5.6. *Multiple poles at the origin.* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2(s+8)}$.

(b) $L(s) = \dfrac{1}{s^3(s+8)}$.

(c) $L(s) = \dfrac{1}{s^4(s+8)}$.

(d) $L(s) = \dfrac{s+3}{s^2(s+8)}$.

(e) $L(s) = \dfrac{s+3}{s^3(s+4)}$.

(f) $L(s) = \dfrac{(s+1)^2}{s^3(s+4)}$.

(g) $L(s) = \dfrac{(s+1)^2}{s^3(s+10)^2}$.

5.7. *Mixed real and complex poles.* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{s+2}{s(s+10)(s^2+2s+2)}$.

(b) $L(s) = \dfrac{s+2}{s^2(s+10)(s^2+6s+25)}$.

(c) $L(s) = \dfrac{(s+2)^2}{s^2(s+10)(s^2+6s+25)}$.

(d) $L(s) = \dfrac{(s+2)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)}$.

(e) $L(s) = \dfrac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$.

5.10. A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}$$

and the characteristic equation $1 + D(s)G(s) = 0$. Let $D(s) = k_p$ at first.

(a) Compute the departure and arrival angles at the complex poles and zeros.

(b) Sketch the root locus for this system for parameter $K = 9.8k_p$. Use axes $-4 \le x \le 4$, $-3 \le y \le 3$;

(c) Verify your answer using MATLAB. Use the command axes ([-4 4 −3 3]) to get the right scales.

(d) Suggest a practical (at least as many poles as zeros) alternative compensation $D(s)$ which will at least result in a stable system.

5.23. For the feedback system shown in Fig. 5.70, find the value of the gain K that results in dominant closed-loop poles with a damping ratio $\zeta = 0.5$.

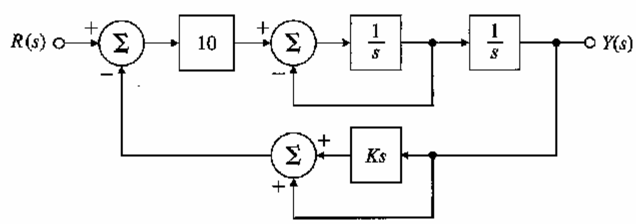

Figure 5.70
Feedback system for Problem 5.23

5.26. Suppose the unity feedback system of Fig. 5.71 has an open-loop plant given by $G(s) = 1/s^2$. Design a lead compensation $D(s) = K\frac{s+z}{s+p}$ to be added in series with the plant so that the dominant poles of the closed-loop system are located at $s = -2 \pm 2j$.

5.31. An elementary magnetic suspension scheme is depicted in Fig. 5.72. For small motions near the reference position, the voltage e on the photo detector is related to the ball displacement x (in meters) by $e = 100x$. The upward force (in newtons) on the ball caused by the current i (in amperes) may be approximated by $f = 0.5i + 20x$. The mass of the ball is 20 g, and the gravitational force is 9.8 N/kg. The power amplifier is a voltage-to-current device with an output (in amperes) of $i = u + V_0$.

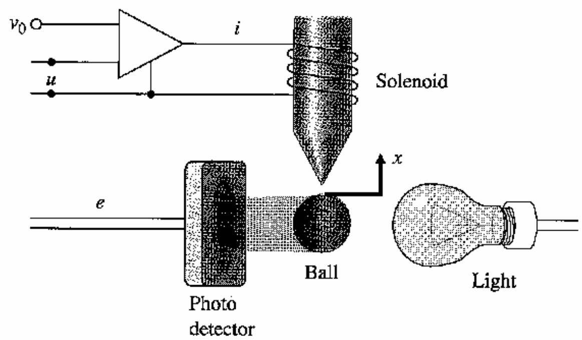

Figure 5.72 Elementary magnetic suspension

(a) Write the equations of motion for this setup.

(b) Give the value of the bias V_0 that results in the ball being in equilibrium at $x = 0$.

(c) What is the transfer function from u to e?

(d) Suppose the control input u is given by $u = -Ke$. Sketch the root locus of the closed-loop system as a function of K.

(e) Assume that a lead compensation is available in the form $\frac{U}{E} = D(s) = K\frac{s+z}{s+p}$. Give values of K, z, and p that yields improved performance over the one proposed in part (d).