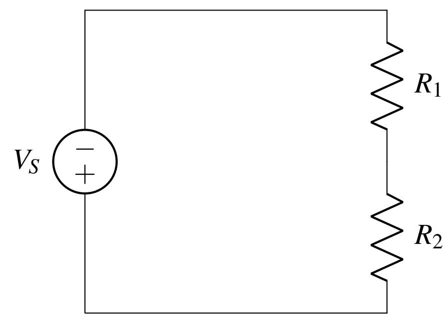
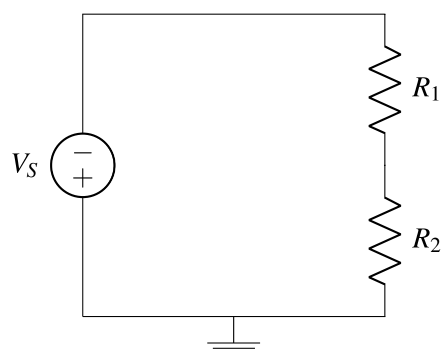
13.1 Analysis Review: Voltage Divider

In order to review the analysis procedure described in the previous note, let's look at a particular circuit named a "voltage divider". As we will see by the end of this note, the voltage divider circuit actually lies at the heart of the resistive touchscreen that we will be studying shortly.
The voltage divider circuit consists of a voltage source (V_s) and two resistors (R_1 and R_2).

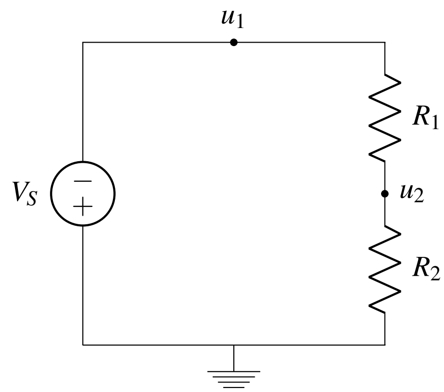


We next use the circuit analysis algorithm developed in the previous note toanalyze this circuit:

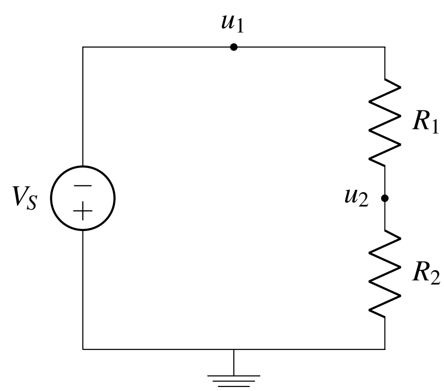
- **Step 1:** Pick a node and label it as ground.



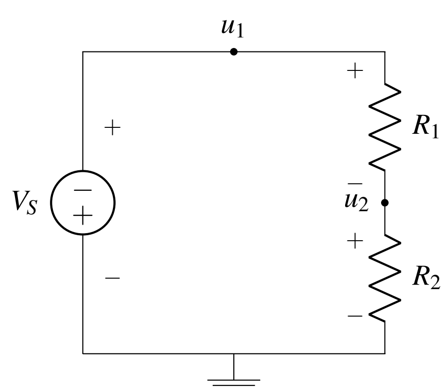
- **Step 2:** Label all remaining nodes as some u_i.



- **Step 3:** Label the current through every non-wire element in the circuit with i_n.



- **Step 4:** Add +/- labels on each non-wire element by following the passive sign convention.



- **Step 5:** Setup the relationship $\mathbf{A}\vec{x} = \vec{b}$:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

- **Step 6:** Use KCL to fill in as many linearly independent rows in $\mathbf{A}$ and $\vec{b}$ as possible. In this circuit, KCL gives us one equation for one of the junctions in the top corners and a second equation at the junction between the resistors:

$$i_1 = -i_2 \rightarrow i_1 + i_2 = 0 \tag{1}$$

$$i_1 = -i_3 \rightarrow i_1 + i_3 = 0 \tag{2}$$

These two equations can fill in two rows in the matrix $\mathbf{A}$ and the vector $\vec{b}$:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ ? \\ ? \\ ? \end{bmatrix}$$

- **Step 7:** Use the I-V relationships of each of the non-wire elements to fill in the remaining rows $\mathbf{A}$ and $\vec{b}$. The I-V relations give us three equations:

$$u_1 - 0 = V_s. \tag{3}$$

$$u_1 - u_2 = i_2 R_1 \rightarrow -i_2 R_1 + u_1 - u_2 = 0. \tag{4}$$

$$u_2 - 0 = i_3 R_2 \rightarrow -i_3 R_2 + u_2 = 0. \tag{5}$$

These three equations can fill in three rows in the matrix $\mathbf{A}$:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -R_1 & 0 & 1 & -1 \\ 0 & 0 & -R_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_s \\ 0 \\ 0 \end{bmatrix}$$

- **Step 8:** Solve the system of equations and determine values of unknown variables. This will result in:

$$i_1 = -\frac{V_s}{R_1 + R_2}$$

$$i_2 = \frac{V_s}{R_1 + R_2}$$

$$i_3 = \frac{V_s}{R_1 + R_2}$$

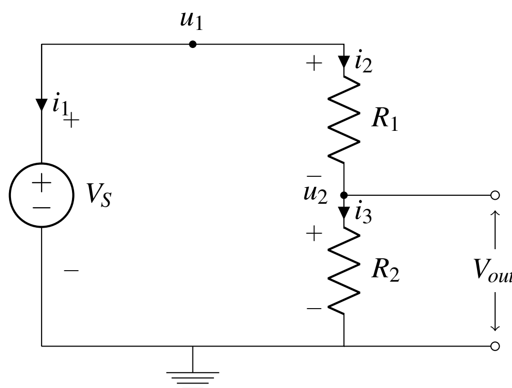
$$u_1 = V_s$$

$$u_2 = \frac{R_2}{R_1 + R_2} V_s$$

Now, if we use a voltmeter to measure the voltage across R_2, which is the same as the difference between u_2 and "0":

$$V_{out} = \frac{R_2}{R_1 + R_2} V_s = \frac{1}{1 + \frac{R1}{R2}} V_s$$

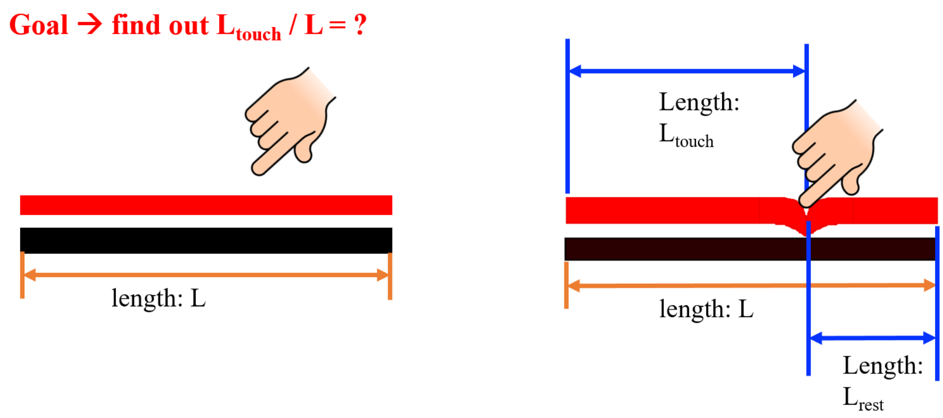
Notice that the reason this circuit is called a "voltage divider" is that we can create any output voltage of $V_{out} = \alpha V_S$ for any $\alpha \in [0, 1]$ (assuming that all of the resistance values are non-negative) by varying the **ratio** of the resistor values R_1/R_2. As we will see shortly, varying this ratio is exactly the mechanism we will use to convert the relative position of a user's touch to a voltage.



13.2 Resistive Touchscreen

In most real systems, an actual touchscreen will be two dimensional in that we would want two coordinates (horizontal and vertical) for the position of a finger press. For the sake of simplicity and building up our understanding in a step-by-step manner, in this note we will simplify the touchscreen in to a 1-D structure (i.e., we will detect only e.g. the vertical or horizontal position of a touch). Before going in to any modeling or analysis, let's first examine the physical structure of a such a 1-D touchscreen and identify the physical quantity that we will be somehow converting in to an electrical value and that is indicative of the position of the touch.

Viewing the cross-section of the 1-D touchscreen (as shown below), the structure consists of two layers: the top layer (red) is flexible, and the bottom layer (black) is non-flexible. Both layers are conductive so current can flow through them (i.e., the top and bottom layers are made from materials with some finite ρ). When we touch the screen, the top part contacts the bottom part at the touchpoint; for this simplifying example, we can assume that the top structure is narrow enough (in the dimension going in to the page) that the touch causes the entire width of the structure to deform uniformly and make contact to the bottom structure. Since our goal was to find the relative position of the touchpoint, it should hopefully be clear from the picture below that the physical quantity we are interested in is L_{touch} (relative to the total length L). In order to understand how we can measure this physical quantity using an electrical circuit, we need to introduce some basic physics that we can use to convert this physical structure in to an electrical model. Once we have that model we can connect some additional components around it to build a complete circuit which we can then analyze (using e.g. the procedure developed in the previous note). It is worth re-emphasizing here that each of these steps is looking at different levels of abstraction - the modeling step converts a physical system in to



an electrical model (as we will see in this particular case, a model that is comprised of wires and resistors), and after additional components are connected to that model to create a complete circuit, we can analyze that circuit without necessarily knowing more details than are captured in the model.

13.3 EE16A Physics

Charge is the basic underlying quantity associated with all electrical systems. Charge can be either positive or negative, although in most materials/systems charge is carried by electrons (which are negative charged). We measure charges with the unit **coulomb (C)**, and typically use the symbol **Q**.

Current is simply a measure of the movement of charge, specifically, the net amount of charge crossing through a surface in a unit time. We usually use the symbol **I** to denote current, and is defined by:

$$I = \frac{dQ}{dt}.$$

The unit for current is **ampere (A)**, which is equivalent to 1 coulomb per second - i.e., $1C = 1A * 1sec$. We typically specify the direction of the net positive charge flow using an arrow. For example, the following arrow

$$\xrightarrow{\quad 5A \quad}$$

represents that net $+5$ coulombs of charges are crossing the surface per second to the right. We could also say that net -5 coulombs of charges are crossing the surface per second to the left.

Voltage: Voltage is defined as the amount of energy needed to move a unit charge between two points. We usually denote voltage with symbol **V**. The unit associated with a voltage is a **Volt (V)**, where 1 Volt is defined such that it will require 1 Joule of energy to move 1 Coulomb (i.e. the unit of charge) between the two points (i.e. $1V = 1J/C$ or $1V * 1C = 1J$). It is important to emphasize that the definition of voltage makes it a relative quantity that can only be measured between two points; an "absolute" voltage is meaningless. As we saw in the analysis procedure, as a shorthand we will sometimes refer to a voltage at a particular point, but if we do so, we are implicitly using some other known point (often, the **ground** that we defined) as a reference against which the voltage at that single point is being measured.

A simple analogy to voltage and the fact that it is a relative quantity can be described using elevation. A mountain's summit could be 9000ft above sea level (where sea level would be the reference of an elevation of 0 ft). Alternatively, we can say that there is 9000ft between the summit and sea level. Instead, if we choose the bottom of the ocean as the reference point (elevation of 0 ft), then there will still be 9000ft between sea level and the summit of the mountain, but the elevation at the mountain referenced to the bottom of the ocean would not be 9000ft (it would be much greater). So, just as a sea level is an arbitrary (although perhaps convenient) reference level for elevation, the ground node we choose as "0V" to measure the rest of the voltage at the rest of the nodes in our circuit against is also an arbitrary reference point.

Resistance: All normal conductors (i.e., materials that allow current to flow through them - you can think of these as a piece of metal for now, although we define them more precisely shortly) require some amount of work to be done to get charge (current) to flow through them. The simplified explanation for this is that when electrons (charge) flow through the conductor, these electrons occasionally collide with the atoms in the conductor and cause the atoms to vibrate with more energy (i.e., generate heat). Since heat is of course just another form of energy, this process implies that energy must be spent to move the electrons through the conductor, and the amount of energy that is spent (per unit charge) is set by the resistance of the structure.

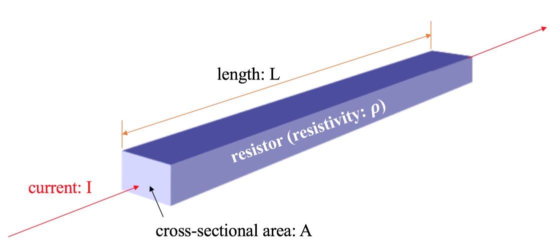
Since as stated previously a voltage captures the amount of energy needed to move a charge between two points, we can capture the behavior describe above by stating that when current flows through a resistor, there is also a voltage drop across that resistor. This statement is known as Ohm's law, which is:

$$V = IR$$

where the resistance R has units of Ohms Ω. Note that this means that 1 Ω = $\frac{1V}{1A}$

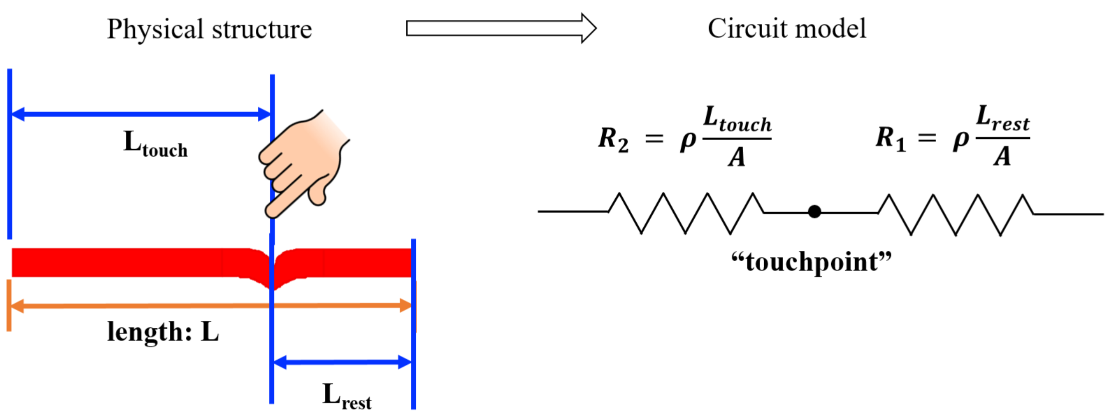
The actual value of resistance for some physical structure is set by only two things: (1) material properties, and (2) its dimensions. For (1), every conducting material has a property known as resistivity ρ; for example, gold has a lower resistivity than steel, and hence for the same physical dimensions, a gold structure will have lower resistance than a steel structure. For (2), let's consider how different dimensions may impact the resistance of a structure. If we increase the length L of the structure (i.e., if we increase the dimension of the structure in the same direction as the current flows through it), this will increase the number of atoms that the carriers would collide with, and hence we can expect the resistance to increase with L. Similarly, if we increase the cross-section area A of the structure (i.e., increase the dimension in directions perpendicular to the current flow), we are allowing more electrons to flow in parallel with each other, and hence we can expect the resistance to decrease with A. Putting all of these together, for a physical structure made out of a material with resistivity ρ and having length L and cross-sectional area A, its resistance is given by:

$$R = \rho \times \frac{L}{A}.$$

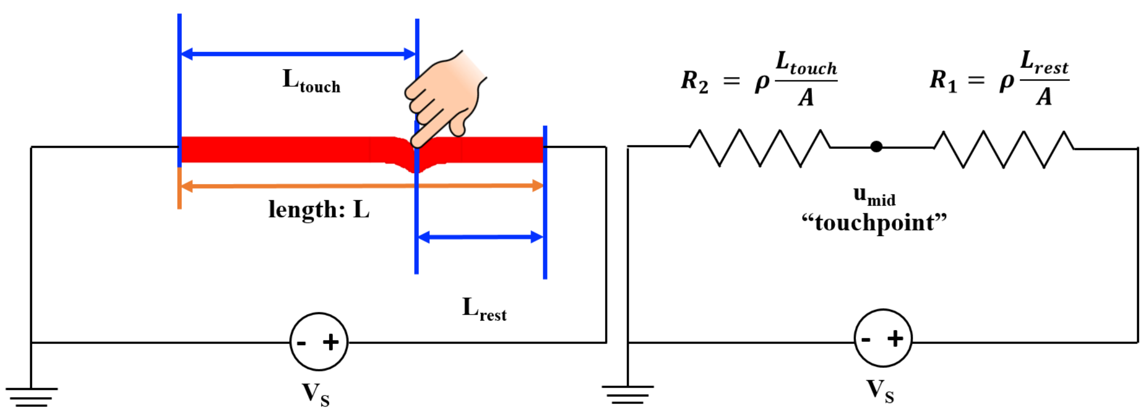


13.4 Resistive Touchscreen Revisited

Now that you've been introduced to some knowledge in circuits, let's review the actual physical structure of the top (red) layer in the 1-D touchscreen.
Since the touchpoint is where we are most interested in, we can divide the top (red) layer into two parts and see what happens at the touchpoint. Given that the top (red) layer has a resistivity ρ and a cross-sectional area A, the resistance of the top layer from the touchpoint to the right-hand end is given by $R_1 = \rho \frac{L_{rest}}{A}$, the resistance of the top layer from the left-hand end to the touchpoint is given by $R_2 = \rho \frac{L_{touch}}{A}$.



We then connect a voltage source V_s to the two ends of the circuit model we just built.



Actually, we have already analyzed this circuit model we just built at the beginning of the note, this is just a voltage divider circuit! First we know that,

$$u_{mid} = \frac{R_2}{R_1 + R_2} \times V_s. \tag{6}$$

Given $R_2 = \rho \frac{L_{touch}}{A}$ and $R_1 = \rho \frac{L_{rest}}{A}$:

$$u_{mid} = \frac{\rho \frac{L_{touch}}{A}}{\rho \frac{L_{rest}}{A} + \rho \frac{L_{touch}}{A}} \times V_s. \tag{7}$$

ρ and A are shared in the denominator and the numerator so they can cancel. Then u_{mid} becomes:

$$u_{mid} = \frac{L_{touch}}{L_{touch} + L_{rest}} \times V_s. \tag{8}$$

What is $L_{touch} + L_{rest}$? It is L, the physical length of the touchscreen.

$$u_{mid} = \frac{L_{touch}}{L} \times V_s. \tag{9}$$

By connecting a voltage source to the two ends of the touchscreen and measure u_{mid}, we can figure out the relative position of L_{touch}.

The relationship we have found between u_{mid} and V_s is very interesting. First, u_{mid} is not dependent on any material property such as ρ and A. The top layer can be built with any material and the relationship between u_{mid} and V_s is still valid. Since there are always some non-idealities in the world, by making u_{mid} independent of any material property, we can make the circuit model we built immune to such non-idealities. We also have the freedom to choose the material ideal for display purpose to build the top layer. This is the beauty of the circuit model we have just built for the touchscreen.

Another thing that has not been explained so far is the function of the bottom (black) layer in the touchscreen. In the following lectures, we will explain how to use the bottom (black) layer to measure u_{mid}.