EECS 16B Designing Information Devices and Systems II
Fall 2017 Miki Lustig and Michel Maharbiz Homework 2

This homework is due September 12, 2017, at 11:59AM.

1. Two Inductors

Consider the circuit below, assume that when $t < 0$, the circuit has reached steady state ($V_1 = 0, V_2 = 0$). At $t = 0$, the switch connected to V_s closes. Assume $V_s = 5\text{V}$, $R_1 = R_2 = 1\text{k}\Omega$, and $L_1 = L_2 = 0.1\text{H}$.

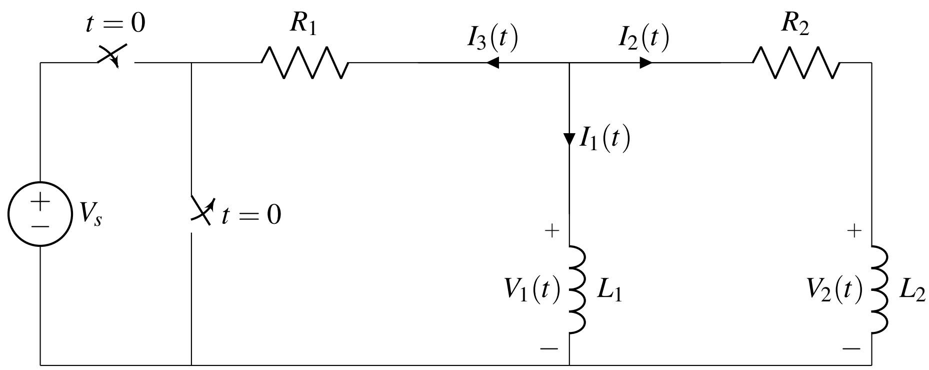


Figure 1: Two Inductor Circuit with Voltage Source

(a) First, use Kirchoff's Laws and the inductor equation ($V = L\frac{dI}{dt}$) to find the second order differential equation for this system in terms of $I_2(t)$, L_1, L_2, R_1, and R_2.

(b) Now cast this second order differential equation into the following form:

$$\frac{d\vec{i}}{dt} = A\vec{i}$$

where

$$\vec{i} = \begin{bmatrix} I_2(t) \\ \frac{dI_2(t)}{dt} \end{bmatrix}$$

Plug in values to get a numerical matrix.

(c) Find the eigenvalues of A. Are they real or complex?

(d) Using the initial conditions, what is the solution to the differential equation?

(e) Sketch the current vs time plots of $I_1(t)$ and $I_2(t)$.

2. Complex numbers

A common way to visualize complex numbers is to use the complex plane. Recall that a complex number z is often represented in cartesian form.

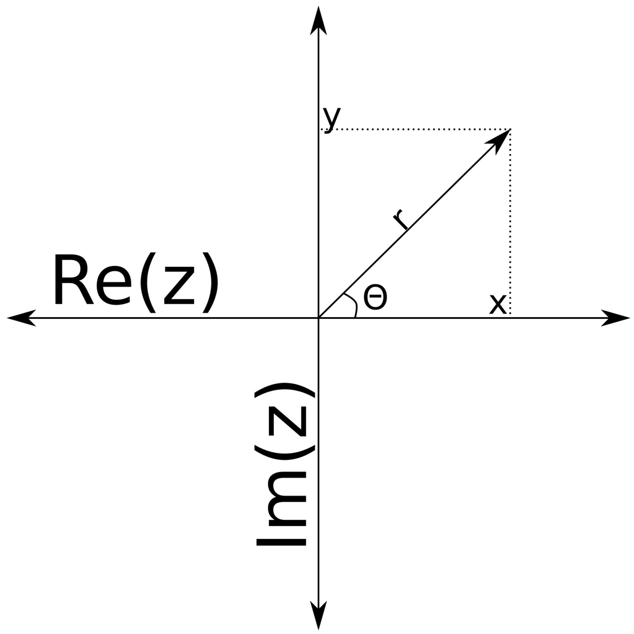


Figure 2: The complex plane

$$z = x + jy \text{ with real}(z) = x \text{ and imaginary}(z) = y$$

See the Figure 2 for how z looks like in the complex plane.

In this question, we will derive the polar form of a complex number and use this form to make some interesting conclusions.

(a) Calculate the length of z in terms of x and y as shown in Figure 2. This is the magnitude of a complex number and is denoted $|z|$ or r. *Hint.* Use the Pythagoras theorem.

(b) Represent the real and imaginary parts of z in terms of r and θ.

(c) Substitute for x and y in z. Use Euler's formula to conclude that,

$$z = re^{j\theta}$$

(d) In the complex plane, draw out all the complex numbers such that $|z| = 1$. What are the z values where the figure intersects the real axis and the imaginary axis? Why do you think the figure is called a *Unit circle*?

(e) If $z = re^{j\theta}$, prove that $z^* = re^{-j\theta}$. Recall that the complex conjugate of a complex number $z = x + jy$ is $z^* = x - jy$.

(f) Show that,

$$r^2 = zz^*$$

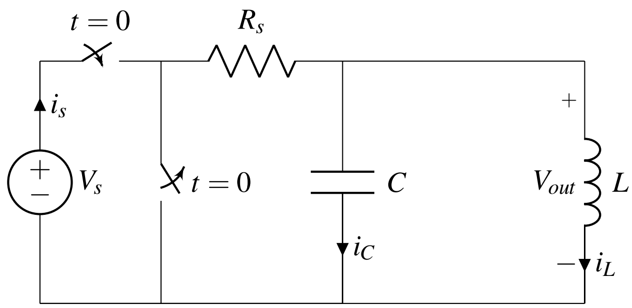
(g) Intuitively argue that,

$$\sum_{k=0}^{3-1} e^{j\frac{2\pi}{3}k} = 0$$

Do so by drawing out the different values of the sum making an argument based on the vector sum.

3. RLC circuit

Now consider the circuit shown below:



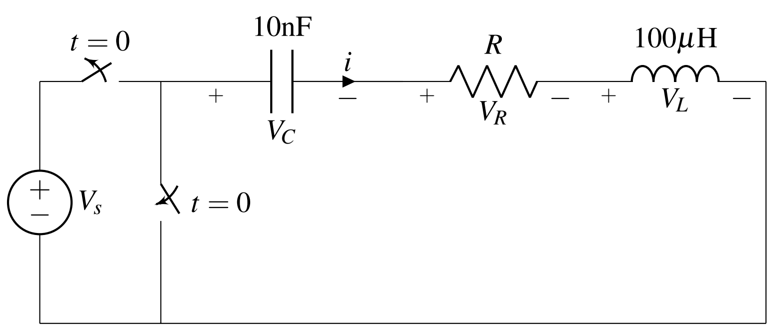
(a) Assuming the circuit reaches steady state for $t < 0$, find the differential equation for V_{out} for $t \geq 0$

(b) What are the initial conditions at $t = 0$ for this differential equation?

(c) Solve the differential equation. Consider all cases (underdamped, critically damped, overdamped)

4. General RLC response types

Consider the following circuit assume this circuit has reached steady state for $t < 0$:



(a) Find the differential equation that describes this circuit for $t \geq 0$ and solve it in terms of V_s, L, R and C.

(b) At what **frequency** is this circuit going to oscillate? Your answer should be in terms of R.

(c) Sketch the transient response of $V_c(t)$ for $t \geq 0$ when $R = 100\Omega$

(d) Sketch the transient response of $V_c(t)$ for $t \geq 0$ when $R = 200\Omega$

(e) Sketch the transient response of $V_c(t)$ for $t \geq 0$ when $R = 1\text{k}\Omega$

Contributors:

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