

Acknowledgements

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The students in the Sp15 pilot offering took detailed lecture notes and wrote them in LaTeX and those formed the basis for much of these official course notes (that continue to evolve). The other instructors in the pilot, namely Babak Ayazifar, Claire Tomlin, and Vivek Subhramaniam also had considerable influence on the ideas here. Furthermore, the faculty who helped put together the ideas for the labs, like Miki Lustig and Laura Waller (imaging) and Tom Courtade (locationing), and Michel Maharbiz (spirit), helped shape the overall cadence of the course that these readings are meant to complement. The undergrads who worked tirelessly over the summer (2015) after the pilot shaped these notes considerably, with particular credit going to Chenyang Yuan, Rachel Zhang, and Siddharth Karamcheti.

The notes really took on much of their current form in the first at-scale semester of Fa15. Anant Sahai took the lead on these notes, with a lot of input from Chenyang Yuan. The other members of the TA group that semester have also contributed. In Sp16, Jennifer Shih worked closely with Anant Sahai and the then 16A instructors Elad Alon and Babak Ayazifar to get these notes actually released. In Su17, Dominic Labanowski, Sajjad Moazeni, Nick Sutardja, Angie Wang, and Amy Whitcombe worked with Elad Alon and Anant Sahai to further develop the 16AB course material. A major new and radically different freshman course doesn't just happen — lots of people in the EECS department worked together for years to make all this a reality.

¹Proper credit here also acknowledges the strong cultural influence of the faculty team that made EECS 70 and shaped it into a required course: Alistair Sinclair, Satish Rao, David Tse, Umesh Vazirani, Christos Papadimitriou, David Wagner, and Anant Sahai, along with pioneering TA's like Thomas Vidick, all helped make and polish the notes for 70. The 16AB+70 sequence all follows the aesthetic that mathematical ideas should have practical punchlines. 16AB emphasizes the rooting of mathematical intuition in physical examples as well as virtual ones.

0.1 Welcome to 16A!

In this class, you will learn about “Designing Information Devices and Systems.” Why are “information devices and systems” important? If this were the 17th century, medical imaging would be something like this:



Figure 1: Rembrandt, “The Anatomy Lesson of Dr. Nicolaes Tulp,” 1632 (with modifications).²

The only way to see inside of the human body was to actually *see* inside of the human body. Thankfully, today non-invasive imaging tools such as CT scanners, X-ray machines, and ultrasound devices exist that can provide this information. In other words, we have *devices*, such as imagers, that provide *information*, such as a visualization of the human body. Often, these devices don’t work alone — they are part of a larger *system* that uses a combination of both physical sensors and signal processing techniques. Today, these “information devices and systems” are everywhere. For instance, smartphones take user input from touchscreens, microphones, accelerometers, and GPS to let you search the web, share pictures, communicate with others, and tell you where you are.

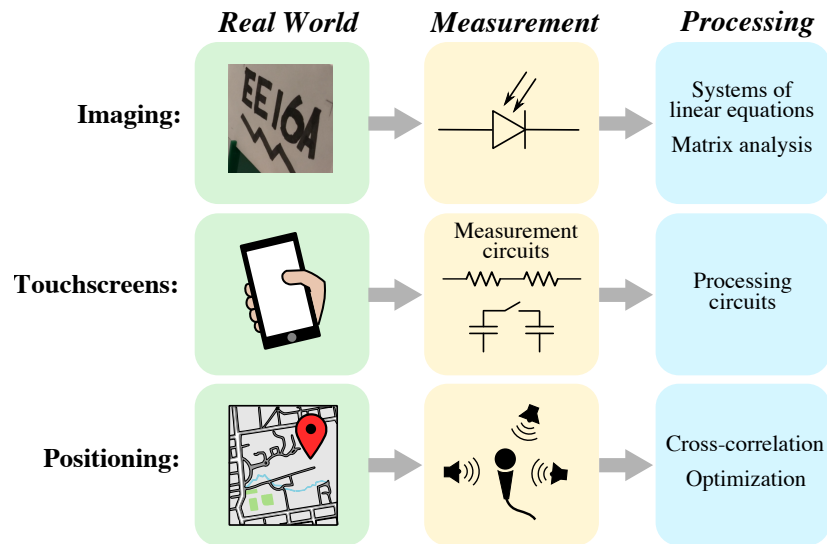
The goal of this class isn’t just to teach you the basics of linear algebra or the basics of circuits, but to help show you how those concepts fit together to enable the complex devices we use today. Of course, doing this will require teaching both basic circuits and linear algebra concepts. Without circuits to interface with the real world, these devices wouldn’t be very useful, and without processing techniques, we wouldn’t be able to extract meaningful information from circuit measurements. Our goal in 16A is to make it clear to you that all of the concepts you will learn about have a direct real-world application (and, in many cases, a multitude of applications). In order to put these concepts together to build a working system, we need to understand design thinking.

²Wikimedia Commons, “File:Rembrandt Harmensz. van Rijn 007.jpg — Wikimedia Commons, the free media repository,” 2016, https://commons.wikimedia.org/wiki/File:Rembrandt_Harmensz._van_Rijn_007.jpg.

0.2 System design

As engineers, we want to solve meaningful problems. Usually, this requires building a system that can **sense** something about the real world, **compute** something useful, and then **communicate** relevant information or **act** on this information to change the environment. In 16A, we will mostly focus on the first two problems — sensing and computing — in the context of three applications that you’ll investigate in lab: imaging, touch sensing, and positioning. 16B builds on the knowledge developed in 16A to explore how we can use the information we compute to interact with the environment.

In general, information devices and systems require (1) a real-world quantity to measure, (2) a means of translating this quantity to the electrical domain,³ and (3) some computation to extract information from the electrical readings. We can represent this flow graphically for the three applications studied in this course:



In an imaging system, our goal is to get a picture of something physical. In lab, you’ll be using a component called a photodiode to convert visible light into a measurable electrical quantity known as current. Modern cameras have millions of these photodiodes — one for each image pixel — but in lab, we’ll explore how to use linear algebra techniques to build an image with only *one* photodiode.

The goal of a touchscreen is to determine the position of one or more fingers from a user. In the second module of this class, you’ll be exploring a few different ways that touchscreens can be built. In one approach, you will use a circuit called a resistive voltage divider to translate the position of a touch into a voltage. In the second approach, we’ll use the fact that the presence or absence of human touch can influence an electrical quantity called capacitance, which is a principle used in most touchscreens that you interact with today.

In a positioning system, the goal is to find your location in a broader physical space. In lab, we will use a microphone to detect sounds from three speakers at fixed locations — translating information to the electrical domain — and then use optimization techniques to compute the microphone’s location from these readings. In real-world GPS systems, these speakers are replaced by satellites at orbiting the earth at known positions.

³Why do we care about electrical signals specifically? First, pretty much any physical quantity that we care about can be translated into a measurable electrical quantity. Second, most processing techniques can be implemented efficiently with computers, which operate on electrical signals. Today (and in the foreseeable future) this computing is done with semiconductors, and thanks to “Moore’s law” — the exponential scaling of semiconductor feature sizes over time — more and more computational processing power has become available via integrated circuits.

0.3 Dealing with complexity

Modern information devices and systems often integrate many sub-systems which are quite complex even independently. For instance, a single smartphone typically has all three of the sub-systems we will study in this class — a camera, touchscreen, and GPS system — in addition to many other features (perhaps a fingerprint reader, voice control, etc.). While the focus of 16A will be on understanding the basics of these sub-systems, approaching even these simpler problems requires **multi-step thinking**.

In previous classes, you may have had homeworks that involved only plugging numbers into an equation to find your answer. In this class, problems will typically require multiple steps — finding an appropriate model for a real-world system, representing this with the right equations, solving this, interpreting the solution, and perhaps using the results to do even more analysis. Our motivation for doing this is to give you practice thinking in a way that can be applied to complex real-world problems. It may be difficult, but in the end you'll learn from it, and the course staff is here to help you along the way.

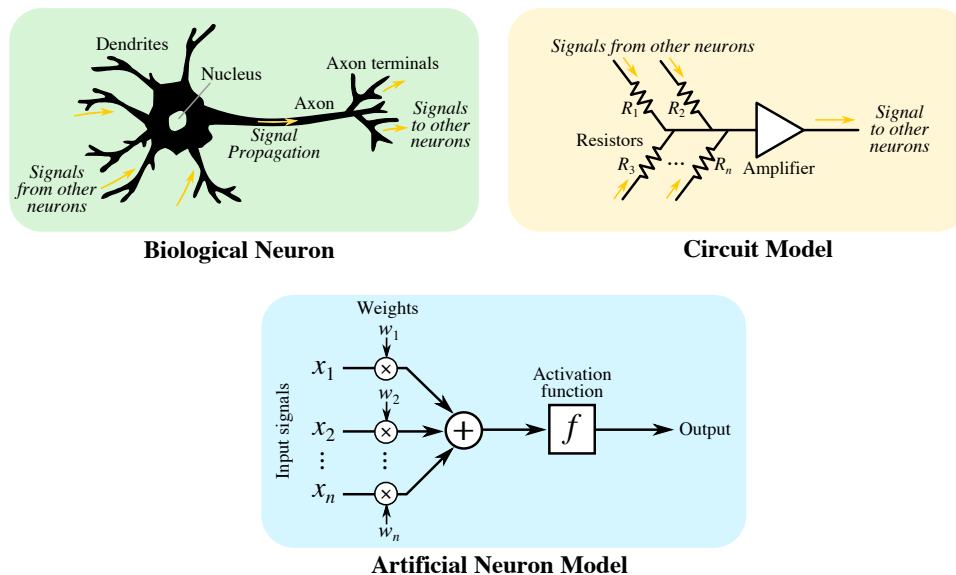
0.4 Why do I have to learn about circuits?

There are many reasons why learning circuits is useful, some of which will likely resonate more strongly with you than others. Here are a few major reasons to consider:

- **Applications:** Can you imagine how useful a smartphone would be without a touchscreen? What about finding your way without GPS tracking technology? Today, we don't think twice about being able to talk to someone halfway across the earth in realtime, but none of these things would be possible without innovations in circuits. Beyond the fact that all computer processing runs on integrated circuits, circuits shape the way we interact with electronic devices to help give them value — to actually interface with the real world, any device is going to require circuits. Moreover, it is difficult to truly create something new without a full understanding of how an actual engineering system is designed at both the hardware and software level. For instance, some companies like Apple practice “vertical integration,” where engineers work on nearly all aspects of a system, from the mechanical design of a product to the software down to the electrical hardware implementation. Without at least a foundational understanding of circuits, it would be difficult to create an innovative system, which is becoming increasingly relevant in industry with the growth of “internet of things” (IoT) based devices in recent years.
- **Understanding primitive design:** Building any type of complex system — be it software, mechanical, electrical, something else, or any combination of these — requires breaking it into a set of simpler sub-systems, which might consist of even more sub-systems. Introducing different layers of abstraction in this manner is an important tool for managing complexity, and circuit design is one of the most powerful ways of introducing you to this process. Just as software programs are typically built from many subroutines constructed from a set of primitive data types (strings, integers, floating-point numbers, lists, etc.), an electrical circuit typically consists of sub-circuits built from simple primitive building blocks (resistors, capacitors, voltage/current sources) that place constraints on each other.
- **Translation to other fields:** Most circuits can be expressed as a system of equations that can be solved using linear algebraic techniques, making circuit analysis just one example of how the linear algebra concepts you'll learn about in this class can be used. In fact, this mathematical approach to

circuit analysis ties directly to other fields of engineering, which can generally be characterized as the study of “effort” and “flow” variables — one variable that applies a force (“effort”), and another variable that moves in response to this (“flow”). What does that mean? In electrical circuit analysis, the “effort” variable is voltage, and the “flow” variable is current. In mechanical engineering, the “effort” variable could be a physical force on an object, and the corresponding “flow” variable would be the velocity of this object. In chemical engineering, “effort” is a chemical potential, while “flow” is molar flow, the amount of a substance that passes into a fixed area over time. While you’ll be learning about these ideas in the context of circuits specifically in 16A, they are relevant to many other applications. Even circuit components like resistors and capacitors have mechanical analogues — resistance behaves like a mechanical damping force such as friction, while capacitance is like the compliance of a spring.⁴

- **Relevance to neural networks:** Because circuit analysis translates to a wide range of fields, we can model many physical systems as electrical circuits, often gaining insight about the system. You may have heard of neural networks, an important machine learning tool that can be used to “learn” tasks such as image and voice recognition from examples instead of explicit programming. Neural networks are modeled after biological neural networks, which are fundamentally circuits operating on electrical signals within a brain:



In a general sense, studying circuits provides you with the tools needed to analyze such networks. More broadly, circuit concepts are relevant to understanding network analysis and signal flows in systems, which can be applied to areas ranging from transportation analysis to social network analysis.

- **Interdisciplinary thinking:** Our last example with neural networks brings up an important point: being exposed to a broad range of disciplines can help you come up with new ideas. In the case of neural networks, biological systems inspired a key modern machine learning tool. Understanding concepts outside of your field gives you a broader range of ideas to pull from when trying to develop innovative solutions.

⁴There’s a whole Wikipedia article on the matter if you want more details: https://en.wikipedia.org/wiki/Mechanical-electrical_analogies