1. OMP Box (Summer 2017 Final)

Your friend who goes to the school on the other side of the bay says that they have several specialized functions and that they need your help to put together an OMP box. You gladly agree to help because you learned a lot of OMP in EE16A. Your friend tells you that there are 100 devices communicating to the OMP box. Their songs are $\vec{s}_0, \vec{s}_1, \dots, \vec{s}_{99} \in \mathbb{R}^{40}$, and **they are all normalized**, i.e., $\|\vec{s}_i\| = 1$ for all $i \in \{0, 1, \dots, 99\}$. At any given time, at most m of them are transmitting simultaneously. We denote their 'messages' as $a_1, a_2, \dots, a_{99}$. If a device, say the i^{th} device, wants to send the message a_i, then it sends $a_i\vec{s}_i$. The OMP box gets a combination of circularly shifted versions of the signals transmitted by these devices and we denote the received signal at the OMP box by $\vec{r}$.

Note: Assume that vectors are zero-indexed, i.e., $\vec{v} = \begin{bmatrix} v[0] \\ v[1] \\ \vdots \\ v[N-1] \end{bmatrix}$.

(a) If the song length is 40 and the number of devices is 100, can the unshifted versions of the songs be perfectly orthogonal to each other? Explain why or why not.

(b) Consider the case where only one device is transmitting. In order to identify this single transmitting device, how many inner products do you need to calculate? Describe what they are.

In the next three parts, you may be writing pseudocode. To do this, you may do the following:

- You may implement helper functions and use them repeatedly in your solution.
- You may define your solution from one part as a function and use that function in a later part.
- In your pseudocode, you may use iteration (`for` and `while` loops).
- Acceptable pseudocode formats:

```
corrVec = corr(v⃗1, v⃗2)            corrVec is corr(v⃗1, v⃗2)
peakId, peakVal = pd(corrVec)     peakId and peakVal returned from pd(corrVec)
bhat = dot(A, lsq(A, b⃗))          bhat = A dot lsq(A, b⃗))
for i in range(10):               for i in (0, 1, ..., 9):
    vecaug(v⃗, i)                      vecaug(v⃗, i)
```

- The functions that you may use:

Name	Inputs	Outputs	Function Description
Correlation `corr`	$\vec{v}_1$ and $\vec{v}_2$	$\mathbf{C}_{\vec{v}_2}^T \vec{v}_1$	Calculates the circular correlation of the $\vec{v}_1$ with $\vec{v}_2$
Peak detection `pd`	$\vec{v}$	$i \leftarrow$ peak index $v[i] \leftarrow$ value of $\vec{v}$ at index i	Outputs the index at which peak occurs as well as the peak value
Circular rotation `circrot`	$\vec{v}$, i	$\vec{v}^{(i)}$	Outputs the vector circulated rotated by i
Vector subtraction `sub`	$\vec{v}_1$ and $\vec{v}_2$	$\vec{v}_1 - \vec{v}_2$	Subtracts two vectors in the given order
Matrix multiplication `dot`	$\mathbf{A}$ and $\vec{v}$	$\mathbf{A}\vec{v}$	Matrix vector multiplication
Least squares `lsq`	$\mathbf{A}$ and $\vec{b}$	$\vec{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\vec{b}$	Least squares solution to the equation $\mathbf{A}\vec{x} = \vec{b}$
Create vector `vec`	$a_0, a_1, \ldots, a_{N-1}$	$\vec{a} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N-1} \end{bmatrix}$	Creates a vector of length N
Vector augment `vecaug`	$\vec{v}$ and i	$\begin{bmatrix} \vert \\ \vec{v} \\ \vert \\ i \end{bmatrix}$	Augments the vector $\vec{v}$ with the new element i
Set augment `setaug`	S and i	$S = S \bigcup \{i\}$	Augments the set S with the new element i
Matrix augment `mataug`	$\mathbf{A}$ and $\vec{v}$	$\left[\begin{array}{c\|c} \mathbf{A} & \vec{v} \end{array} \right]$	Augments matrix $\mathbf{A}$ with column vector $\vec{v}$
Threshold reached `thres`	$\vec{v}$ and threshold k	$I = \mathbb{I}\{\\|\vec{v}\\| > k\}$	Outputs 1 if norm of the vector is greater than the threshold k and 0 otherwise

Table 1: Table of functions available.

(c) Suppose that $m = 1$, i.e., you know that at most one device is transmitting at any given time. Write pseudocode using the functions given to you that will output which of the devices was transmitting and an estimate for what it was transmitting. The inputs to your algorithm are the received signal vector $\vec{r}$ and the dictionary of songs $\mathscr{S} = \{\vec{s}_0, \vec{s}_1, \ldots, \vec{s}_{99}\}$. The outputs of your algorithm are the index of the transmitting device i, the shift that the corresponding song experienced N_i, and the message estimate $\hat{a}_i$.

(d) Suppose that $m = 4$, i.e., you know that at most four devices are transmitting at any given time. However, you are only interested in the 'loudest' device, i.e., the device with the largest $\|a_i\|$. Would the pseudocode from part (c) return the best estimate for the 'loudest' device? Explain why or why not.

(e) Let $m = 4$ and suppose that you are interested in the two 'loudest' devices. Write pseudocode using the functions given to you that will output the indices of the two 'loudest' devices and estimates what the two 'loudest' devices were transmitting. The inputs to your algorithm are the received signal vector $\vec{r}$ and the dictionary of songs $\mathscr{S} = \{\vec{s}_0, \vec{s}_1, \ldots, \vec{s}_{99}\}$. The outputs of your algorithm are the indices of the two loudest devices $\{i_1, i_2\}$, the shifts that the respective songs experienced $\{N_{i_1}, N_{i_2}\}$, and the message estimates $\{\hat{a}_{i_1}, \hat{a}_{i_2}\}$.