Problems for Section 2.6

2.29. Figure 2.55 shows a simple pendulum system in which a cord is wrapped around a fixed cylinder. The motion of the system that results is described by the differential equation

$$(l + R\theta)\ddot{\theta} + g \sin\theta + R\dot{\theta}^2 = 0,$$

where

l = length of the cord in the vertical (down) position,

R = radius of the cylinder.

(a) Write the state-variable equations for this system.

(b) Linearize the equation around the point $\theta = 0$, and show that for small values of θ the system equation reduces to an equation for a simple pendulum, that is,

$$\ddot{\theta} + (g/l)\theta = 0.$$

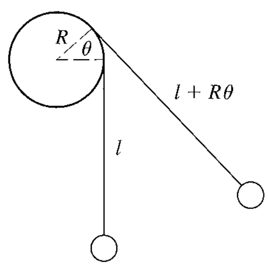

Figure 2.55
Motion of cord wrapped around a fixed cylinder

2.30. A schematic for the satellite and scientific probe for the Gravity Probe-B (GP-B) experiment is sketched in Fig. 2.56. Assume that the mass of the spacecraft plus helium tank, m_1, is 2000 kg and that the mass of the probe, m_2, is 1000 kg. A rotor will float inside of the probe and will be forced to follow the probe with a capacitive forcing mechanism; however, this will have no effect on m_2. The spring constant of the coupling, k, is 3.2×10^6. The viscous damping, b, is 4.6×10^3.

(a) Write the equations of motion for the system consisting of masses m_1 and m_2 using the inertial position variables, y_1 and y_2.

(b) The actual disturbance, u, is a micrometerorite and the resulting motion is very small. Therefore, rewrite your equations with the scaled variables $z_1 = \frac{y_1}{10^6}$, $z_2 = \frac{y_2}{10^6}$, and $v = 1000u$.

(c) Put the equations in state-variable form using the state $\mathbf{x} = [z_1 \quad \dot{z}_1 \quad z_2 \quad \dot{z}_2]^T$, the output $y = z_2$, and the input an impulse, $u = 10^{-3}\delta(t)$ N·sec on mass m_1.

(d) Using the numerical values, enter the equations of motion into MATLAB in the form

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u, \qquad (2.131)$$

$$y = \mathbf{H}\mathbf{x} + Ju \qquad (2.132)$$