1. **Eigenvalues and Special Matrices – Visualization**

   The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

   (a) Does the identity matrix in $\mathbb{R}^n$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

   (b) Does a diagonal matrix $\begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}$ in $\mathbb{R}^n$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

   (c) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (d) Does a reflection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (e) If a matrix $\mathbf{M}$ has an eigenvalue $\lambda = 0$, what does this say about its null space? What does this say about the solutions of the system of linear equations $\mathbf{M}\vec{x} = \vec{b}$?

   (f) Does the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

   Hint: What is the rank of the matrix?

2. Steady State Reservoir Levels

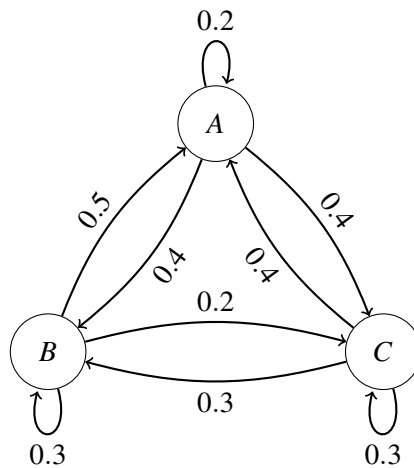We have 3 reservoirs: A, B and C. The pumps system between the reservoirs is depicted in Figure 1.

Figure 1: Reservoir pumps system.

(a) Write out the transition matrix representing the pumps system.

(b) Assuming that you start the pumps with the water levels of the reservoirs at $A_0 = 129, B_0 = 109, C_0 = 0$ (in kiloliters), what would be the steady state water levels (in kiloliters) according to the pumps system described above?

Hint: If $\vec{x}_{ss} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix}$ is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs A, B and C with A_{ss}, B_{ss} and C_{ss} kiloliters of water, respectively, and apply the pumps once?

Hint II: Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.