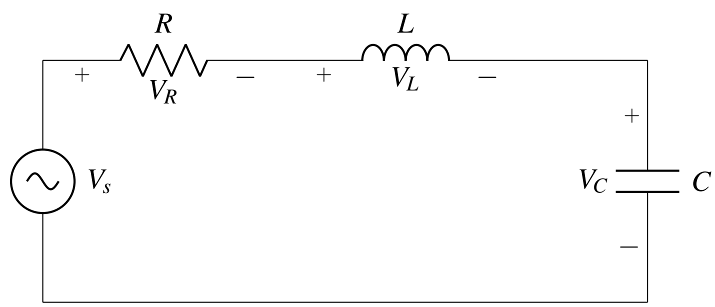
This homework is due September 26, 2017, at Noon.

1. **RLC circuit**

   In this question, we will take a look at an electrical systems described by second order differential equations and analyze it using the phasor domain. Consider the circuit below where V_s is a sinusoidal signal, $L = 1\text{mH}$ and $C = 1\text{nF}$:



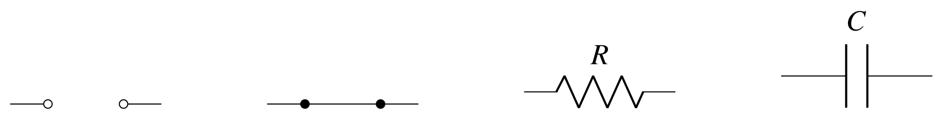
   (a) Transform the circuit into phasor domain.
   (b) Solve for the transfer function $H_C(\omega) = \frac{\widetilde{V}_C}{\widetilde{V}_s}$ in terms of R, L, and C
   (c) Solve for the transfer function $H_L(\omega) = \frac{\widetilde{V}_L}{\widetilde{V}_s}$ in terms of R, L, and C
   (d) Solve for the transfer function $H_R(\omega) = \frac{\widetilde{V}_R}{\widetilde{V}_s}$ in terms of R, L, and C
   (e) Sketch the Bode plots of $H_C(\omega)$, $H_L(\omega)$, and $H_R(\omega)$ when $R = 2\text{k}\Omega$
   (f) Draw the Bode plot of $H_R(\omega)$ two more times, but change R to be 20Ω, then $200\text{k}\Omega$.
   (g) Find ω_{c1} and ω_{c2} of $H_R(\omega)$ for $R = 20\Omega$, $R = 2\text{k}\Omega$ and $R = 200\text{k}\Omega$
   (h) Which of the three values of R gives the largest bandwidth? Which gives the highest Q?
   (i) Graph the actual frequency response of $H_R(\omega)$ for $R = 20\Omega$, $R = 2\text{k}\Omega$ and $R = 200\text{k}\Omega$. What is different between the Bode plots and the actual responses? Label on each graph what type of damping the circuit experiences.

2. **Circuit Design**

   In this problem, you will find a circuit where several components have been left *blank* for you to fill in.

   Assume the op-amp is *ideal*.

   You have at your disposal *only one of each* of the following components (not including R_1 and R_2):



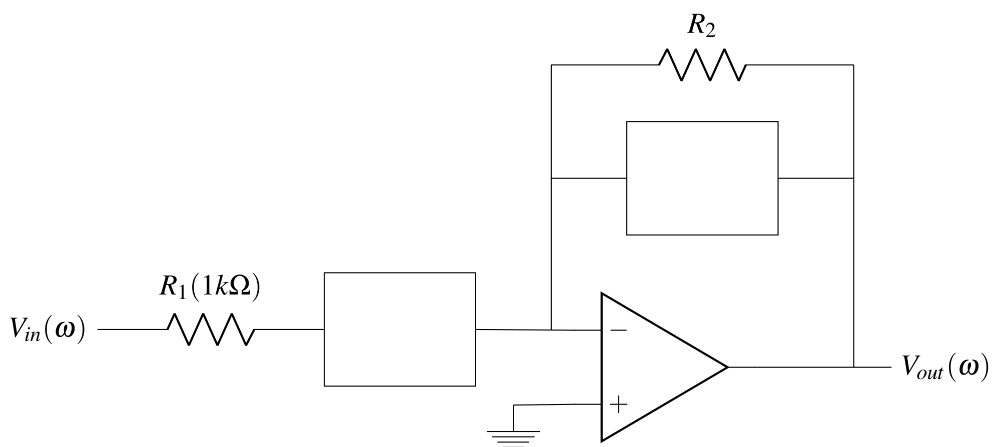
(a) an open circuit

(b) a short circuit

(c) a resistor (you choose from the value $R = 1\text{k}\Omega, 15\text{k}\Omega, 30\text{k}\Omega$)

(d) a capacitor (you choose from the value $C = 0.5\mu\text{F}, 1\mu\text{F}, 2\mu\text{F}$)

Consider the circuit below. The voltage source $v_{in}(t)$ has the form $v_{in}(t) = v_0 \cos(\omega t + \phi)$. The labeled voltages $V_{in}(\omega)$ and $V_{out}(\omega)$ are the phasor representation of $v_{in}(t)$ and $v_{out}(t)$. The transfer function $H(\omega)$ is defined as $H(\omega) = V_{out}(\omega)/V_{in}(\omega)$.



(a) Let R_1 be $1k\Omega$. **Fill in the boxes and determine the value of** R_2 so that
   - It is a high-pass filter.
   - $|H(\infty)| = 10$.
   - $|H(10^3)| = \sqrt{50}$.
   - R_2 must be one of the three values listed above

(b) Draw the Bode plot of this transfer function.

3. Bode plots to Transfer Functions

(a) Find the transfer function $H(\omega)$ that corresponds to the magnitude plot below. The phase of $H(\omega)$ is 90° at $\omega = 0$

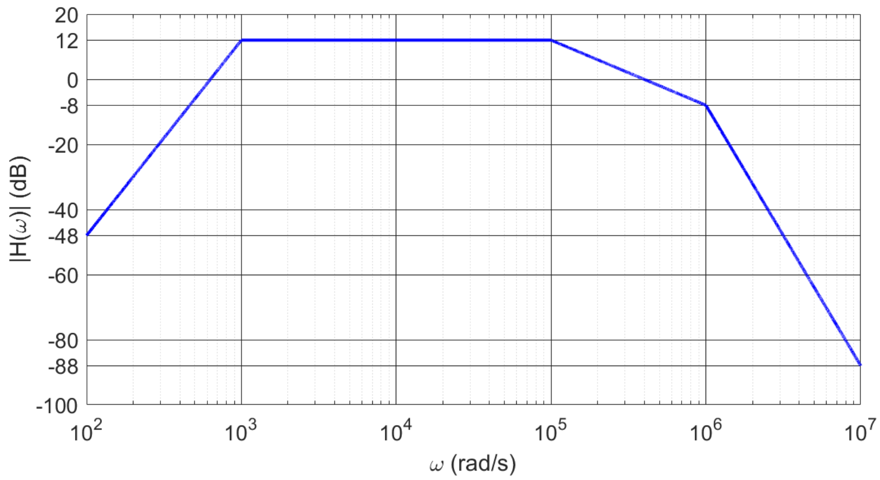


Figure 2: Magnitude graph for 3 (a)

(b) Find the transfer function $H(\omega)$ that corresponds to the magnitude plot below. The phase of $H(\omega)$ is 0° at $\omega = 0$

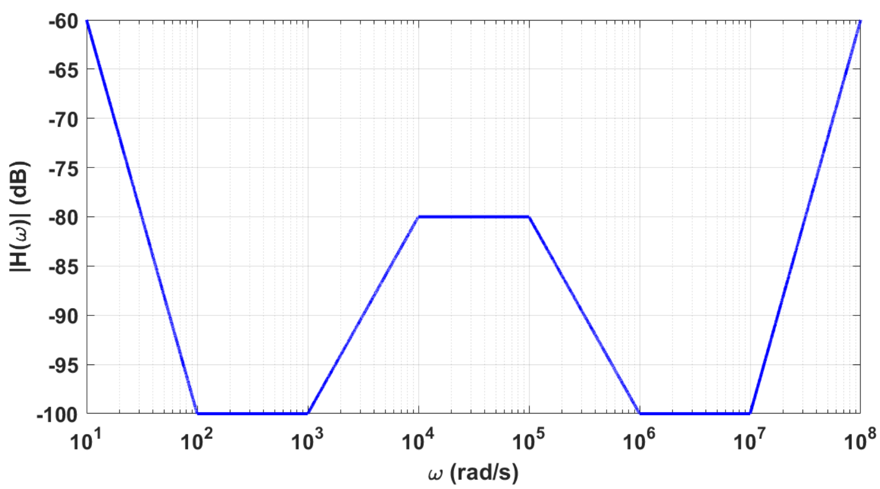


Figure 3: Magnitude graph for 3 (b)

(c) Using Python or MATLAB, graph the bode plot for the transfer function

$$H(\omega) = \frac{(500 + j\omega)(10^3 + j\omega)^2}{(j\omega)(10^2 + j\omega)^3}$$

4. LTE Filter

We are interested in creating a filter to receive LTE signals (a.k.a 4G signals often used for phones). There are many different frequencies at which signals are transmitted (called bands), so we're going to focus on LTE band 1. Band 1 transmits with a center frequency of 2.140 GHz, and a bandwidth of 60 MHz.

(a) Find ω_{c1} and ω_{c2} for LTE band 1.

(b) In order to receive a good signal, we need to ensure that $|H(\omega)|$ is small enough outside the pass band. Otherwise, noise from neighboring transmission bands will distort the signal we're trying to receive. Let's say we did some analysis, and found that at 200MHz away from the center frequency (1.940 GHz and 2.440 GHz), our $|H(\omega)|$ has to be 80dB lower than the pass band gain. Using your frequency response and Bode plot knowledge, determine the minimum number of poles and zeros required to get the required magnitude drop.

(c) Assuming we have a pass band gain of 1, give an expression for $H(\omega)$

(d) Draw the Bode plot of $H(\omega)$

5. Color organ filter design

Consider another microphone similar to the one which we were analyzing last week. As before, you obtain the data by playing a uniform tone with varying frequencies, and measuring the resultant peak-to-peak voltages using an oscilloscope. Below is the data obtained from your experiments:

Input frequency (Hz)	Output peak-to-peak (V)
10	1.4
20	1.5
40	1.5
60	1.6
100	2.2
160	2.3
320	2.4
640	2.5
1200	5
2500	5
5000	5
10000	4.9

(a) Now that you know how to design filters and basic op-amp amplifiers, please design the filters and amplifiers (coloured yellow and red respectively) for the color organ circuit below. You may use one op-amp per coloured block. Please draw the schematic-level representation (with op-amps and resistors/capacitors) of your designs for the filters and amplifiers, and show your work for the any values for Rs and Cs. If possible, choose 'realistic' values for resistors (1kOhm to 500kOhz) and capacitors (1nF to 200uF) or justify your reasoning.

Don't forget about the filter between the two parts of the band-pass filter! (Why is it necessary?)

For the purposes of this problem, consider the following frequency ranges for each filter:

- Low pass filter - less than 100 Hz
- High pass filter - more than 2400 Hz
- Band pass filter - between 300 and 600 Hz

In the lab, you will use different frequencies for your cutoff as each microphone will be different.

For the purposes of this homework problem, to ensure that the LED turns on properly, the output (as measured **before** the 10Ω resistors) in the desired ranges should have a peak-to-peak of about 5V, and any output outside the desired ranges caused by identical-amplitude signals from the other ranges should be less than 1.5V peak-to-peak.[1]

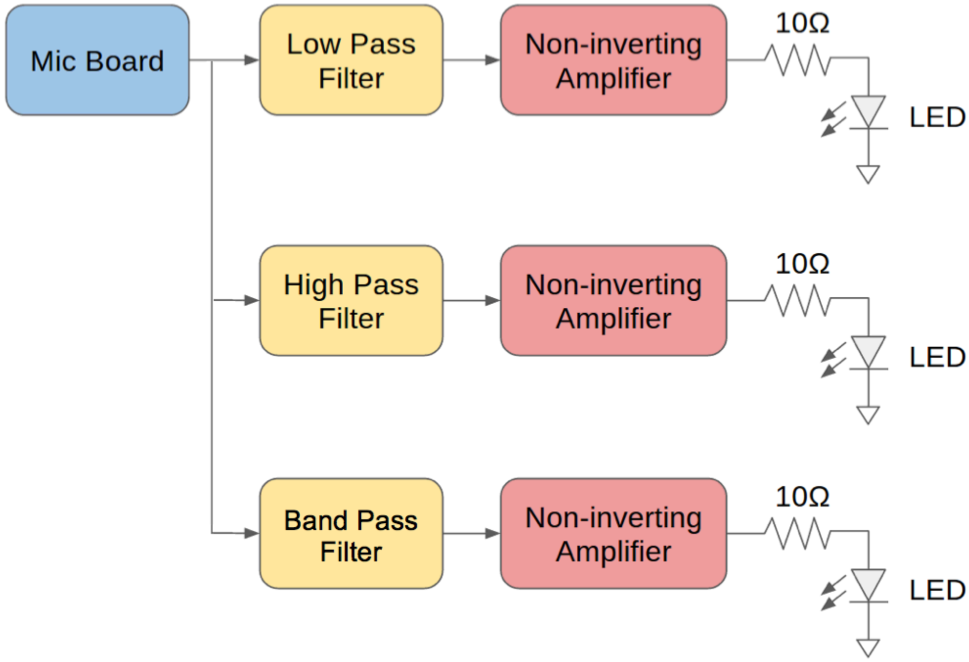


Figure 4: Color organ architecture

(b) For each of the waveforms from the microphone, determine the output voltage for each filter right before the 10Ω resistors (V_{out}).

- $V_l(t) = 0.75\cos(2\pi\,50\text{Hz})$
- $V_b(t) = 1.25\cos(2\pi\,450\text{Hz})$
- $V_h(t) = 2.5\cos(2\pi\,3000\text{Hz})$

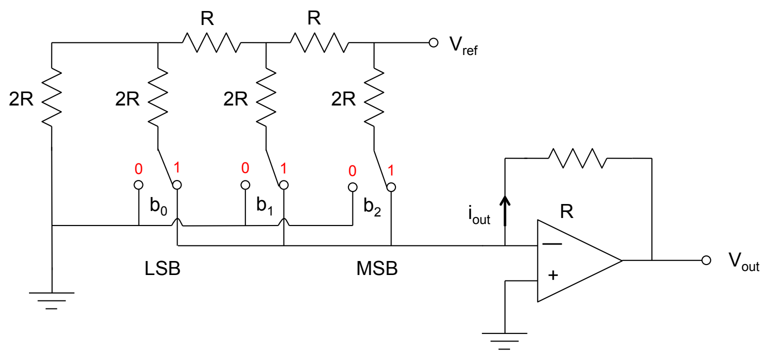
(c) Finally, we will get a rough idea for how the color organ will function (on paper) before we finally put those microphones to good use in the lab. Given the above definitions for V_l, V_b, and V_h, plot the output voltage for the low pass filter in red, bad pass filter in green, and high pass filter in blue for the following microphone output waveform for $0 \leq t \leq 4$.

$$V_{\text{mic}}(t) = \begin{cases} V_l(t) & \text{for } t < 1.5 \\ V_b(t) & \text{for } t \geq 1.5 \text{ and } t \leq 2 \\ V_h(t) & \text{for } t > 2 \end{cases}$$

6. Multiplying DAC

Previously, you analyzed one way of using a R-2R ladder to convert a digital binary number into an analog voltage. In this problem, we will examine a different way of using the R-2R ladder to make a DAC.

[1] Note that LEDs will only turn on when the voltage across it is greater than say **positive** 0.7V. If you wanted to detect and turn on the LED all the time instead of half the time, you would use either a mini rectifier, some kind of peak detection circuit, or digital processing, all of which is beyond the scope of the course.



Let $b_0, b_1, b_2 = \{0, 1\}$ (that is, either 1 or 0) and let $V_{\text{ref}} = V_{\text{DD}}$. If a switch b_i is on (1), then the current in that branch with the 2R resistor is allowed to flow into the output wire. Otherwise, the current flows into the ground. Notice that with the op-amp's negative feedback, the amount of current flowing through the 2R resistors is constant regardless of the switch states.

As before, (b_2, b_1, b_0) represents a 3-bit binary (unsigned) number where each of b_i is a binary bit.

(a) Find the current i_{out} as a function of b_2, b_1, b_0.

(b) Finally, solve for V_{out} using the current i_{out}. Express your final answer in terms of V_{DD} and the binary bits b_2, b_1, b_0. What do you notice? What is different compared to the previous R-2R DAC in homework 1, and if there is a difference, what can we do to remedy it?

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