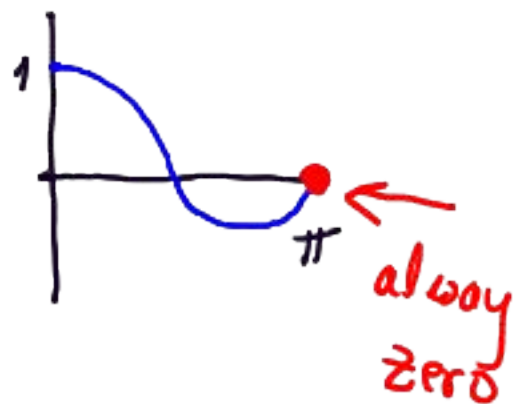
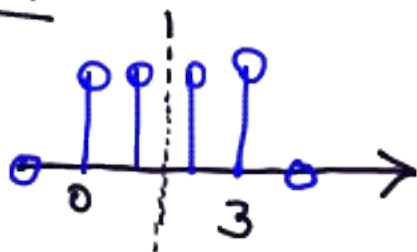
$$h[n] = h[M-n]:$$

Type I ( $M$  even)



$$A(e^{j\omega}) = h\left[\frac{M}{2}\right] + 2 \sum_{k=1}^{\frac{M}{2}-1} h\left[\frac{M}{2}-k\right] \cos(\omega k)$$

Type II ( $M$  odd)

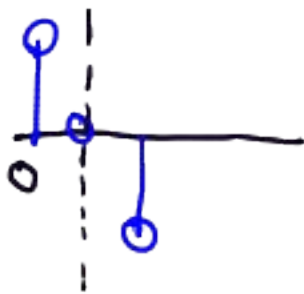


$$A(e^{j\omega}) = \text{In the text}$$

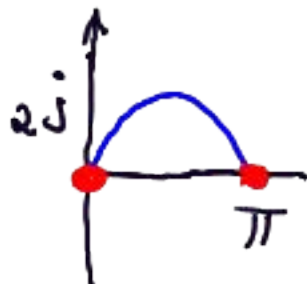
$$h[n] = -h[M-n]$$

(5)

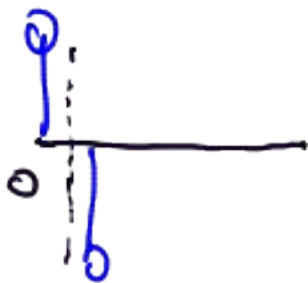
Type III (M even)



$$A(e^{j\omega}) = j 2 \sum_{k=0}^{\frac{M}{2}-1} h[\frac{M}{2}-k] \sin(\omega k)$$



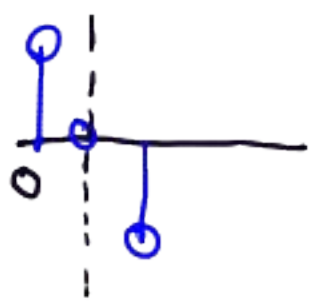
Type IV (M odd)





$$h[n] = -h[M-n]$$

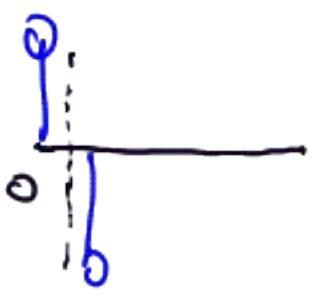
Type III (M even)



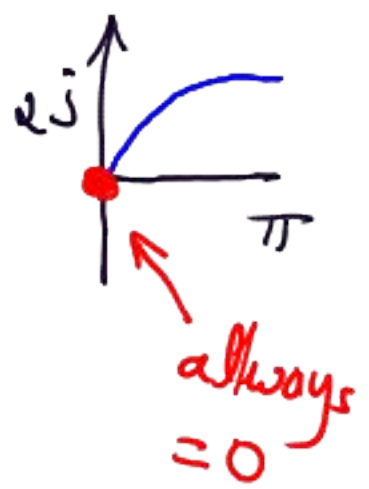
$$A(e^{j\omega}) = j 2 \sum_{k=1}^{M/2} h[\frac{M}{2}-k] \sin(\omega k)$$



Type IV (M odd)



$$A(e^{j\omega}) = \text{see text}$$



# Zeros of GLP system

⑥

Type I, II:  $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n} =$$

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$$= \sum_{n=0}^M h[M-n] z^{-n} =$$

# Zeros of GLP system

(6)

Type I, II:  $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^M h[n] z^{-n} =$$

$$= \sum_{n=0}^M h[M-n] z^{-n} = z^{-M} \sum_{n=0}^M h[M-n] z^{M-n}$$

# Zeros of GLP system

(6)

Type I, II:  $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^M h[n] z^{-n} =$$

$$= \sum_{n=0}^M h[M-n] z^{-n} = z^{-M} \sum_{n=0}^M h[M-n] z^{\underbrace{M-n}_{\equiv k}}$$

$$= z^{-M} \sum_{k=0}^M h[k] z^k$$

# Zeros of GLP system

(6)

Type I, II:  $h[n] = h[n-M]$

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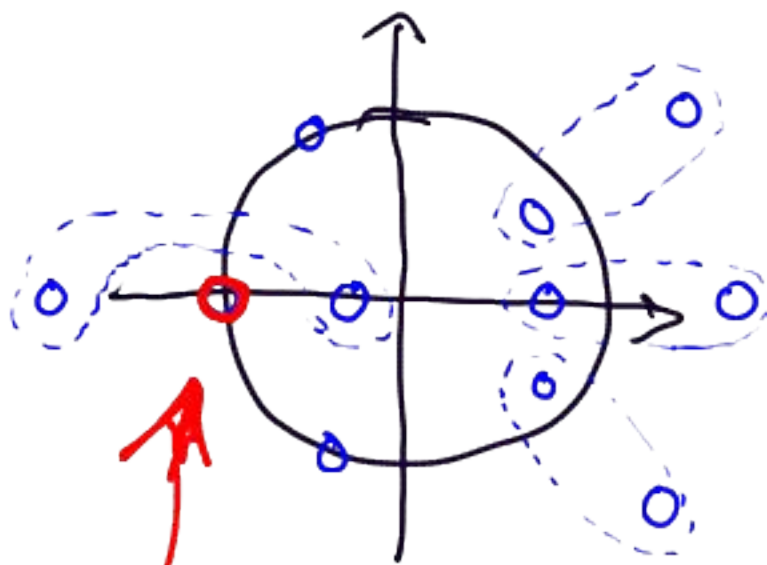
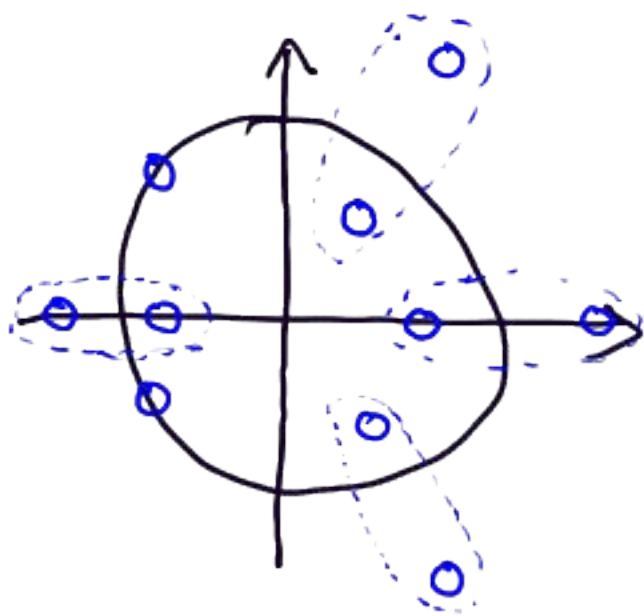
$\underbrace{M-n}_{\equiv k}$

$$= z^{-M} \sum_{k=0}^M h[k] z^k$$

$$\Rightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

$$H(z) = z^{-M} H(z^{-1}) \quad \text{Type I, II}$$

(7)



$$H(-1) = 0 \quad \text{Type II (Never high-pass)}$$

→ FOR GLP, IF  $a = re^{j\theta}$  is a zero  
 $\frac{1}{a^*}$  is also a zero

# Zeros of GLP system

(6)

Type I, II:  $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^M h[n] z^{-n} =$$

$$= \sum_{n=0}^M h[M-n] z^{-n} = z^{-M} \sum_{n=0}^M h[M-n] z^{M-n}$$

*green annotations:*  
under  $z^{M-n}$  is  $\equiv k$   
under  $h[M-n]$  is  $\equiv k$

$$= z^{-M} \sum_{k=0}^M h[k] z^k$$

$$\Rightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

for type II: *odd*

$$H(-1) = (-1)^M H(-1) = -H(-1) \Rightarrow \boxed{H(-1) = 0}$$



similarly, can show for

⑧

type III, IV

$$H(z) = -z^{-M} H(z^{-1})$$

$$H(1) = 0 \rightarrow \text{Never low-pass}$$

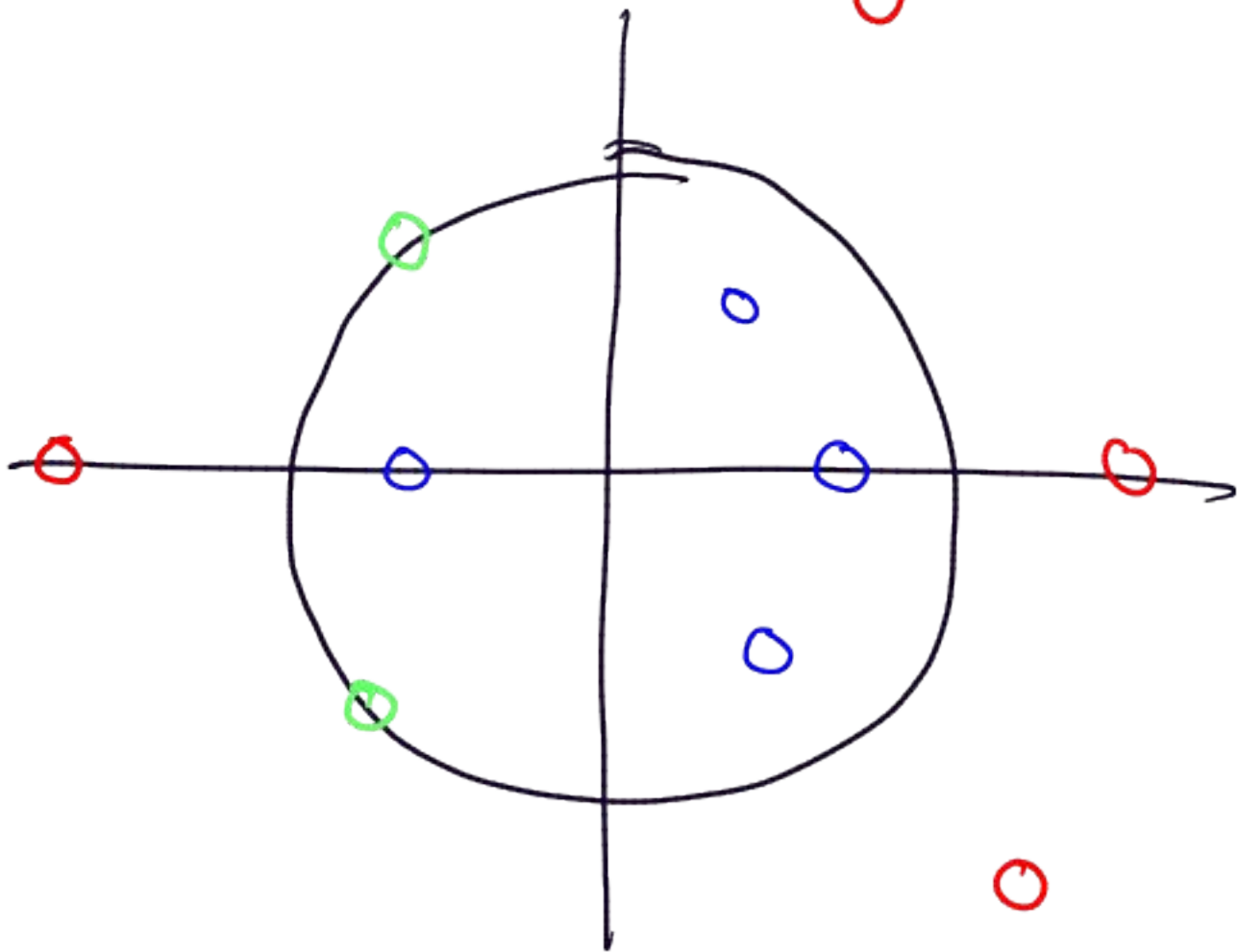
for type III

$$H(-1) = 0$$

only band pass

# Relation of FIR GHP to min-phase systems

(9)



$$H(z) = H_{\min}(z) H_{\max}(z) H_{\text{all}}(z)$$

$\uparrow$                        $\uparrow$   
minimum phase      maximum phase

10

There is much more to  
learn, ... but...

This is

The END

(for now)

# What's Next:

(11)

EE 121 → digital communication

EE 46 → Prob. & Random processes.

EE 127A → optimization

EE 145B → Image Proc. & tomography

## More advanced

EE 225A (EE 123 + EE 46) + +

EE 225B // ⇒ 2D +

CS 270 Vision

EE 224A, 226A, 227A, 229, CS 281A + B

EE 225E → Principles of MRI



# Principles of Magnetic Resonance Imaging EEc225E / BIOEc265



MRI has revolutionized diagnostic medicine.  
Ever wondered how it really works?

Spring 2012

TT, 2-3:30 Cory 299

Prerequisites: Either EE 120 or BioEC165/EI Eng C145B

Units: 3

Instructor: Michael (Miki) Lustig

Magnetic Resonance Imaging (MRI) is a non-invasive imaging modality. Unlike Computed Tomography (CT) that uses x-ray, MRI does not use any ionizing radiation and is considered safe. MRI provides a large number of flexible contrast parameters, which give excellent soft-tissue contrast.

The class will cover:

### Fundamentals of MRI:

- Multi-dimensional Fourier Transforms and linear systems
- Nuclear Magnetic Resonance Physics
- Imaging Sequences
- Contrast Generation
- Image reconstruction
- MRI Hardware and Software
- Imaging tradeoffs and image artifacts

### Advanced Topics:

- Rapid imaging
- Parallel Imaging
- Emerging research opportunities (High-field, dynamic imaging, functional imaging, hyperpolarization, compressed sensing)

Class includes hands-on Matlab labs for sequence design and reconstruction, tour to an MRI facility and guest lecture by a radiologist.

\* This year, for the first time students will perform imaging experiments on a physical earth-field MRI system

