# EECS 16A   Designing Information Devices and Systems I
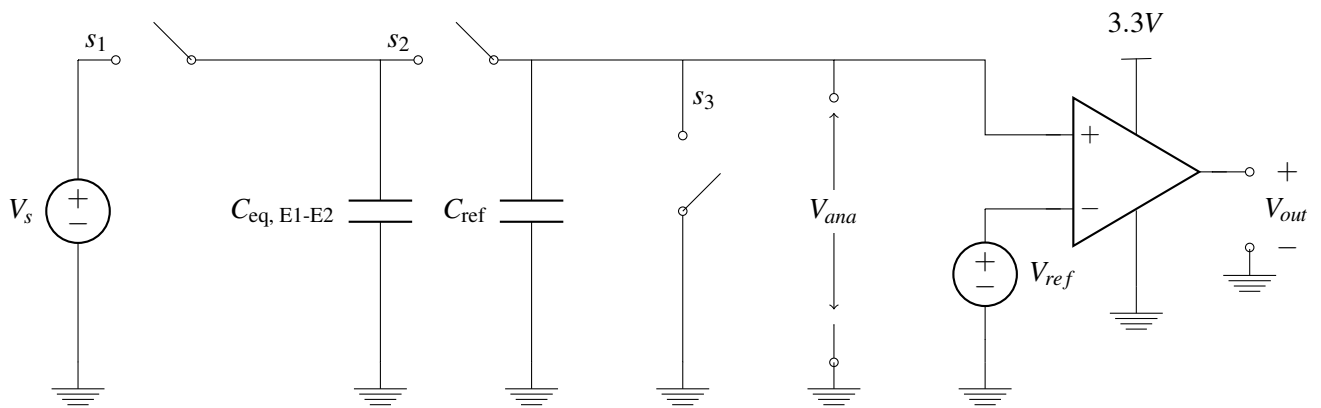# Fall 2017     Official Lecture Notes     Note 19- Draft

## 19.1   Comparator Review

Remember that the high level goal for a comparator is to take an analog voltage output and interface it with a digital signal processor, e.g. to read a voltage divider output using your Launchpad (in lab).

Examine the circuit below of a comparator connected to our capacitive touchscreen model:



Note some of the simplifications we have made in the model. For one, there are separate ground labels instead of a "bottom" wire connecting all the ground nodes. Functionally this is the same, but as circuits get more complicated drawing a large bottom wire can impair visual inspection. Also note that, instead of drawing a full voltage source symbol for the rail of the op-amp, we use a new symbol with a horizontal line and a voltage number. Again, this is functionally the same, but means we don't have to add potentially confusing additional elements to our model. If the label says $3.3V$, you can assume that it is the positive terminal of a voltage source with its negative terminal connected to the circuit's ground.

Refer to Note 18 for more details on the functionality of this circuit. But remember: If the analog voltage $V_{ana}$ is greater than $V_{ref}$, the output voltage $V_{out}$ will be $3.3V$. If $V_{ana} < V_{ref}$, $V_{out}$ will be $0V$. Recall for $V_{ana}$, we have:

$$V_{ana} = \frac{C_{\text{eq, E1 - E2}}}{C_{\text{eq, E1 - E2}} + C_{\text{ref}}} \times V_s.\tag{1}$$

When there is touching, $C_{\text{eq, E1 - E2}} = C_{\text{E1 - E2}} + \Delta C$ in which $\Delta C > 0$, we have:

$$V_{\text{ana, touch}} = \frac{C_{\text{E1-E2}} + \Delta C}{C_{\text{E1-E2}} + \Delta C + C_{ref}} \times V_s \tag{2}$$

When there is no touching:

$$V_{\text{ana, no-touch}} = \frac{C_{\text{E1-E2}}}{C_{\text{E1-E2}} + C_{ref}} \times V_s \tag{3}$$

For any finite capacitance $\Delta C > 0$, $V_{\text{ana, touch}}$ is always greater than $V_{\text{ana, no-touch}}$.

A second important question is that **how should we set up $C_{ref}$?**

The two open questions here are *how do you pick the value for $V_{ref}$* and *how do you pick the value for $C_{ref}$?* These two questions are intrinsically tied together. Typically they must be chosen together, where $C_{ref}$ can be picked almost arbitrarily and that choice subsequently drives our selection of $V_{ref}$. One constraint on $C_{ref}$ is that we don't make it much larger than $C_{eq}$. Recall that $V_{ana} = \frac{C_{\text{eq, E1 - E2}}}{C_{\text{eq, E1 - E2}} + C_{\text{ref}}} \times V_s$. We realize that $V_{ana}$ is associated with $C_{\text{ref}}$. As long as we do not set $C_{\text{ref}}$ to some value much greater (for example, 1000 times greater) than $C_{eq,E1-E2}$ so that $V_{ana}$ becomes too small (for our op-amp to pick up the signal), we can get some reasonable $V_{ana}$, which then allows us to set up $V_{ref}$ according to $V_{\text{ana, touch}}$ and $V_{\text{ana, no-touch}}$. Practically, we will set up one of $C_{ref}$, $V_{ref}$ first, and then set up the other value according to $V_{ref} = \frac{1}{2}(V_{\text{ana, touch}} - V_{\text{ana, no-touch}})$. It turns out that making $C_{ref}$ much larger than $C_{eq}$ will limit the "swing" in the voltage, giving us less tolerance for noise or variation (among other potential problems).

So if $C_{ref}$ is chosen almost arbitrarily, is there a "best" value to place $V_{ref}$ at? By best, we mean most robust to variation/noise. It turns out yes there is a "best" place, and this place really is exactly halfway in between the two points $V_{ana,touch}$ and $V_{ana,no-touch}$ as shown in the following graph:
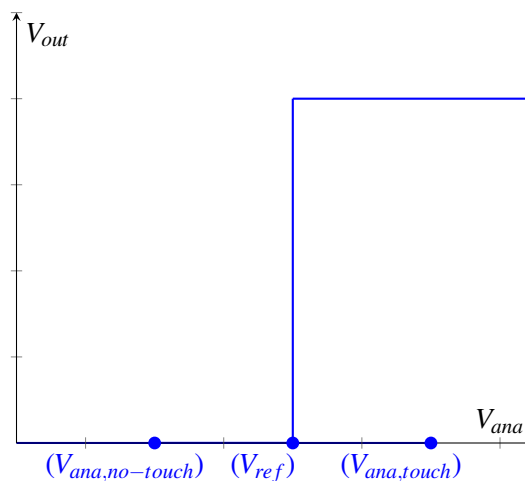
## 19.2 Design example – DAC

Your Launchpad has a component called a digital to analog converter (DAC) that translates digital signals into an output analog voltage. We'll use a simple in a simple design problem. Our goal for this design is to use a digital analog converter (DAC) to build a home audio system.



The DAC takes in digital bits and converts them into an analog signal. Then this signal is fed into a speaker, as illustrated on the right. The maximum voltage the DAC can produce is 3.3V, and the minimum voltage the DAC can produce is 0V. However, you want the input voltage to the speaker to be between 0V and 10V (in order to make the speaker loud enough). So somehow you want to be able to map voltages from 0 to 3.3V to voltages from 0 to 10V. Suppose that we can model the speaker as an 8Ω resistor connected to ground and we model the DAC with its Thevenin equivalent with voltage $V_{TH}$ and thevenin resistance $R_{TH} = 1k\Omega$. What if we connect the DAC and the speaker directly? We will have the following circuit
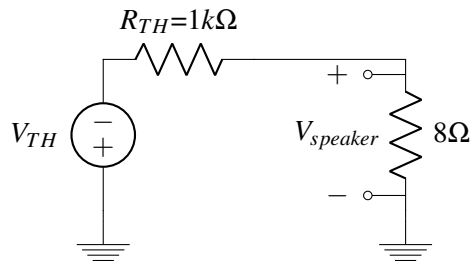


Now we can see that

$$V_{speaker} = \frac{8}{8+1000} \times V_{TH} \tag{4}$$

which is much smaller than $V_{TH}$. This definitely won't do what we would like.

What we really want in between the DAC and the speaker is something that provides a gain of roughly 3 (10/3.3) and acts as a buffer, i.e., that can measure the output voltage of the DAC, $V_{DAC}$, without affecting it, and for the output, we want to be able to draw any current the meet the spec. So we want something like the following
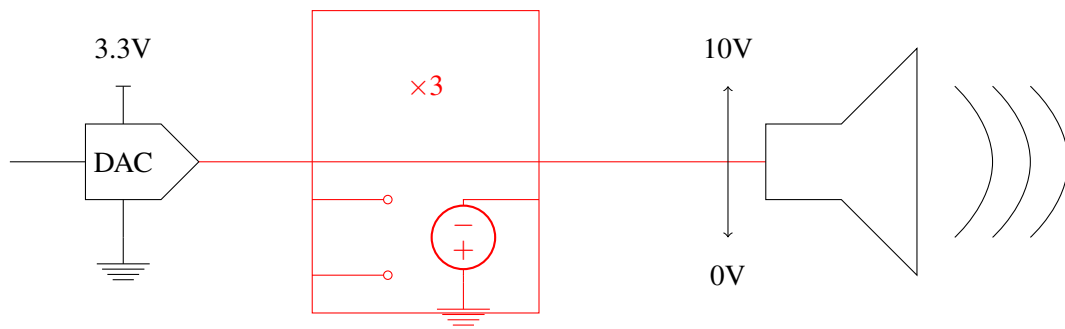
This looks just like the internal of an op amp! Now notice that based on what we know so far about op amps as comparators, we can't just scale the voltage linearly as we we would have wished: if we connect an ideal op amp (infinite internal gain) with $V_{DD} = 10V$ and $V_{SS} = 0V$, the output voltage would either be $10V$ or $0V$, but not something in between. To achieve what we would like, we need another tool, which we will introduce in the next section.

## 19.3 Negative feedback

Negative feedback is used just about everywhere, including electronics, biology, mechanics, robotics, and more. The basic idea is that we measure an output, compare it to where we "want to be " (i.e. some target output), then adjust the inputs to get closer to that desired output. Let's turn this high level description into a more mathematical one.

Concretely, we want to get a certain known gain out of our op amp. Currently we have an op-amp with some very large uncertain internal gain. We can describe this problem using a block diagram; a collection of drawing (mathematical in nature) that operate on quantities of interest using simplified representations.

Let's take a look at a generic block diagram for negative feedback systems.
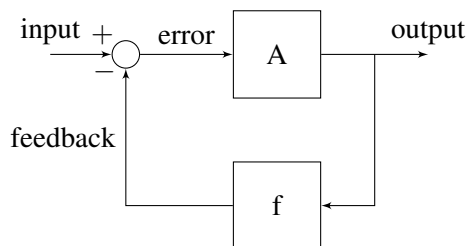


The idea is we take the difference between the input and a scaled version (multiplied by $f$) of the output, which we call feedback, and apply gain $A$ on it to again produce the output.

Observe that if the feedback signal for some system (e.g. noise) is increased), the error (input - feedback) signal will decrease (move down), which then causes the output to go down as well, i.e., the loop has ability to suppress the original change in the feedback signal!

Now we can kind of get an intuitive idea of how negative feedback can be useful. When we want to get a system to have a desired output, negative feedback loops can help re-adjust to the value of the desired output when theoutput is too high or too low relative to the target value.
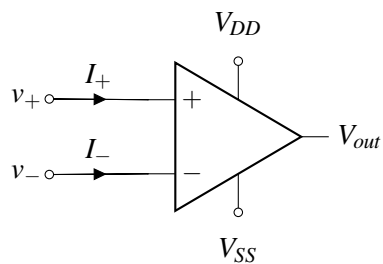
Now let's see how negative feedback loops can be realized in op amps. Consider the following circuit:
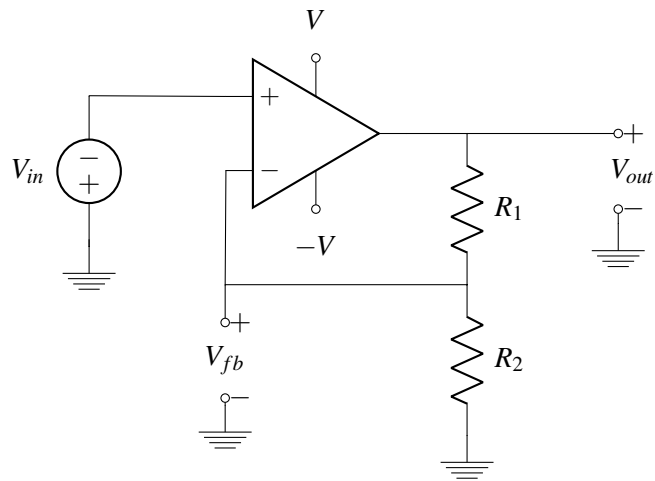


To help analyze circuits of this kind, we will introduce two "golden rules" that we could use to make our lives simpler.
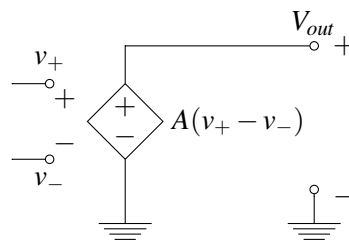
# 19.4   Golden Rules

Recalling the op amp symbol from previous lecture



For an ideal op amp, the "golden rules" are

- (1) $I_+ = I_- = 0$. Now let's think about why this is the case. Recall that the equivalent op amp circuit is

Notice that there is no closed circuit connected to the positive or negtiave input terminal of the op amp. Thus, no current can flow into the positive or negative input terminal. **Note that this rule holds regardless of whether there is negative feedback or not.**

- (2) $V_+ = V_-$: We will explain later why this is true, but intuitively this means that the "error signal" going into the op amp must be zero. One important thing to note now is this rule only holds when there is negative feedback.

$A \to \infty$ implies the Golden Rules. We will see this in homework.

Now let's use the golden rules to analyze the circuit we saw earlier:
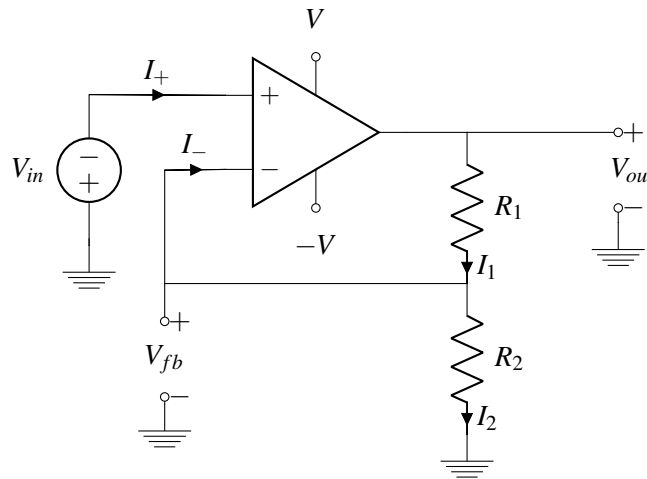


We know that by applying KCL at the junction between $R_1$ and $R_2$, we have

$$I_1 = I_2 + I_-. \tag{5}$$

By the first golden rule, we know that $I_+ = I_- = 0$. Hence,

$$I_1 = I_2. \tag{6}$$

Now let's apply the second golden rule, $V_+ = V_-$. Using this, we have

$$V_{in} = V_{fb}. \tag{7}$$

Now we can solve for $I_2$ using Ohm's law, $V_{fb} = I_2 R_2$, hence

$$I_2 = \frac{V_{fb}}{R_2}. \tag{8}$$

Using Ohm's law on $R_1$, we also have $V_{out} - V_{fb} = I_1 R_1$. Hence,

$$I_1 = \frac{V_{out} - V_{fb}}{R_1}. \tag{9}$$

However, we know that $I_1 = I_2$, which gives us the following relationship

$$I_1 = \frac{V_{out} - V_{fb}}{R_1} = \frac{V_{fb}}{R_2} = I_2, \tag{10}$$

which is equivalent to

$$I_1 = \frac{V_{out} - V_{in}}{R_1} = \frac{V_{in}}{R_2} = I_2. \tag{11}$$

Moving terms around, this gives us

$$V_{out} = V_{in}\left(1 + \frac{R_1}{R_2}\right). \tag{12}$$

Notice that here the ratio $\frac{V_{out}}{V_{in}}$ only depends on the ratio $\frac{R_1}{R_2}$. This is a great property since it is rather difficult to produce resistors with a particular absolute resistance. As long as the two resistors are produced with the same error rate $\varepsilon$, i.e., they have resistance $(1+\varepsilon)R_1$ and $(1+\varepsilon)R_2$, the ratio between their resistance will remain the same

$$\frac{(1+\varepsilon)R_1}{(1+\varepsilon)R_2} = \frac{R_1}{R_2}. \tag{13}$$

# 19.5 Second golden rule revisited

Recall that in the last section, we stated the second golden rule $V_+ = V_-$, i.e., the voltage at the positive input terminal (relative to ground) and the voltage at the negative input terminal (relative to the same ground) are the same when there is negative feedback. Now we would like to justify why this is the case. We return to the block diagram we drew earlier for a general negative feedback loop, but now focus on its application in circuits.
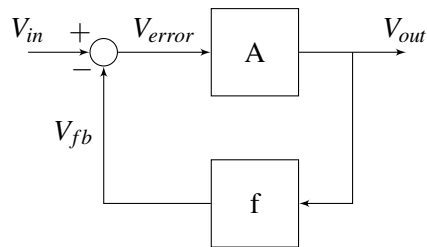


Observe that if $V_{fb}$ remains unchanged and $V_{in}$ goes up, then $V_{error}$ goes up. Then since $A$ is a positive number, $V_{out}$ also goes up, which causes $V_{fb}$ to go up. In other words, the magnitude of $V_{error}$ goes down, meaning that the system is going to stablize itself. Now what if we change the minus sign to a plus sign in the diagram, i.e., changing the system into a positive feedback system. With a similar logic, you could verify that if $V_{in}$ goes up, $V_{fb}$ goes up, but $V_{error}$ goes up, which further causes $V_{out}$ to go up. We see that it is not possible to stabilize the system. Let's look at the negative feedback op amp circuit we've seen earlier,
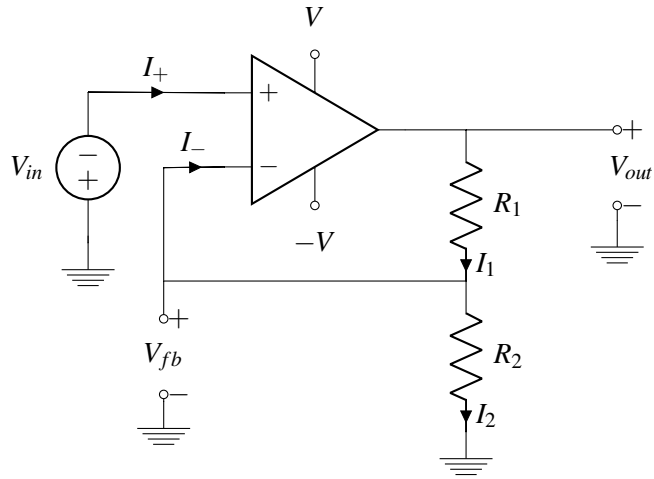
We know that when $V_{in}$ increases, $V_{out}$ also increases since $V_{out} = A\left(V_{in} - V_{fb}\right)$. When $V_{out}$ increases, $V_{fb} = \frac{R_2}{R_1+R_2}V_{out}$ also increases, which then cause $V_{error}$, and hence, $V_{out}$ to go down. (Note in this case, $f = \frac{R_2}{R_1+R_2}$ in the block diagram.) Now let's derive why $V_+ = V_-$ in this case. We know that in the above circuit, $V_+ = V_{in}$ and $V_- = V_{fb}$. Let's redraw the block diagram.
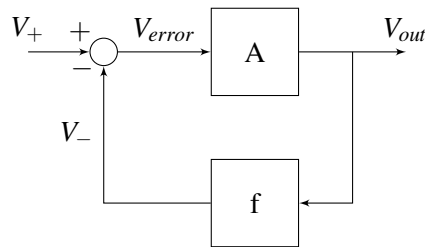


Now when the system stablilizes, we have

$$V_{error} = V_+ - V_- \tag{14}$$

$$V_{out} = AV_{error} = A\left(V_+ - V_-\right) \tag{15}$$

$$V_- = fV_{out} \tag{16}$$

Combining the last two equations, we have

$$V_{out} = A\left(V_+ - fV_{out}\right), \tag{17}$$

which gives us

$$V_{out}\left(1 + Af\right) = AV_+. \tag{18}$$

Finally, we have

$$V_{out} = \frac{A}{1 + Af}V_+. \tag{19}$$

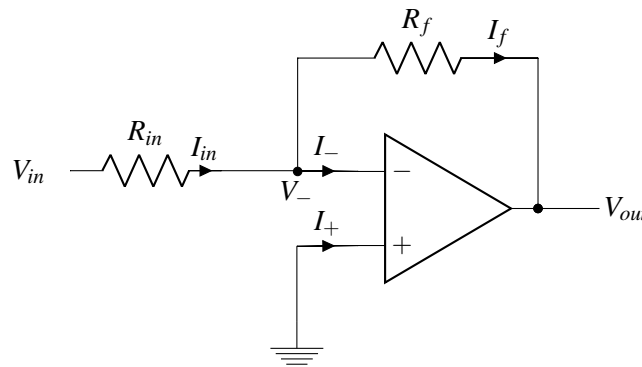Hence,

$$V_- = fV_{out} = \frac{fA}{1 + Af}V_+. \tag{20}$$

Now we know that the gain $A$ is very large, hence $fA$ is very large. Hence, the ratio

$$\frac{fA}{1+Af} \approx 1. \tag{21}$$

Thus, when $A \rightarrow \infty$ which is what we assume for an ideal op amp, $V_+ = V_-$.

## 19.6 Inverting op amp

Let's apply what we've learned so far about Golden rules and negative feedback to the following op amp circuit:

Given an ideal op-amp (with power rails of sufficiently large magnitude), what is $V_{out}$ if we input an arbitrary voltage of $V_{in}$?

The first golden rule says that $I_- = I_+ = 0$. Hence using KCL at the node labelled with voltage $V_-$, we have

$$I_{in} = I_- + I_f = 0 + I_f = I_f. \tag{22}$$

We have

$$I_{in} = I_f. \tag{23}$$

Now, let's apply the second golder rule, $V_+ = V_-$. Since the positive input terminal is connected to ground, $V_- = 0$. Hence, we have

$$V_+ = V_- = 0. \tag{24}$$

By Ohm's law,

$$I_{in} = \frac{V_{in} - V_-}{R_{in}} = \frac{V_{in}}{R_{in}} \tag{25}$$

$$I_f = \frac{V_- - V_{out}}{R_f} = -\frac{V_{out}}{R_f}. \tag{26}$$

Since $I_{in} = I_f$, we have

$$\frac{V_{in}}{R_{in}} = -\frac{V_{out}}{R_f}. \tag{27}$$

Moving terms around, we have

$$V_{out} = -\frac{R_f}{R_{in}}V_{in}. \tag{28}$$

Observe that the output voltage is a multiple of the input voltage with a scaling factor of $-\frac{R_f}{R_{in}}$. In addition, notice that the $V_{out}$ and $V_{in}$ are of opposite signs. This type of circuit is what we call an **inverting amplifier**.

## 19.7 More complicated op amp example

Now let's take a look at a slightly more complicated op amp circuit example with two voltage sources:
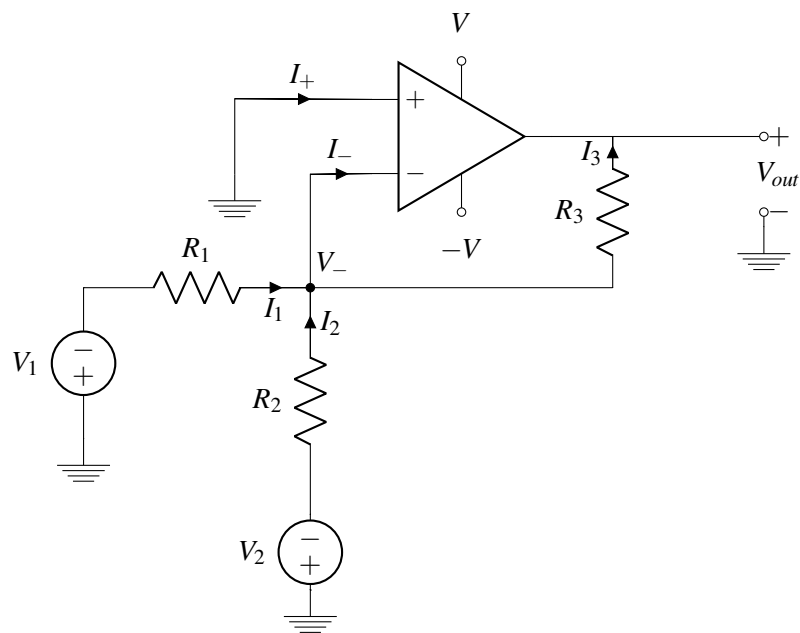


First, let's apply the first golden rule, $I_- = I_+ = 0$. Applying KCL at the node labelled $V_-$, we have

$$I_1 + I_2 = I_- + I_3 = 0 + I_3 = I_3. \tag{29}$$

Hence, we have

$$I_1 + I_2 = I_3. \tag{30}$$

Now by the second golden rule and the fact that the positive input terminal is connected to ground, we have

$$V_+ = V_- = 0. \tag{31}$$

Applying Ohm's law at each of the three resistors, we have

$$I_1 = \frac{V_1 - V_-}{R_1} = \frac{V_1}{R_1} \tag{32}$$

$$I_2 = \frac{V_2 - V_-}{R_2} = \frac{V_2}{R_2} \tag{33}$$

$$I_3 = \frac{V_- - V_{out}}{R_3} = -\frac{V_{out}}{R_3} \tag{34}$$

$$\tag{35}$$

Plugging in the above result to the KCL equation $I_1 + I_2 = I_3$ derived previously, we have

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_{out}}{R_3}. \tag{36}$$

Multiplying both sides by $R_3$, we have

$$V_{out} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2, \tag{37}$$

relating the final output with the two inputs $V_1$ and $V_2$.