EECS 16B Designing Information Devices and Systems II
Fall 2017 Miki Lustig and Michel Maharbiz **Homework 5**

This homework is due October 10, 2017, at noon.

1. 1D linear approximations in continuous systems

Linearization is an incredible tool when it comes to studying systems with non-linear dynamics. (This is when system matrix A is dependent on the state variables.) We overcome this by fixing a point in state space, often denoted as x_0, and approximating the transitions about that point. To better understand this, we will work through a 1D example.

(a) Consider an arbitrary function of $f(x)$ whose derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ is well defined. Construct a function of the form,

$$g(x) = mx + b$$

that approximates $f(x)$ in a neighborhood around a particular point x_0. m will be related to $\frac{\mathrm{d}f}{\mathrm{d}x}$.
Hint. Recall the definition of a derivative.

(b) We will study the following system.

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t) = f(x) \text{ where } f(x) = -2\sin\left(\frac{1}{3}x\right)$$

What is $\frac{\mathrm{d}f}{\mathrm{d}x}(x)$?

(c) What are the equilibrium points for this system?

(d) Construct a linear approximation $g(x)$ of $f(x)$ about the point $x_0 = 0$.

(e) Using the above approximation, can you solve the system,

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t) = f(x) \approx g(x)$$

from $x(0) = 1$.

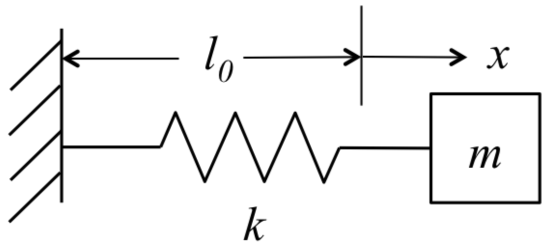
Note. This approximation is valid for points x close to x_0. We will explore this "closeness" when we study state feedback.

2. Spring and mass

Lets look at a mechanical spring-mass system governed by differential equations similar to those of electrical circuits.

Remember from physics that the motion of a mass is subject to Newton's second law $F = ma$ where $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ and $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ and that springs generate force according to $F_{sp} = -k\Delta x$ where k is the spring's stiffness. We set x to be 0 when the spring is at its rest length l_0 so that $\Delta x = x$. There is no gravity in this problem.

(a) Find a differential equation in terms of x and its derivatives that describes the motion of the mass. What order is this differential equation?

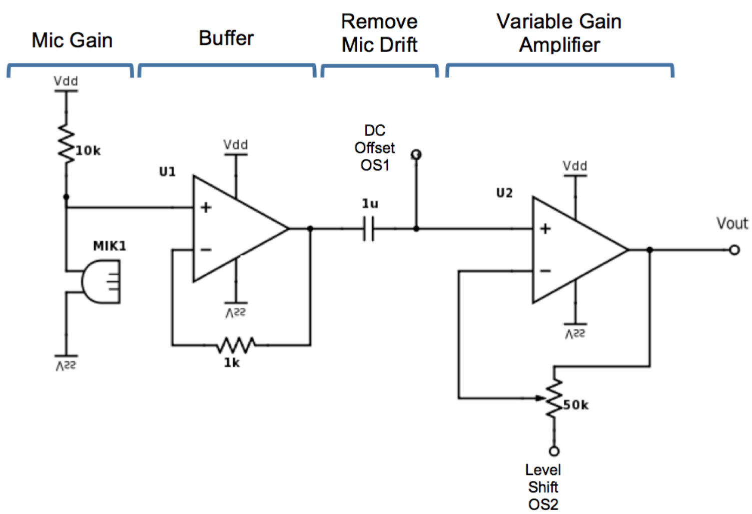


(b) Write the state space model for this system as $\dot{\vec{x}} = A\vec{x}$. What is your state vector? What are the units of the elements of the A matrix?

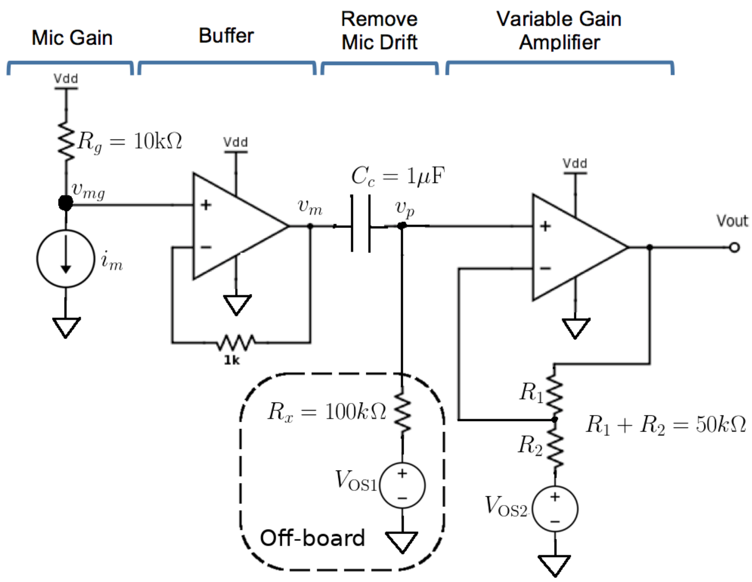
(c) Find the eigenvalues of this system by solving $|A - \lambda I| = 0$. Is this system stable?

3. Microphone circuit

In this problem, we will analyze and understand the circuit on the mic board of the SIXT33N robot car. Here is the schematic for the circuit as shown in lab:



For analysis purposes, we will transform it slightly into the following circuit by adding a few elements for modelling the system such as R_x (food for thought: after completing this problem, think about why R_x is necessary for this circuit to function) as well as some reference labels to assist with the analysis. We will go over each stage in more detail below.



Please **leave your answers in symbolic form** - e.g. write R_g instead of "10kOhm" in your expressions, unless otherwise asked. Justify all results and work.

We will assume that **all op-amps are ideal** in our analysis.

(a) Let us start with the **mic gain** stage.

The type of microphone we are using is called an electret microphone[1]. For circuit analysis purposes, we can model it as a current source[2], i_m.

Although the primary component of the microphone is an AC signal corresponding to the sound wave $\tilde{i_m}(t)$ (denoted as $\tilde{i_m}$), it turns out that the microphone also has bit of DC current, $I_{m,\text{DC}}$. As a result, our microphone current i_m is the sum of the DC current and the AC signal:

$$i_m = \tilde{i_m} + I_{m,\text{DC}}$$

Given our model so far, we'd like to find the output voltage of the mic gain stage. **Find an expression for v_{mg} in terms of i_m and V_{dd}.**

Next, we will analyze the **buffer** stage. Since this is an ideal op-amp in negative feedback, no current is drawn across the 1k resistor, so $v_m = v_{mg}$.

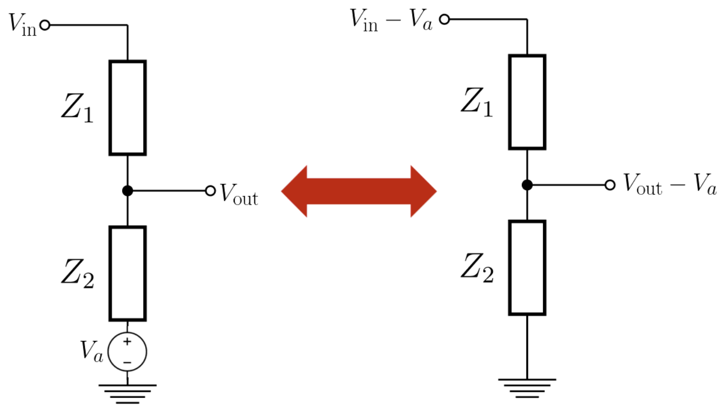
(b) We will now analyze the **remove mic drift** stage. The off-board resistor R_x forms the second part of a high-pass RC circuit.

However, there is just one small catch - what do we do about the V_{OS1} source? Can we still apply all the RC circuit analysis techniques we've learned so far?

Hint: There is a technique called virtual ground which can help us here. This is the idea that for some types of circuit analysis, ground is just a reference point which is relative. Convince yourself that following circuits in the diagrams below are equivalent:

[1] Like this one: `https://www.sparkfun.com/products/8635`

[2] This is beyond the scope of the class, but if you're curious, this circuit is the underlying circuit inside the microphone which can be modelled as a current source: `https://upload.wikimedia.org/wikipedia/commons/5/57/Electret_condenser_microphone_schematic.png`



For calculating the cutoff frequency, you can use $C_c = 1\mu$F and $R_x = 100\text{k}\Omega$.

What is the transfer function $H(\omega) = \dfrac{v_p - V_{\mathbf{OS1}}}{v_m - V_{\mathbf{OS1}}}$**? What is the cutoff frequency of the RC (**R_x **and** C_c**) filter in this stage?** Express your $H(\omega)$ symbolically and express your cutoff frequency symbolically first, then numerically.

(c) What is the gain $|H(\omega)|$ for the microphone voltage v_m's DC drift ($\omega = 0$) and for some tone in the audio frequency range ($\omega = 2\pi 440$ Hz)? Based on your results, what happens to the DC drift/offset, and what happens to the audio frequency tone? In general, what happens to audio frequency tones (say from about 20 Hz to 20 kHz)?

(d) Recall that our microphone itself had some DC bias $I_{m,\text{DC}}$. Let's see what happens to it after this RC circuit.

Express v_m in terms of $\tilde{i_m}$, $I_{m,\text{DC}}$, R_g, and any other symbolic terms as needed.

(e) Based off your answers in the previous section, what does the RC filter do to the $\tilde{i_m}$ term and $I_{m,\text{DC}}$ term? Express v_p in terms of $\tilde{i_m}$, $I_{m,\text{DC}}$, R_g, V_{OS1}, and any other symbolic terms deemed necessary. We can assume that the AC signal component of the microphone current $\tilde{i_m}$ contains only frequencies in the range of 20 Hz to 20 kHz (audio frequencies).

Hint: one of the terms should disappear. Which one, and why? Justify your answer. Hint 2: there is an offset involved in the final answer. Again, think about it and justify your answer.

(f) Now, we will analyze the **variable gain amplifier** stage. Using your knowledge of op-amps, express V_{out} symbolically in terms of R_1, R_2, v_p, and any other symbolic terms deemed necessary.

Hint: The virtual ground technique from part (b) may be very useful here.

(g) Finally, express V_{out} (assume $V_{\text{OS1}} = V_{\text{OS2}}$ for this problem) in terms of $\tilde{i_m}$, R_1, R_2, R_g, and any other symbolic terms deemed necessary.

(h) In lab, we will generate V_{OS1} and V_{OS2} using resistive dividers. However, in reality, resistors have up to $\pm 5\%$ tolerance (meaning that each resistor can be 5% greater or less than the advertised value) and so building two idental resistive dividers may not produce identical voltages. Let us analyze and see what happens for our circuit.

Let ΔV be the difference between V_{OS1} and V_{OS2}: $V_{\text{OS1}} - V_{\text{OS2}} = \Delta V$. In addition, let $\tilde{i_m} = 0$ for this subproblem (i.e. no microphone signal) since we want to see what happens to the offset even in the absence of a microphone signal.

Express V_{out} in terms of V_{OS2}, R_1, R_2, ΔV, and any other symbolic terms deemed necessary.

You may notice that really bad things happen to your output voltage! (Exactly what the bad things are, you'll have to do the problem.) This is why in lab we cannot connect V_{OS2} directly, and must instead use a potentiometer to finely adjust the voltage divider which creates V_{OS2} so that $V_{\text{OS2}} = V_{\text{OS1}}$.

4. Redo Problem 1 on the midterm

(a)

(b)

(c)

(d)

5. Redo Problem 2 on the midterm

6. Redo Problem 3 on the midterm

(a)

(b)

(c)

(d)

(e)

7. Redo Problem 4 on the midterm

(a)

(b)

(c)

(d)

(e)

8. Redo Problem 5 on the midterm

(a)

(b)

(c)

EE 16B Midterm 1
Fall 2017

Name:____

SID #:____

(after the exam begins add your SID# in the top right corner of each page)

Discussion Section and TA:______________________________

Discussion Section and TA:______________________________

Lab Section and TA:____________________________________

Name of left neighbor:__________________________________

Name of right neighbor:_________________________________

Instructions:

Show your work. An answer without explanation is not acceptable and does not guarantee any credit.

Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.

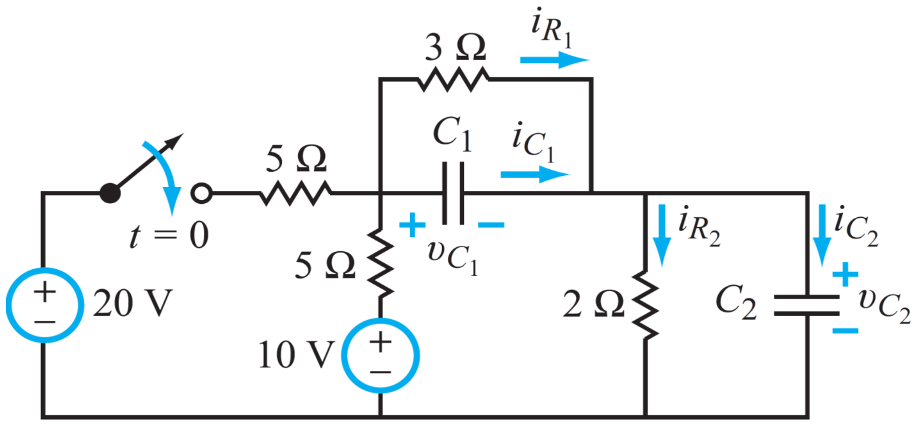
Do not remove pages, as this disrupts the scanning. If needed, cross out any parts that you don't want us to grade.

PROBLEM	MAX
1	n
2	n
3	n
4	n
5	n

"Bueller?... Bueller?... Bueller?"

- Ferris Bueller's Day Off

Problem 1 *Warm up* (15 points)
Consider the following circuit.



a) What is $i_{C1}(t=0)$? Show your work!

Solution:

b) What is $i_{R1}(t=0)$? Show your work!

Solution:

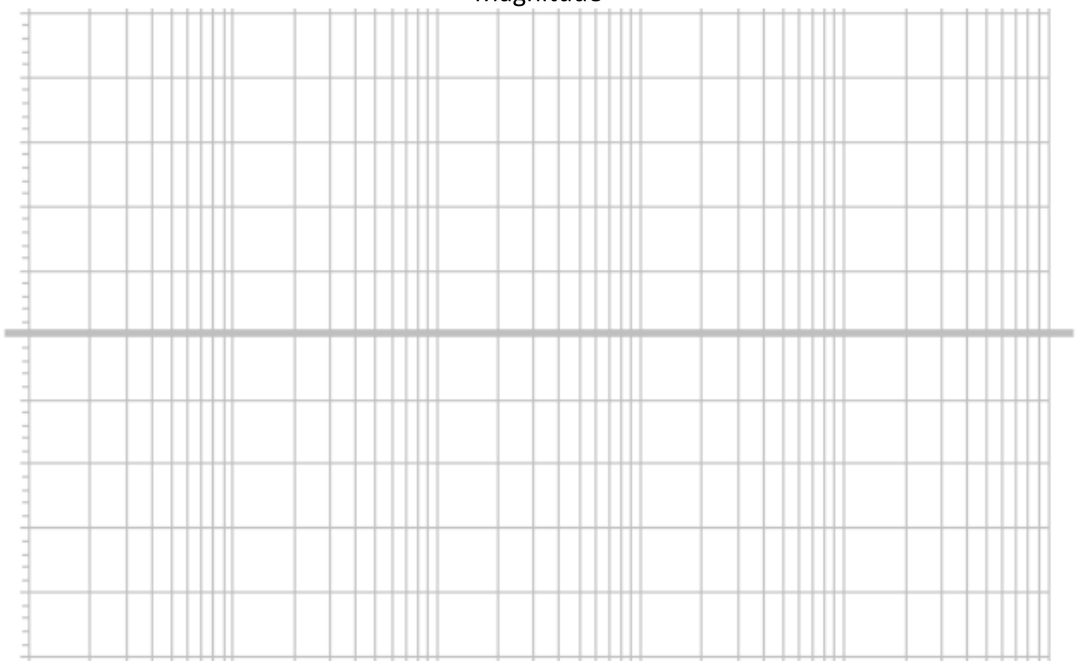
c) What is $v_{C1}(t=\infty)$? Show your work!

Solution:

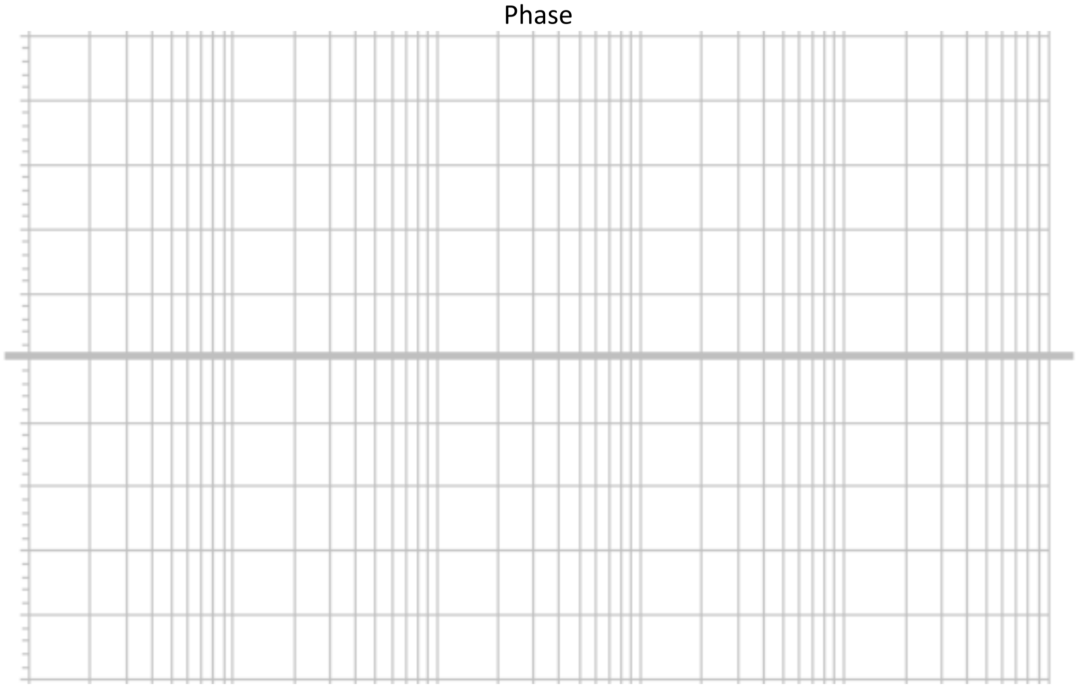
d) Generate a Bode magnitude and phase plot for the following transfer function. Properly label all axes.

$$\mathbf{H}(\omega) = \frac{j100\omega}{10 + j\omega}$$

Magnitude



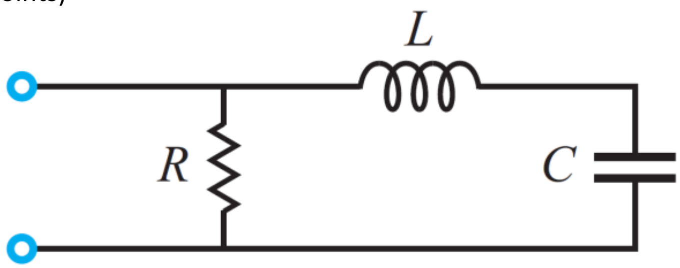
Phase



"Ash: I can't lie to you about your chances, but... you have my sympathies."

- *Alien*

Problem 2 *Resonance* (n points)



At what frequency or frequencies does the impedance across the two terminals become purely real? You must show your work to get full credit.

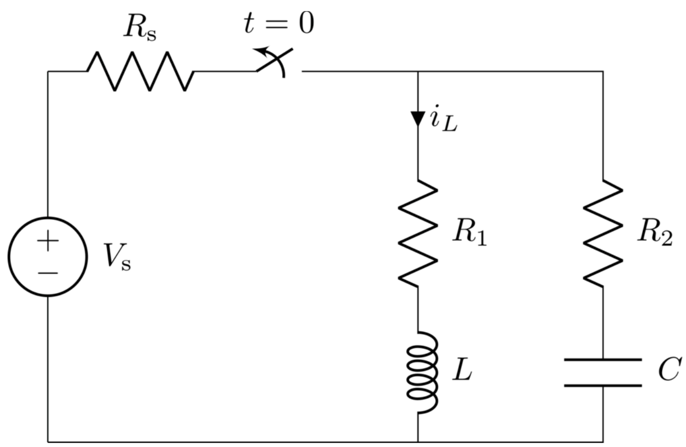
Solution:

"O-Ren Ishii: You didn't think it was gonna be that easy, did you?
The Bride: You know, for a second there, yeah, I kinda did.
O-Ren Ishii: Silly rabbit.
The Bride: Trix are...
O-Ren Ishii: ...for kids."

- Kill Bill Vol. 1

Problem 3 *Time, time, time…* (n points)

Consider the circuit below. Assume the switch was closed for all time until t = 0, when it was opened.



a) Provide a symbolic equation in one variable *that can be solved* to determine $v_C(t)$ for t ≥0.

Solution:

b) What is $v_C(0)$? Show your work.

Solution:

c) What is $i_L(0)$? Show your work.

Solution:

d) What is $v_C(\infty)$? Show your work.

Solution:

e) What is $i_L(\infty)$? Show your work.

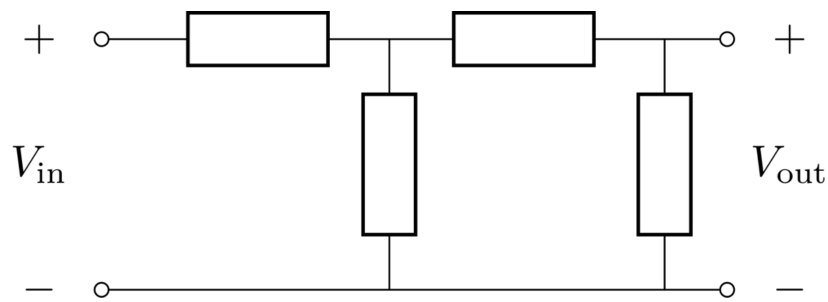
Solution:

"There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable.
There is another theory which states that this has already happened."
— *Douglas Adams, The Restaurant at the End of the Universe*

Problem 4 (n points)

You have three components on your workbench: a resistor, an inductor and a capacitor. Their values are R = 5 Ω, L = 20 mH, and C = 0.5 μF. You also have a short circuit and an open circuit. Wire them up inside the box below, making sure to connect all four wires below to your circuit such that the box acts as a bandpass filter.

WE STRONGLY SUGGEST YOU SHOW YOUR SYMBOLIC WORK BELOW (FOR PARTIAL CREDIT IF WRONG)!



a) What is ω_0 (the natural frequency) of your circuit?

Solution:

b) What are the -3 dB (cutoff) frequencies?

Solution:

c) What is the bandwidth of your filter?

Solution:

d) What is the Q?

Solution:

e) Is it possible to double the magnitude of Q by changing the values of L and/or C, while keeping ω_0 and R unchanged?

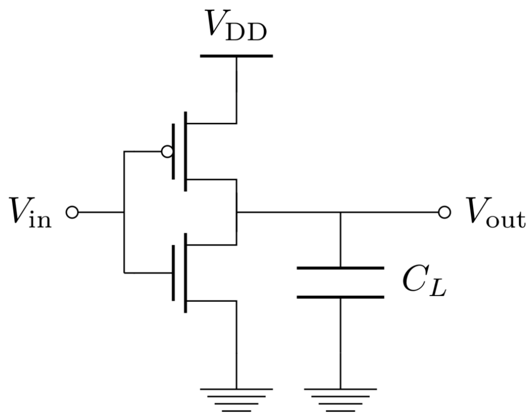
Circle one: YES NO

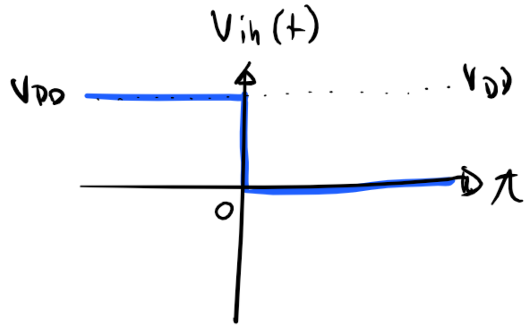
If yes, propose such values. If no, why not?

Solution:

"And so it goes..."
— *Kurt Vonnegut Jr., Slaughterhouse-Five*

Problem 5 (n points)
Consider the circuit below. Please apply the "switch with resistor" model of a transistor when solving this problem. Assume R_{DS} is the 'on' resistance and $|V_{th,n}| = |V_{th,p}| << V_{DD}$.





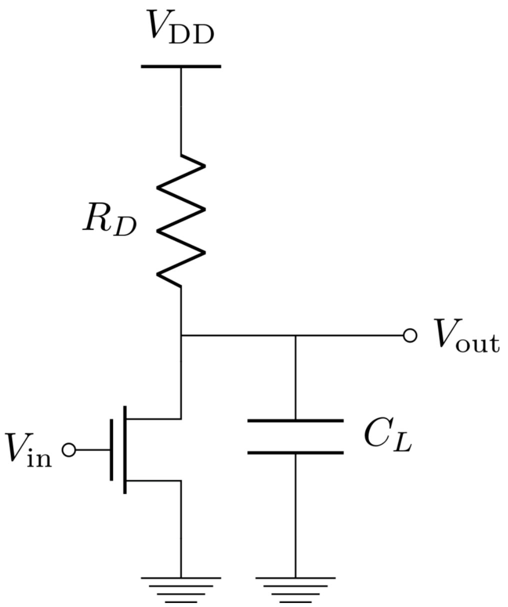
a) If $v_{in}(t)$ is as plotted above, please provide a differential equation in $v_{out}(t)$. (5 points)

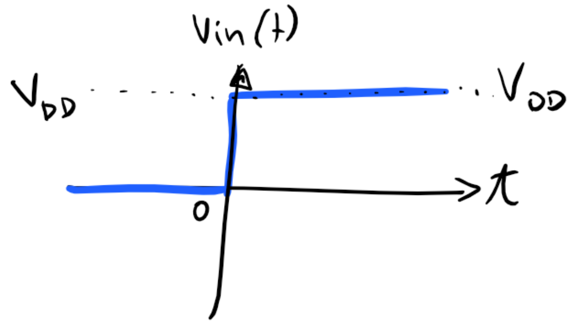
Solution:

b) Please provide an expression for $v_{out}(t)$. (5 points)

Solution:

c) Consider the circuit below. Please apply the "switch with resistor" model of a transistor when solving this problem. Assume $|V_{th,n}| = |V_{th,p}| << V_{DD}$.

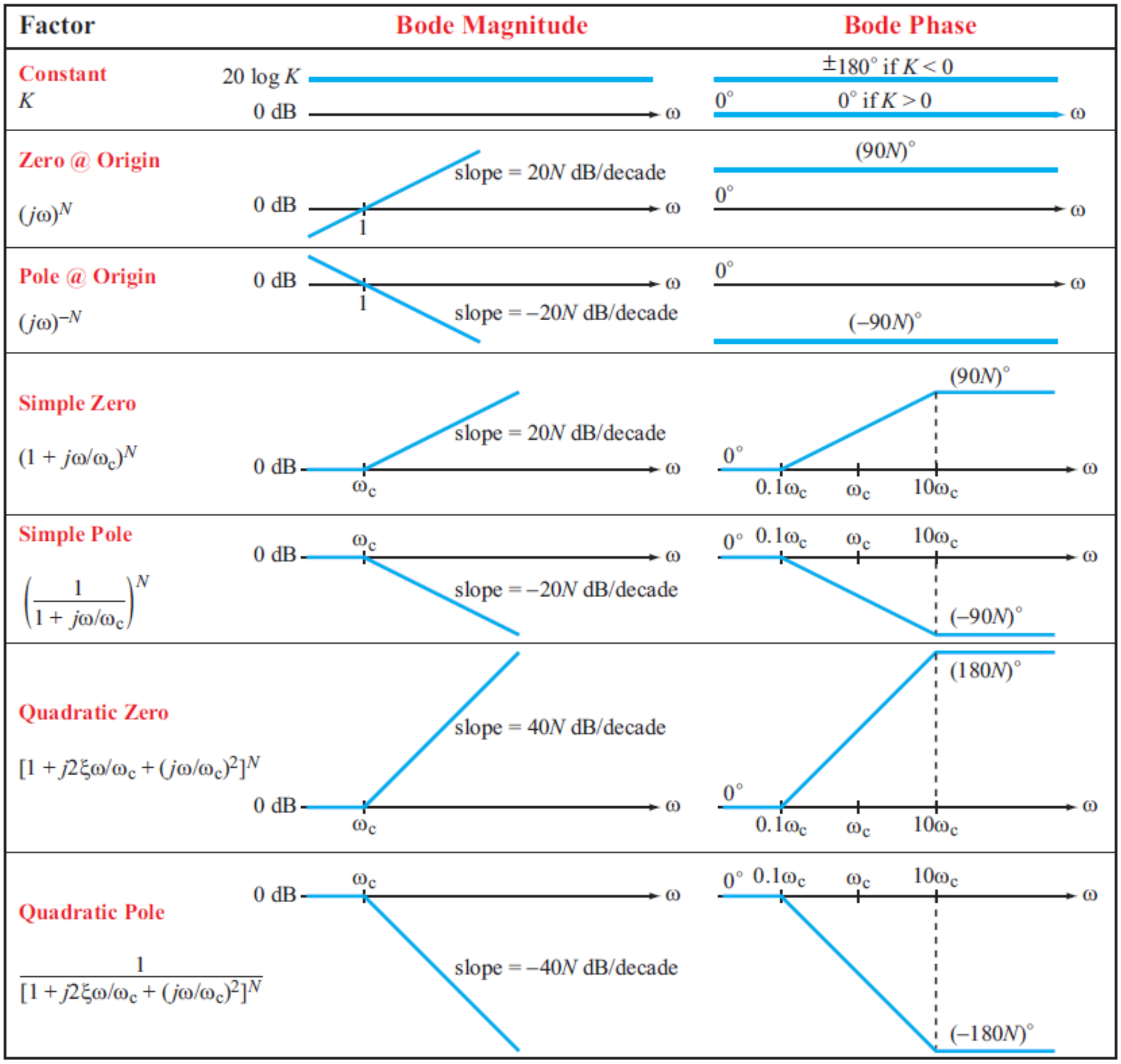




If $v_{in}(t)$ is as plotted above, please provide a differential equation in $v_{out}(t)$. (5 points)

Solution:

Extra Space


Factor
Bode Magnitude
Bode Phase
Constant
K
20 log K
0 dB
ω
±180° if K < 0
0°
0° if K > 0
Zero @ Origin
(jω)^N
slope = 20N dB/decade
1
(90N)°
Pole @ Origin
(jω)^−N
slope = −20N dB/decade
(−90N)°
Simple Zero
(1 + jω/ωc)^N
ωc
0.1ωc
10ωc
(90N)°
Simple Pole
(1 / (1 + jω/ωc))^N
slope = −20N dB/decade
(−90N)°
Quadratic Zero
[1 + j2ξω/ωc + (jω/ωc)^2]^N
slope = 40N dB/decade
(180N)°
Quadratic Pole
1 / [1 + j2ξω/ωc + (jω/ωc)^2]^N
slope = −40N dB/decade
(−180N)°

Contributors:

- Siddharth Iyer.
- Justin Yim.
- Edward Wang.