This homework is optional but in scope.

1. **DFT**

   (a) Compute the DFT coefficients of $x_1[n] = \cos(\frac{2\pi}{6}n)$ where $n \in \{0, 1, \ldots, 5\}$.

   (b) Plot the time domain representation of $\vec{x}_1$. Plot the magnitude, $|X[n]|$, and plot the phase, $\angle X[n]$, for its DFT-basis representation.

   (c) Compute the DFT coefficients of $x_2[n] = \cos(\frac{4\pi}{6}n)$ where $n \in \{0, 1, \ldots, 5\}$.

   (d) Plot the time-domain representation and the magnitude and phase for the DFT-basis representation of $\vec{x}_2$.

   (e) How about the general case, $x_p[n] = \cos(\frac{2\pi}{6}pn)$, where $n \in \{0, 1, \ldots, 5\}$?

   (f) Compute the DFT coefficients of $\vec{s} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^T$.

   (g) Compute the DFT coefficients of $y_1[n] = \cos(\frac{2\pi}{6}n - \pi)$ where $n \in \{0, 1, \ldots, 5\}$.

2. **LTI filter**

   Suppose we apply the length $L = 100$ input

   $$u[n] = \cos(0.1\pi n) + \cos(0.4\pi n), \quad n = 0, 1, \ldots, 99 \tag{1}$$

   to a finite impulse response filter whose impulse response is

   $$h[n] = \begin{cases} \frac{1}{6} & n = 0, \ldots, 5 \\ 0 & \text{otherwise.} \end{cases}$$

   We want to find the output $y[n]$, $n = 0, \ldots, 104$ from the DFT.

   (a) Find the 105-point DFT of $u[n]$ by adding 5 zeros to the length-100 signal given above and using the DFT command `numpy.fft.fft(x, norm="ortho")`. Plot the magnitude $|U[k]|$ against the frequency variable $\omega \in [0, 2\pi]$. (Recall that the integer k corresponds to the frequency $\frac{2\pi}{105}k$.)

   (b) Find the 105-point DFT of $h[n]$ by adding 99 zeros to the length-6 impulse response given above. Plot the magnitude $|H[k]|$ against the frequency variable $\omega \in [0, 2\pi]$. Given this frequency response how do you think each frequency component of $u[n]$ will be affected when the filter is applied?

   (c) Remember, the DFT coefficients of the output are

   $$Y[k] = \sqrt{105}H[k]U[k], \ k = 0, \ldots 104.$$

   Find $y[n]$ using the inverse DFT command `numpy.fft.ifft(x, norm="ortho")` Plot both $u[n]$ and $y[n]$ versus time n and explain how the filter modified $u[n]$.
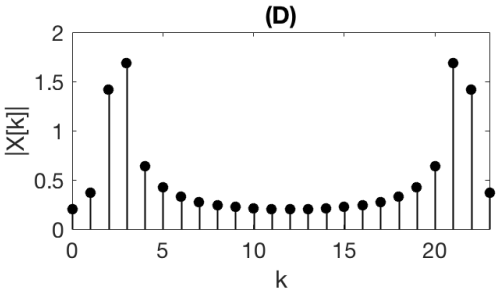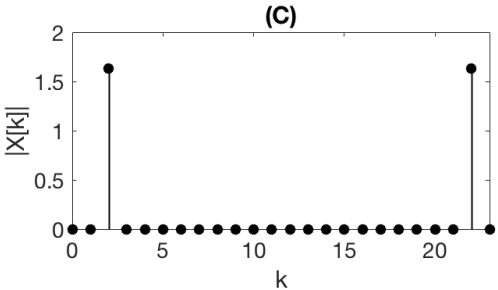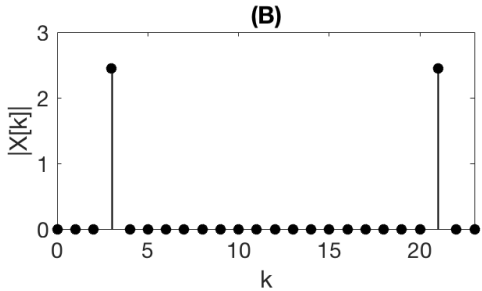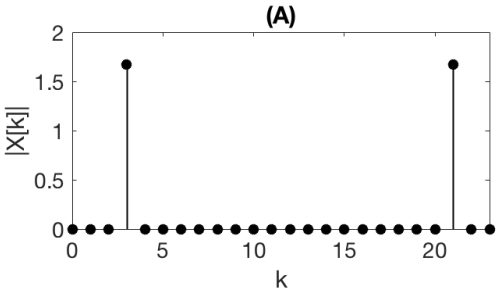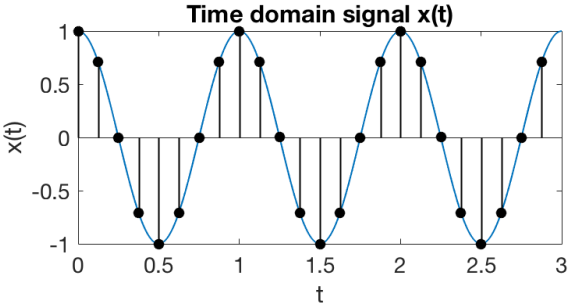
3. DFT Sampling Matching

**Select the correct answer from the multiple choice options provided and give some justification.**

(a) A sampled time domain signal and its DFT coefficients are given below:
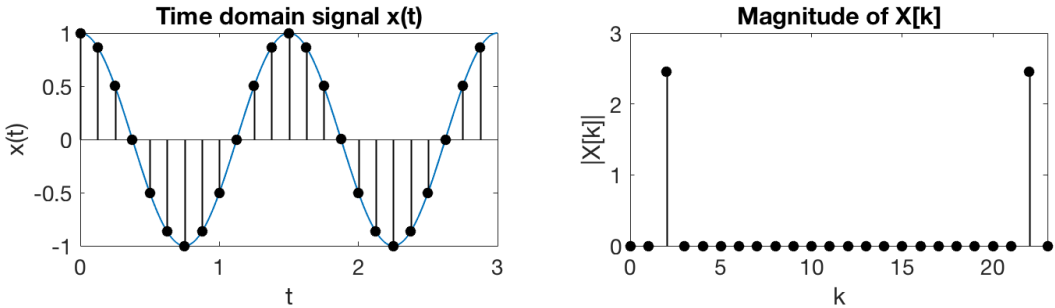

Now given the following time domain signal, which of the options below shows the correct DFT coefficient magnitudes?

(b) Given the time domain signal below, which of the options below shows the correct DFT coefficient magnitudes?

Time domain signal x[n]

(A)

(B)

(C)

(D)

(c) Given the time domain signal below, which of the options below shows the correct DFT coefficient magnitudes?

Time domain signal x(t)

(A)

(B)

(C)

(D)
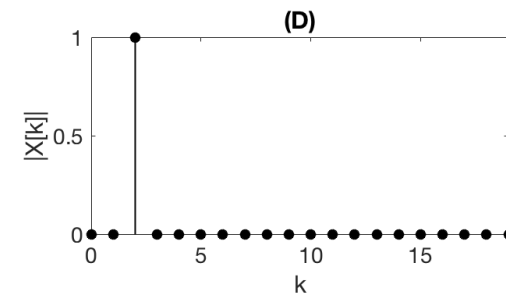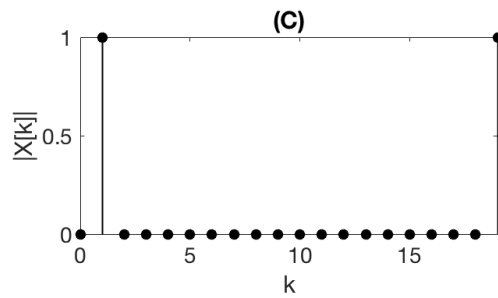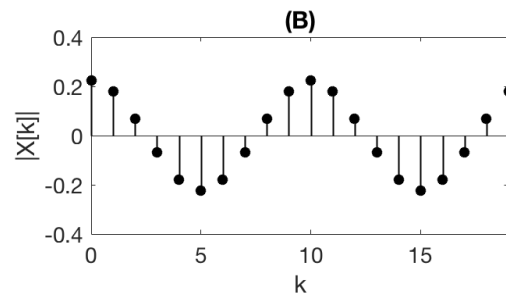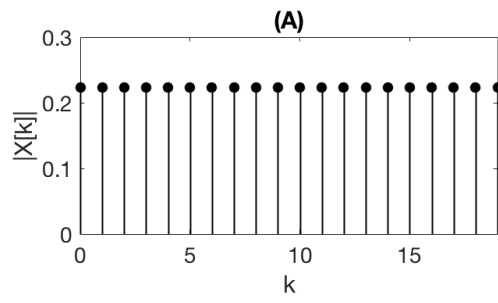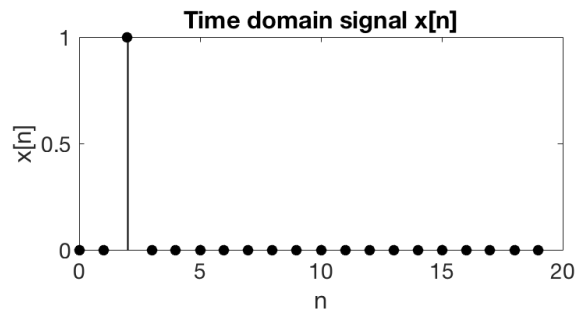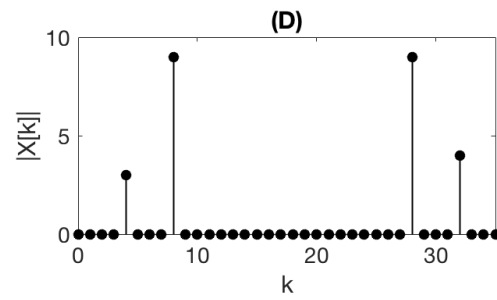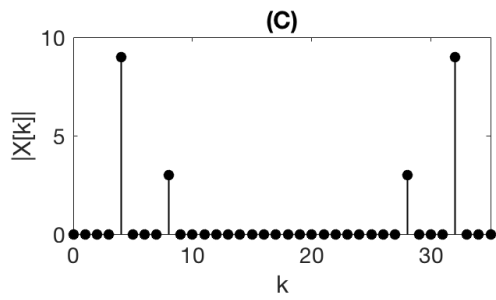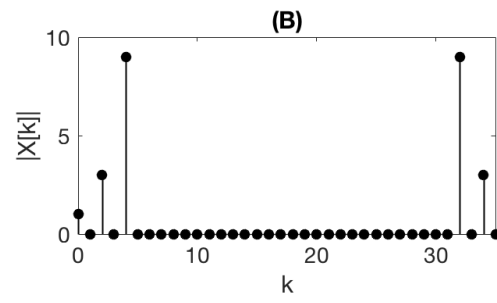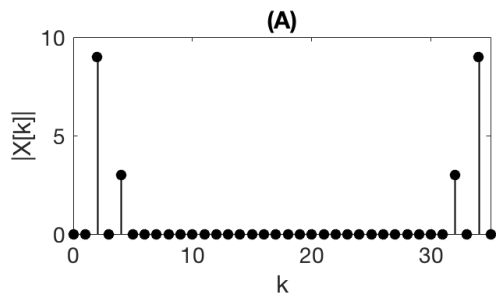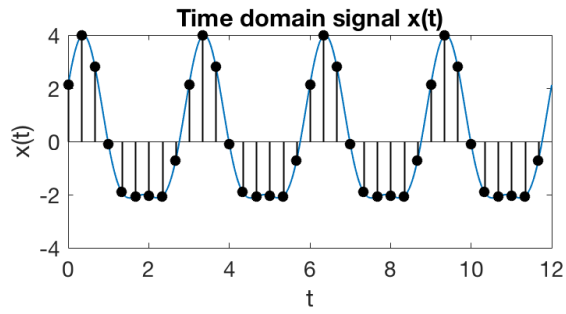
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