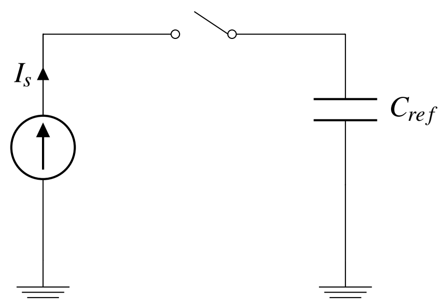
Design Example Continued

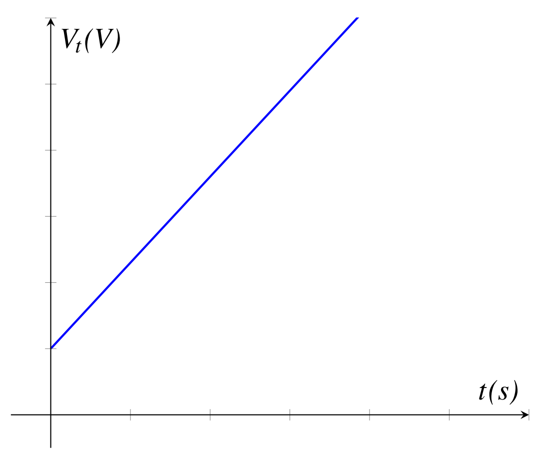
Continuing our analysis for "countdown timer" circuit.



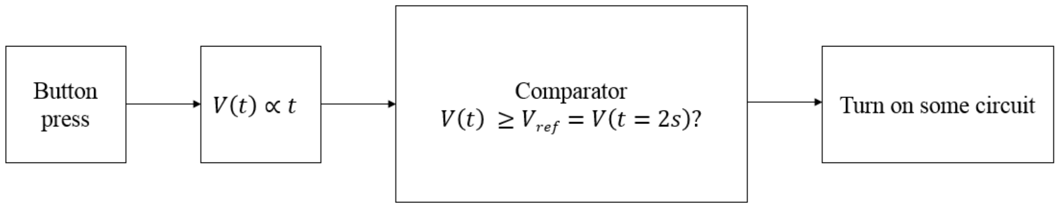
We know for a capacitor C:

$$I = C\frac{dV}{dt} \quad (1)$$

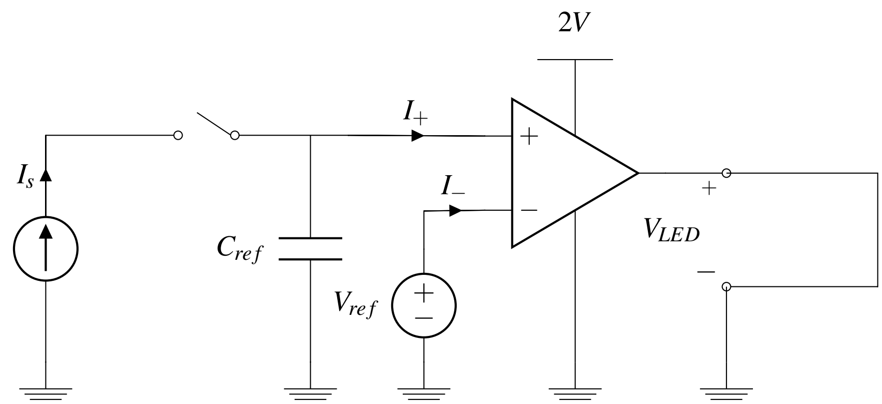
There is a linear relationship between the voltage across capacitor $V(t)$ and charging time t.



When a button is pressed, we want to turn on some circuit after 2s. Given that voltage is linearly dependent on charging time, we can use a comparator and a reference voltage V_{ref} to decide if 2s has already passed after the button press to decide whether or not to turn on some circuit.

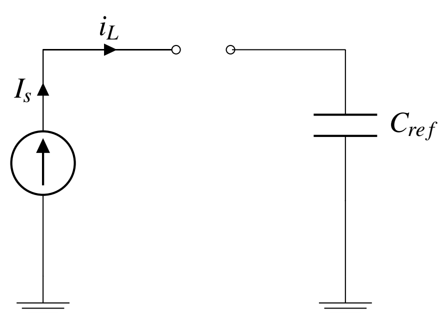


Adding an op-amp comparator and a voltage source V_{ref} to the "countdown timer" circuit:



Obviously, V_{ref} should be set equal to the voltage of C_{ref} after charging for 2s. After 2s since the button is pressed, if the voltage across C_{ref} becomes higher than V_{ref}, the comparator outputs $2V$ to the output of the op-amp to turn on the LED.

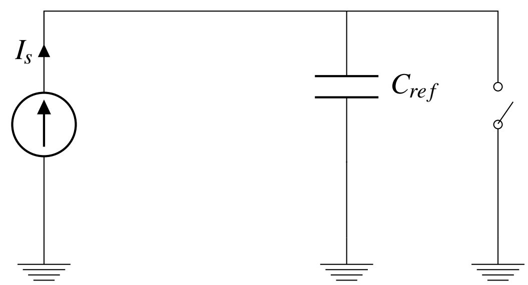
Step 4: design verfication Now, let's actually analyze the design completely to make sure it works. Before the button is pressed, the circuit on the current source side looks like:



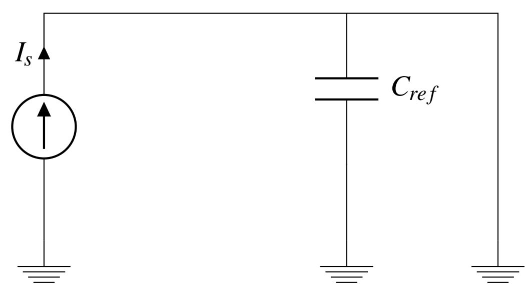
According to KCL,

$$I_s = I_L \tag{2}$$

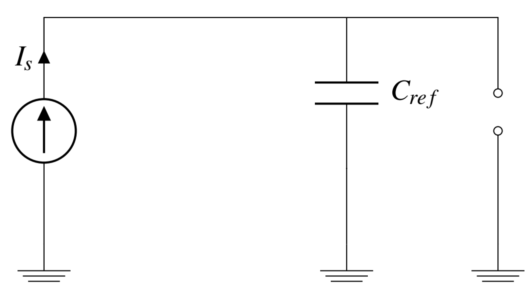
I_s is the constant current supplied by the current source, I_L is the current flow into the switch. Before touch, I_L must equal 0 since there is an open circuit. However, the current source guarantees that I_s is nonzero. It is very easy to see that mathematically this is problematic, how do we solve the problem? We can add another switch in the circuit:



Before touch, the switch is on and can be replaced by a wire:



After touch, the switch is off and can be replaced by an open circuit:



In either cases, there is a loop in the circuit for I_s to flow. Now, there is a remaining mystery to be answered, how do we build that current source?

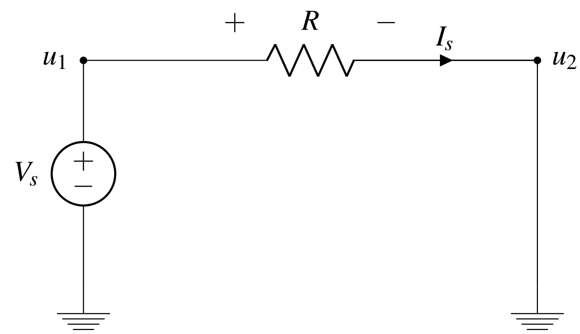
22.1 "Almost" current source

In this section, we will use resistors, voltage sources and op amps to build a current source. We know by Ohm's law,

$$I = \frac{V}{R} \tag{3}$$

If we have a voltage source V_s, we can scale it by a resistance value R, then we should get a constant current I_s.
Now let's use our design procedures to build a current source.

Step 1 restate design goal: we want to build a current source that can output constant current regardless of whatever elements we hook up to it.
Step 2 Let's now take a voltage source and connect it to a resistor to output a current:
Attemp #1

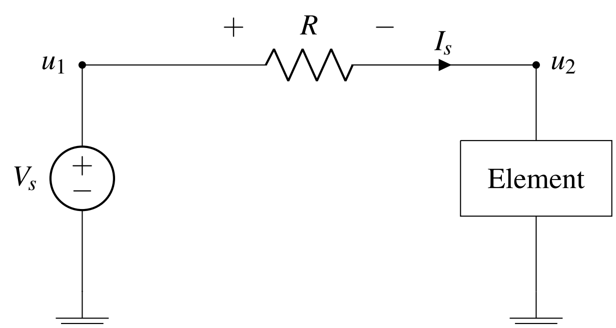


In the above circuit,

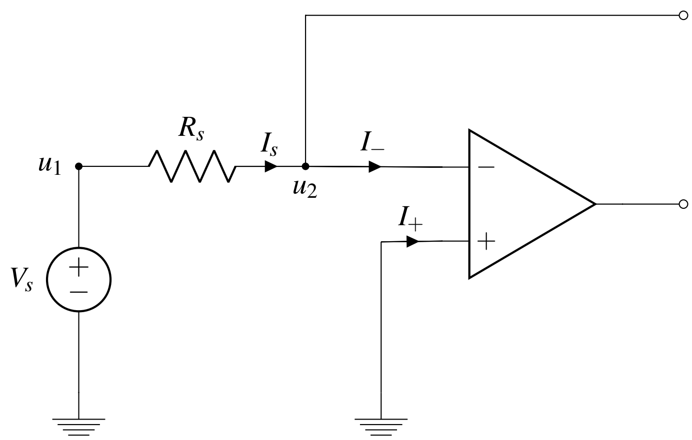
$$u_1 = V_s \tag{4}$$

$$I_s = \frac{V_s}{R} \tag{5}$$

However, if we hook up an element between u_2 and ground, we can tell immediately that the current through R_s is no longer the constant $I_s = \frac{V_s}{R}$.



Recall that it is important to guarantee a constant current output from the current source regardless of the element added to the circuit.
Attempt #2 Although our attempt 1 has failed, we have learned an important lesson: if we can somehow set u_2 to $0V$ without physically connecting it to ground. The current through R will always equal $\frac{V_s}{R}$ ($I_R = \frac{u_1-u_2}{R} = \frac{V_s-0}{R}$)! According to golden rule #2, we can set both V_+ and V_- to $0V$ if an op amp circuit is in negative feedback. Indeed, we will now use an op amp to build a current source!



According to golden rules:

$$I_- = 0 \tag{6}$$

$$V_- = V_+ = 0V \tag{7}$$

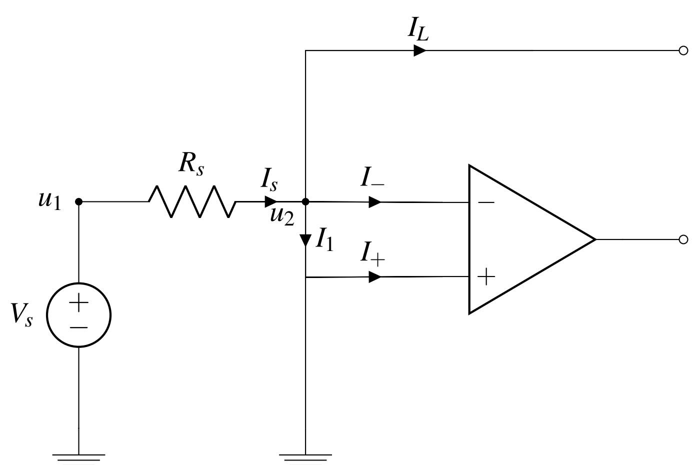
We also know that the current across R will always be:

$$I_s = \frac{u_1 - u_2}{R} \tag{8}$$

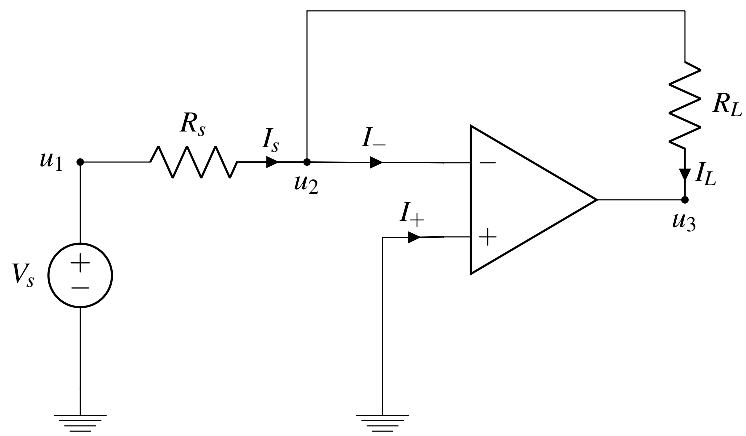
Solving the above equations, we can get the value of I_s:

$$I_s = \frac{V}{R} \tag{9}$$

By setting u_2 (V_-) to $0V$ by using a negative feedback circuit, we have succesfully built a current source!
It is important to keep in mind setting u_2 to $0V$ by using a negative feedback circuit is very different from physically connecting the node of u_2 to ground. If we physically connect u_2 to ground by adding a wire between V_- and V_+:



If we physcially connect both V_- and V_+ to ground, I_L will become $0A$ because it is shorted by the wire between V_- and V_+. Instead, now we have $I_1 = I_s$. So we must not physically connect V_- to ground.
Let's now hook up a resistor R_L to the circuit and prove that current flow through R_L is constant:



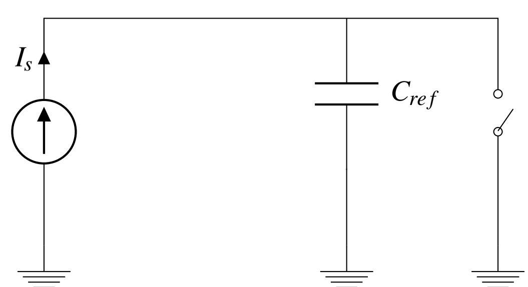
According to KCL:

$$I_L = I_s = \frac{V}{R_s} \tag{10}$$

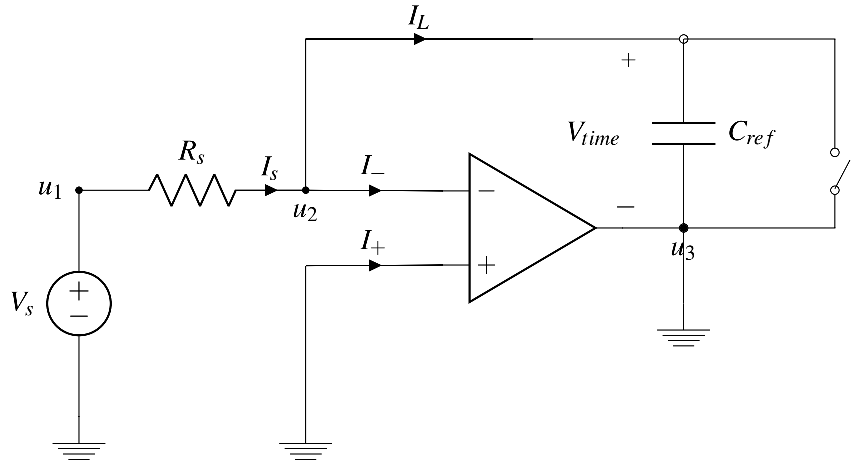
From I_L equation, we can see immediately that I_L is not affected by changes in R_L. How does the circuit maintain the constant current flow through R_L?
The op-amp outputs a negative V_{out} to maintain a constant current flow through R_L. In other words, the voltage drop across R_L is always given by

$$V_{R_L} = \frac{V}{R_S} \times R_L \tag{11}$$

Now let us plug in the current source to the following circuit:



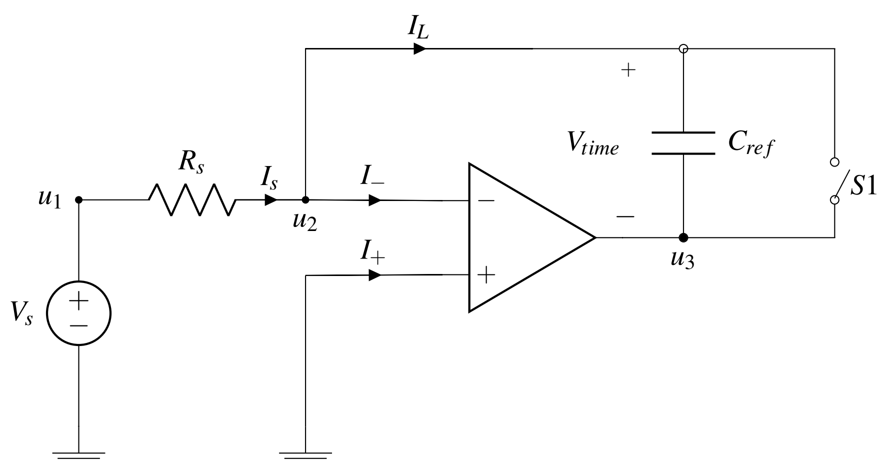
The circuit becomes:



Now u_3 is connected to ground. By doing this, the voltage across C_{ref} becomes:

$$V_{time} = u_2 - u_3 = 0V - 0V = 0V \tag{12}$$

There is even a worse problem, recall that there is an controlled voltage source inside the op-amp, this op-amp wants to set u_3 to some nonzero value $A \cdot V_c$ but u_3 is also manually connected to $0V$. The fix to solve this problem is to simply get rid of the ground connection.



$$I_s = C_{ref}\frac{dV}{dt} \tag{13}$$

$$I_s = C_{ref}\frac{d(u_2 - u_3)}{dt} \tag{14}$$

According to golden rule #2, $u_2 = 0V$.

$$I_s = C_{ref}\frac{d(0V - u_3)}{dt} = C_{ref}\frac{d(-u_3)}{dt} \tag{15}$$

Solving the above equation:

$$u_3(\mathrm{t}) = -\frac{I_s}{C_{ref}} \times t + u_3(\mathrm{t} = 0\mathrm{s}) \tag{16}$$

$u_3(\mathrm{t})$ is associated with the initial value $u_3(\mathrm{t} = 0\mathrm{s})$. Before touch, the switch $S1$ is on, which sets $u_3(\mathrm{t} = 0\mathrm{s})$ to $0V$. Therefore,

$$u_3(\mathrm{t}) = -\frac{I_s}{C_{ref}} \times t = -\frac{V_s}{R_s C_{ref}} \times t \tag{17}$$

Note there is a term $R_s C_{ref}$ in the denominator, what is the unit of $R_s C_{ref}$?

$$\text{Unit for } R_s C_{ref} = \frac{V}{A} \times CV = \frac{C}{A} = \text{second} \tag{18}$$

It is good to know that this multiplication result of RC is very useful and common in the timing analysis for circuits, this is an indicator of how fast a circuit is.
There are 2 important points to keep in mind when this current source:

- Do not connect the circuit element that we want to supply the constant current to with ground externally. Doing so may force V_{out} of the op-amp to $0V$ and lead to nonidealities.
- The circuit element we hook up to the current source must still keep the op-amp circuit in its negative feedback state. Being in negative feedback allows us to set the node u_2 to $0V$ without physically connecting it to ground and hence allows a constant current output.