**1. Mechanical Gram-Schmidt (Fall 2016 Final)**

(a) Use Gram-Schmidt to find an orthonormal basis for the following three vectors.

$$\vec{v}_1 = \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} \sqrt{2} \\ 0 \\ -\sqrt{2} \end{bmatrix}$$

(b) Express $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ as vectors in the basis you found in part (a).

**2. Gram-Schmidt Properties**

(a) If we are performing the Gram-Schmidt method on a set of vectors, does the order in which we take the vectors matter? Consider the following set of vectors.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Perform Gram-Schmidt on these vectors first in the order $\vec{v}_1,\ \vec{v}_2,\ \vec{v}_3$ and then in the order $\vec{v}_3,\ \vec{v}_2,\ \vec{v}_1$. Do you get the same answer?

(b) What happens when we perform Gram-Schmidt on a set of $n$ vectors $\{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n\}$, where only $n-1$ of them are linearly independent?

**3. Orthonormal Projections**

(a) Suppose the $n \times m$ matrix $\mathbf{A}$ has linearly indpendent columns. The vector $\vec{y}$ in $\mathbb{R}^n$ is not in the subspace spanned by the columns of $\mathbf{A}$. Show that the projection of $\vec{y}$ onto the subspace spanned by the columns of $\mathbf{A}$ is $\mathbf{A}(\mathbf{A}^{\mathbf{T}}\mathbf{A})^{-\mathbf{1}}\mathbf{A}^{\mathbf{T}}\vec{y}$.

(b) Now suppose we perform gram schmidt on $\mathbf{A}$ to get a new matrix $\mathbf{Q}$. Show that the projection of $\vec{y}$ onto the subspace spanned by the columns of $\mathbf{Q}$ is now $\mathbf{Q}\mathbf{Q}^{\mathbf{T}}\vec{y}$.