## EE128 Homework \#2

Due on 9/22/04 (before the lab session)
(1) The following state equation describes the dynamics of a coupled two-tank system. The variable $\mathrm{q}, \mathrm{h} 1$, and h 2 represent, respectively, the incoming flow rate into the first tank, water level of the first tank, and the water level of the $2^{\text {nd }}$ tank. (This system is as shown in Figure 2.54 but without the hole A.)
(1.a) For $\mathrm{q}=1$, find the equilibrium state
(1.b) Linearize the system at the equilibrium state.
(1.c) Find the transfer function between $q$ and $h_{2}$ base on the linearized equation.

$$
\begin{aligned}
& \dot{h}_{1}=q-\sqrt{h_{1}-h_{2}} \\
& \dot{h}_{2}=\sqrt{h_{1}-h_{2}}-0.5 \sqrt{h_{2}}
\end{aligned}
$$

(2) The following state equation describes the dynamics of the population of foxes and rabbits in an idealized ecosystem where 1 is a normalized number.

$$
\begin{aligned}
& \dot{f}=f(r-1) \\
& \dot{r}=r(1-f)
\end{aligned}
$$

(2.a) Find all the equilibrium states of the system.
(2.b) Linearize the system about each of the equilibrium states found in (2.a)
(2.c) Use Simulink to plot the changing of the foxes and rabbits population for the following 3 initial conditions. You may turn in computer printout of the responses or hand sketch the responses.

$$
\left[\begin{array}{l}
f \\
r
\end{array}\right]=\left[\begin{array}{l}
1 \\
0
\end{array}\right], \quad\left[\begin{array}{l}
f \\
r
\end{array}\right]=\left[\begin{array}{l}
0 \\
1
\end{array}\right], \quad\left[\begin{array}{l}
f \\
r
\end{array}\right]=\left[\begin{array}{l}
2 \\
1
\end{array}\right]
$$

(3) Textbook problem 2.29
(4) Textbook problem 3.20 (b)
(5) Find the following transfer functions: $\mathrm{Y}(\mathrm{s}) / \mathrm{U}(\mathrm{s}), \mathrm{Y}(\mathrm{s}) / \mathrm{N}(\mathrm{s})$ and $\mathrm{E}(\mathrm{s}) / \mathrm{N}(\mathrm{s})$ of the system described by the block diagram on the right.

(6) Consider the following suspension system. Assume $u(t)=0$, i.e., the bottom part is kept stationary.
(6.a) Find the transfer function from the force (f) to the displacement of the mass (x).
(6.b) For mass $=500 \mathrm{~kg}$, determine the value of the spring K and the damper B such that with a step input $f(\mathrm{t})$, the step response of the system $(\mathrm{x}(\mathrm{t}))$ settles down within $1 \%$ of the final value in 1 second and with $5 \%$ overshoot.
(6.c) What is the final $x(t)$ value if a 50 kg object is placed on top of the mass? (Hint: This is the DC gain of the system.)
(6.d) Does the overshoot tend to increase or decrease if the mass increases? Explain why.
(7) Consider the system in (6)
(7.a) Find the transfer function from $\mathrm{u}(\mathrm{t})$ to $\mathrm{x}(\mathrm{t})$. (Hint: $u(t) \rightarrow U(s)$ and $\dot{u}(t) \rightarrow s U(s))$

(7.b) With the value of $K$ and $B$ found in (6.b), what is the percentage overshoot of the step response of the system from $u(t)$ to $x(t)$ ? (You may use Matlab to plot the step response and find the overshoot by inspection.)

