EE16B
Designing Information Devices and Systems II

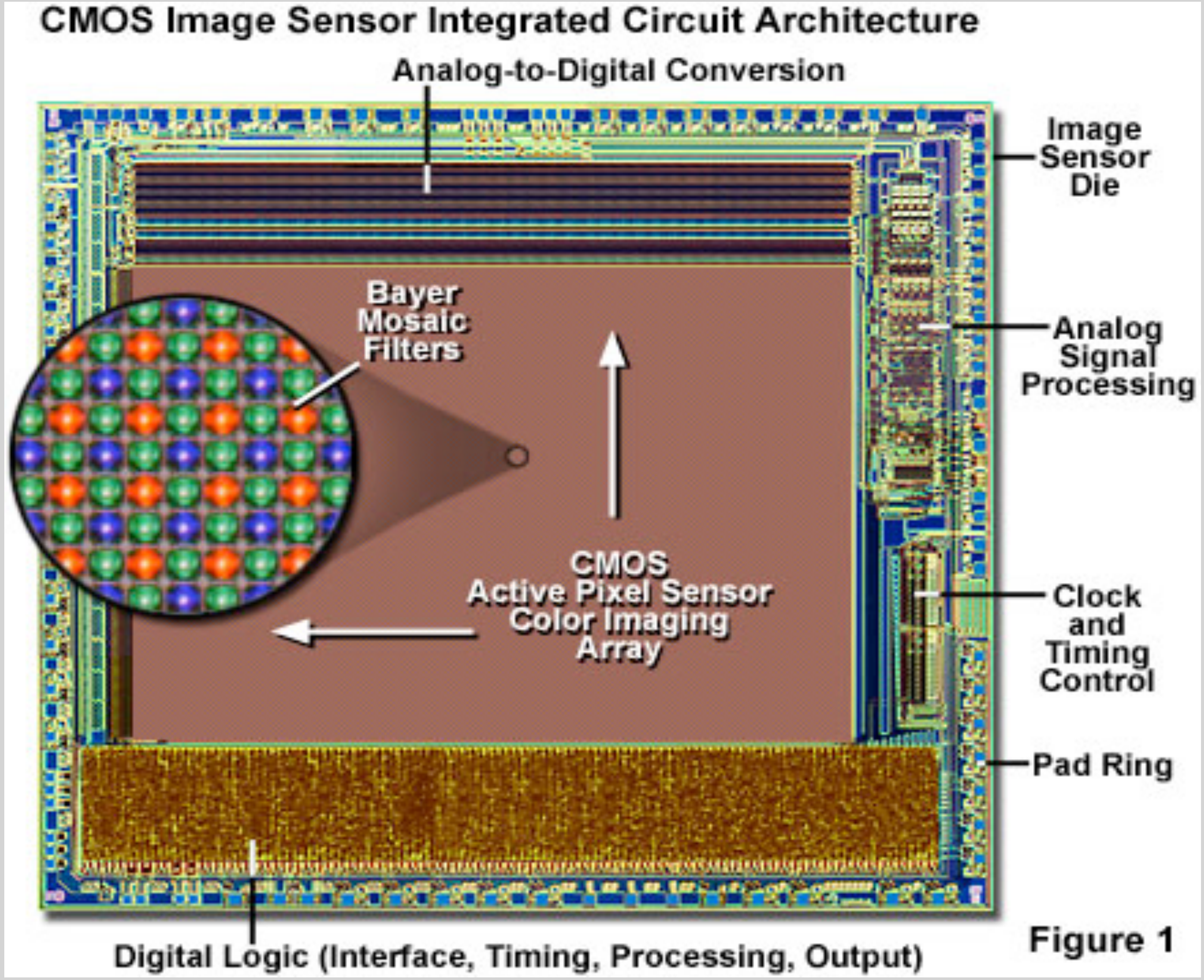
Lecture 10B
Sampling and Interpolation
Polynomial Interpolation

Sampling and Interpolation

- (Digital) Signal Processing - Only going to touch the surface
  - EE120, EE123, EE145A, EE121, EE225A, EE225B, CS 194-026, CS280

The Real World	Acquisition System (Physics/Physical)	Analog to Digital Conversion	Digital "magic": Processing, manupulation, interpretation Storage	Reconstruction & Digital to Analog Conversion
RF Audio Light Energy …	Conversion of X to electric Analog Filtering Amplification Modulation Microphone CCD/CMOS Accelerometers Antennas Radio-receivers …	Discrete values Discrete representation	Digital filtering Decoding Coding Compression Learning Rendering …	Audio Light RF Magnetic Display Communications …

Example Digital Imaging Camera

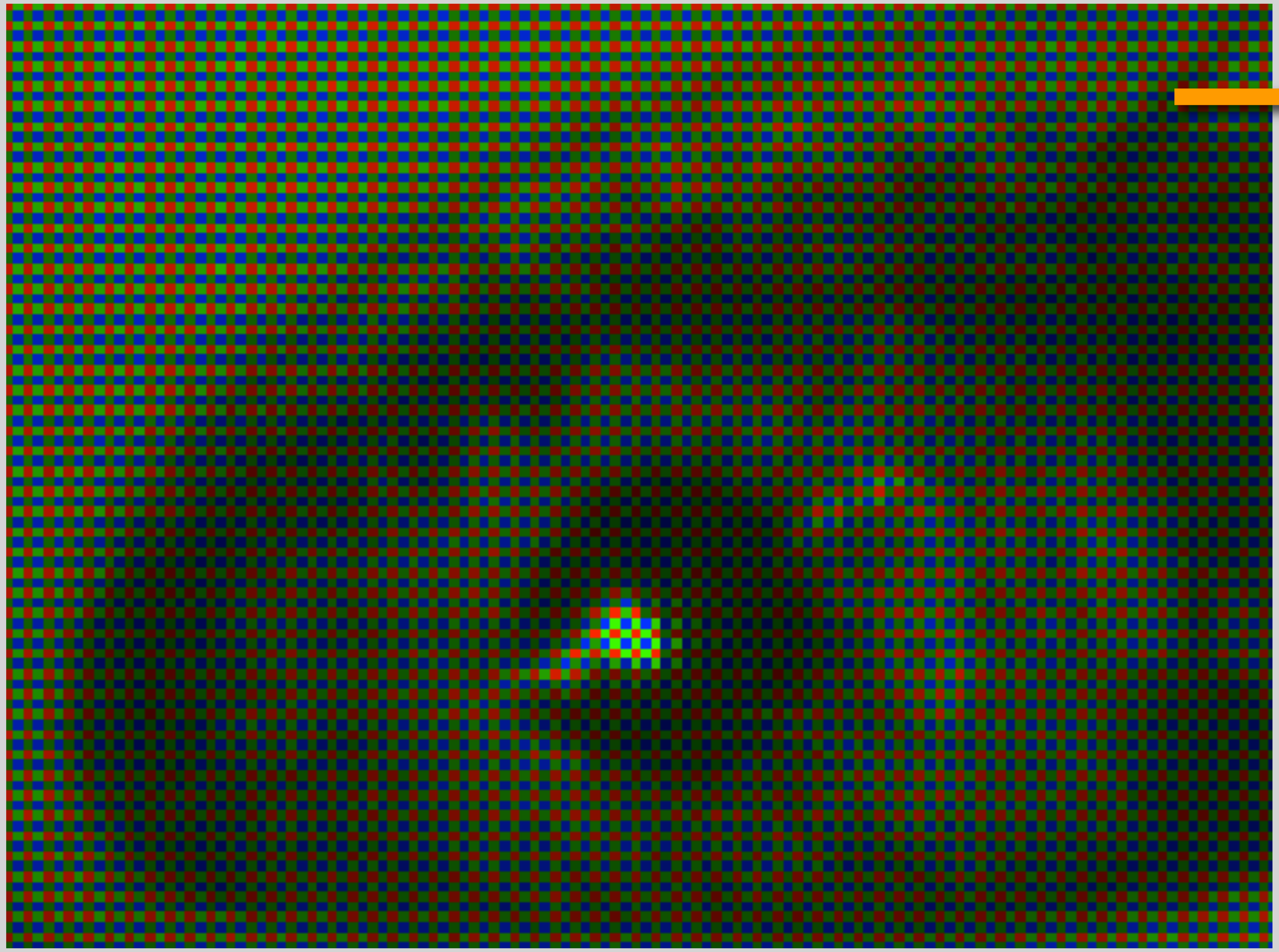


Focus/exposure Control → preprocessing → white-balancing → demosaic → Color transform → Post-processing → Compression

http://micro.magnet.fsu.edu/primer/digitalimaging/cmosimagesensors.html

Example: Digital Camera

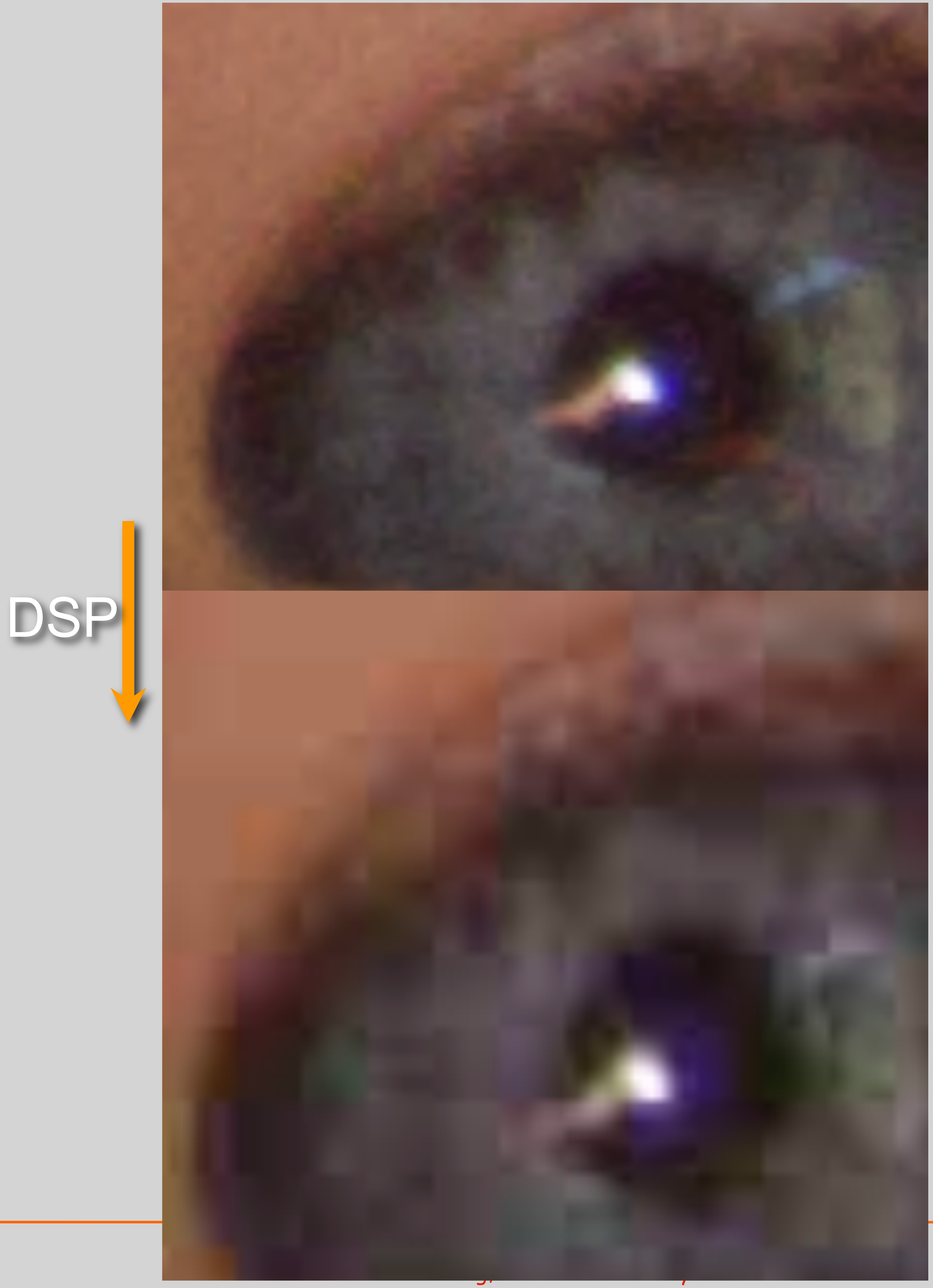
DSP



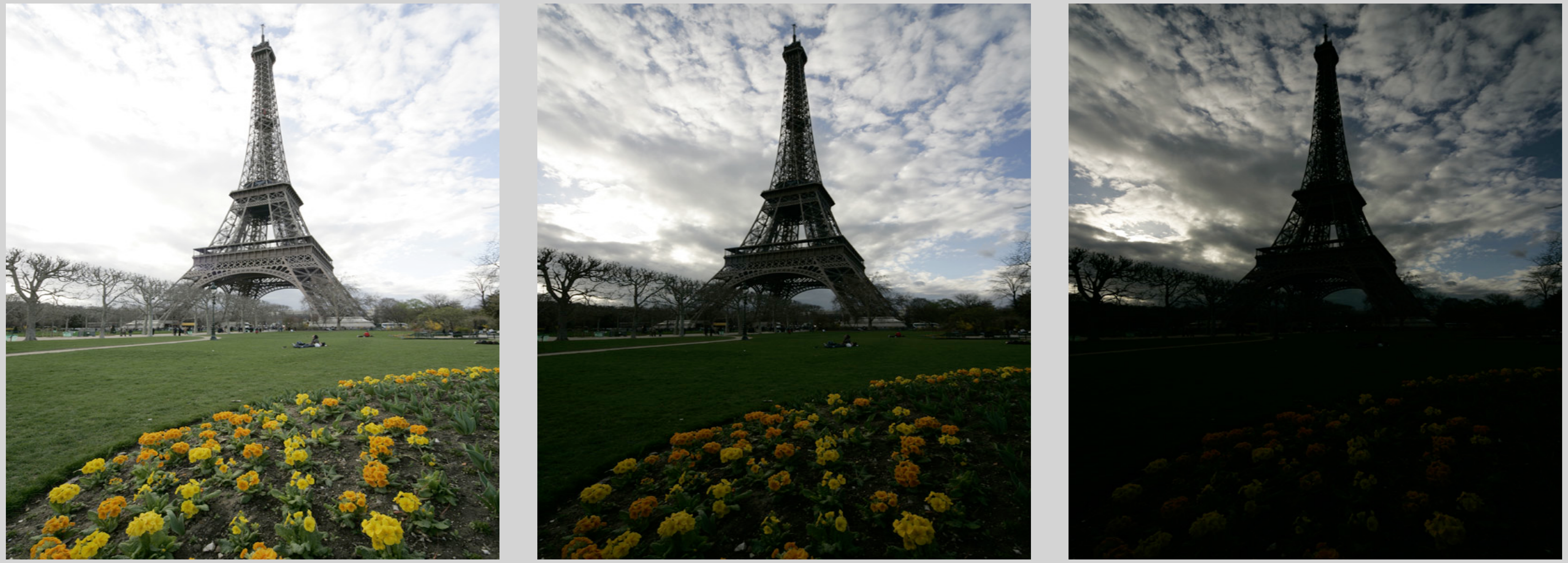
Example: Digital Camera

- Compression of 40x without perceptual loss of quality.

- Example of slight overcompression: difference enables x60 compression!



Computational Photography



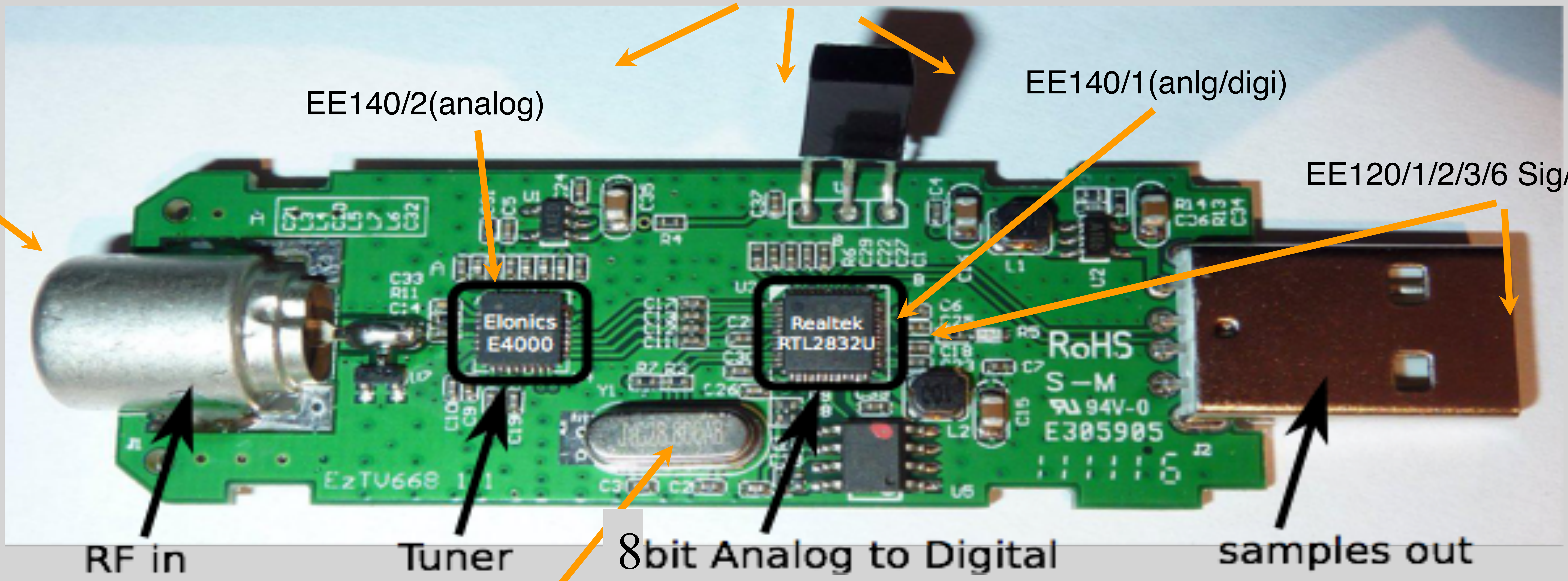
DSP

Implemented in all smart phones (HDR)

*www.hdrsoft.com

Software-Radio

- Inexpensive TV dongle based on RTL2832U and E4000 /820T chipset can be used as SDR

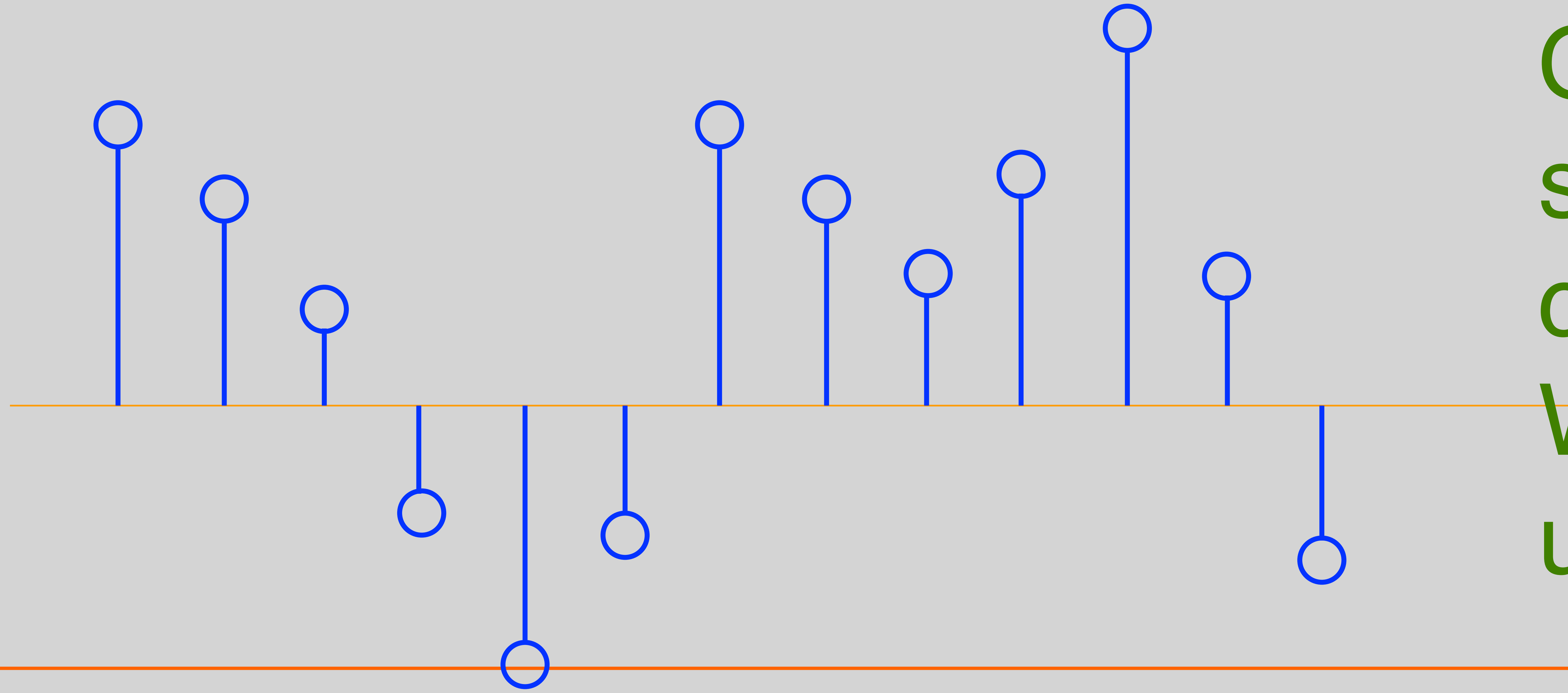


SDR Demo

Interpolation

- Given data points (x_i, y_i) i=1,2,… n find a continuous function that exactly matches the points.

$$y = f(x) \qquad \Rightarrow f(x_i) = y_i, \qquad i = 1,\ 2,\ 3\ , \cdots\ ,\ n$$



Q: If points are samples of a continuous-time. What would be the units?

Interpolation

- Given data points (x_i, y_i) i=1,2,… n find a continuous function that exactly matches the points.

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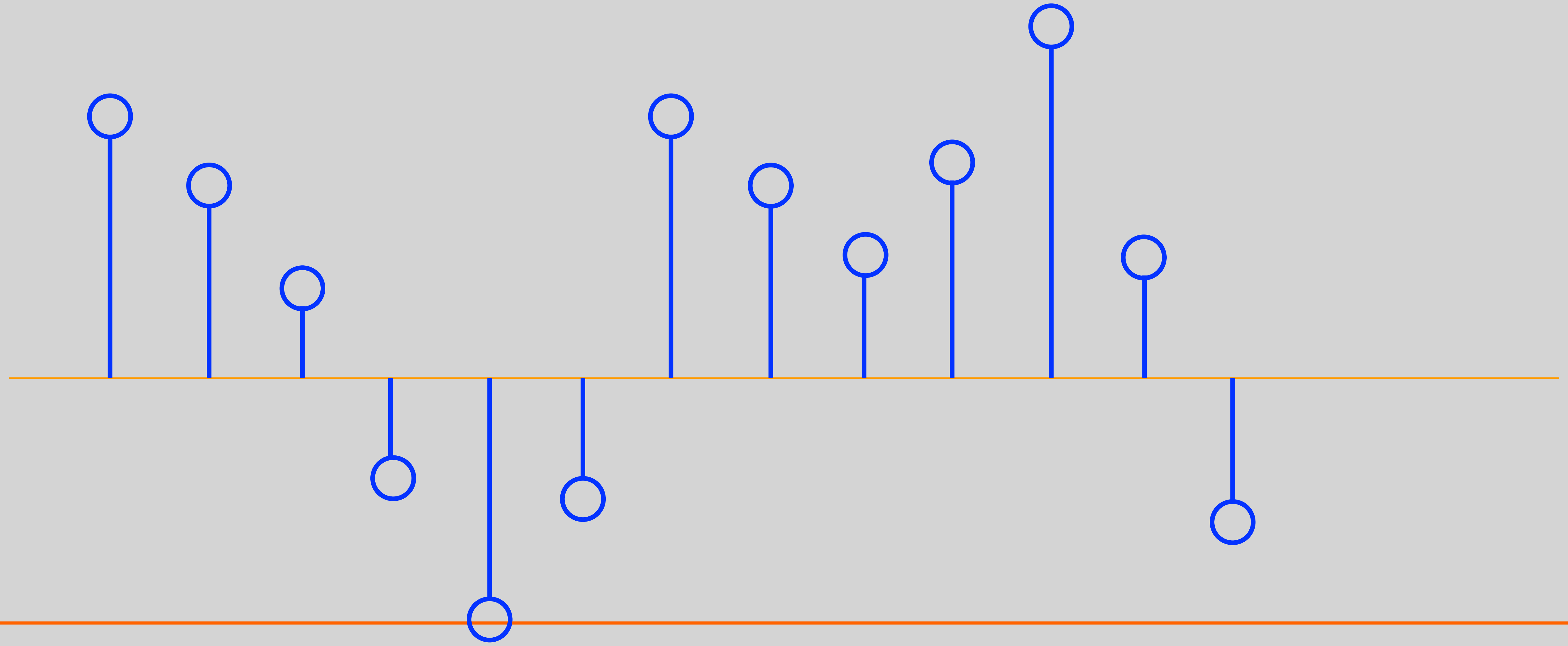
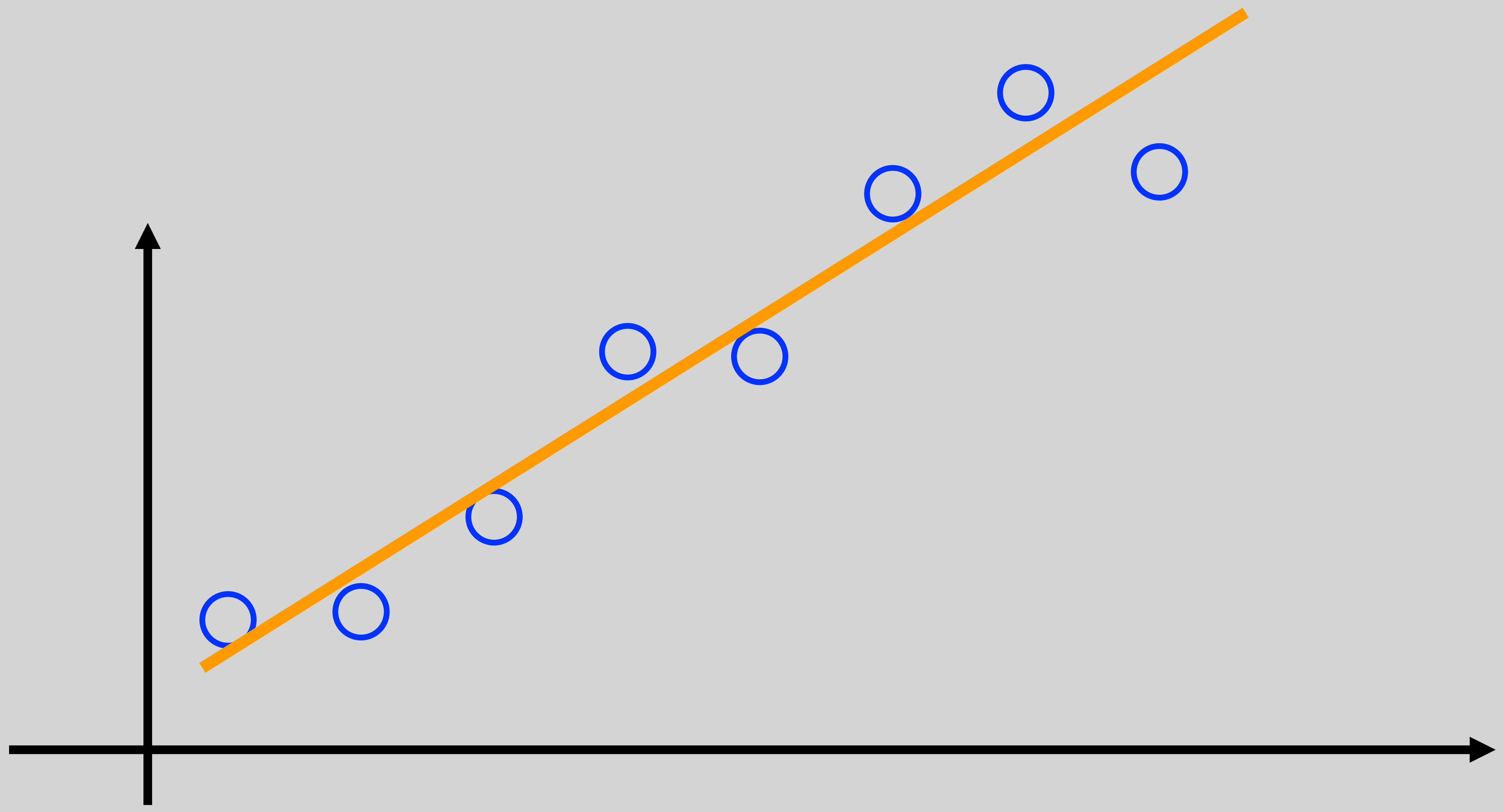
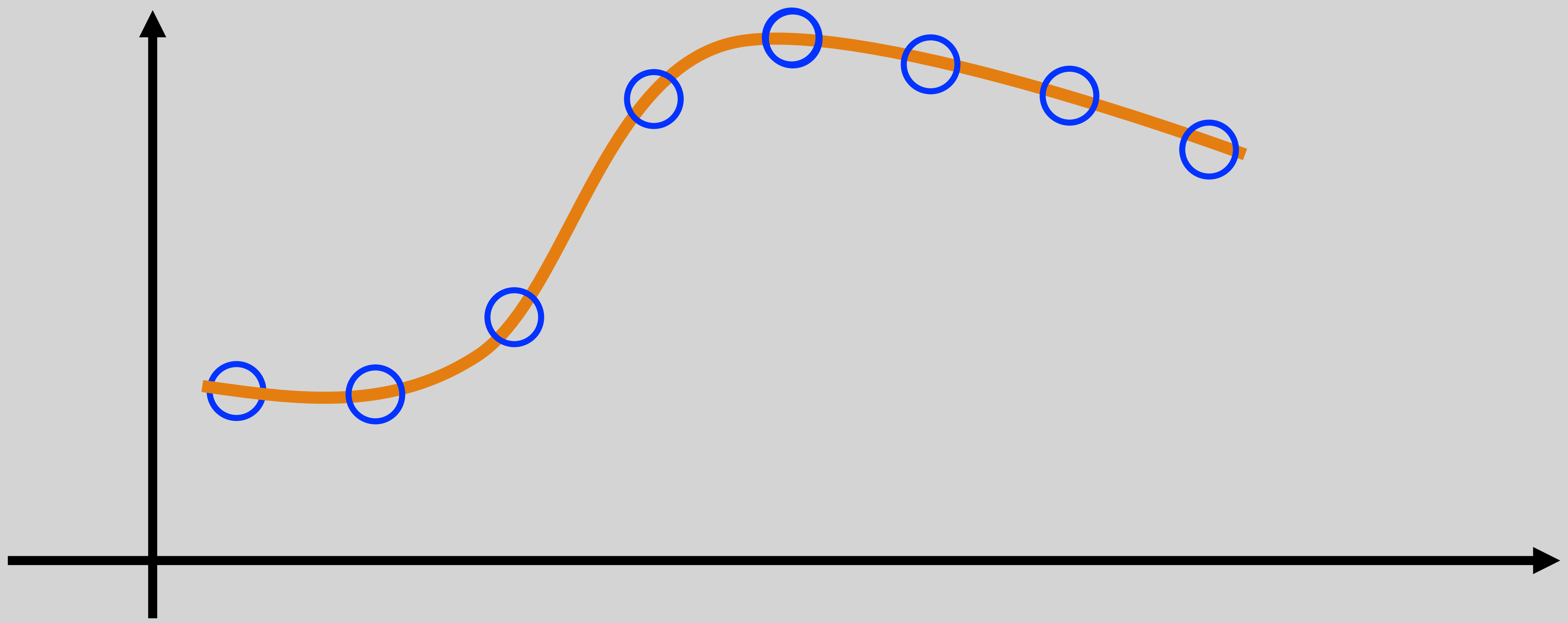


Regression Vs Interpolation

regression



interpolation

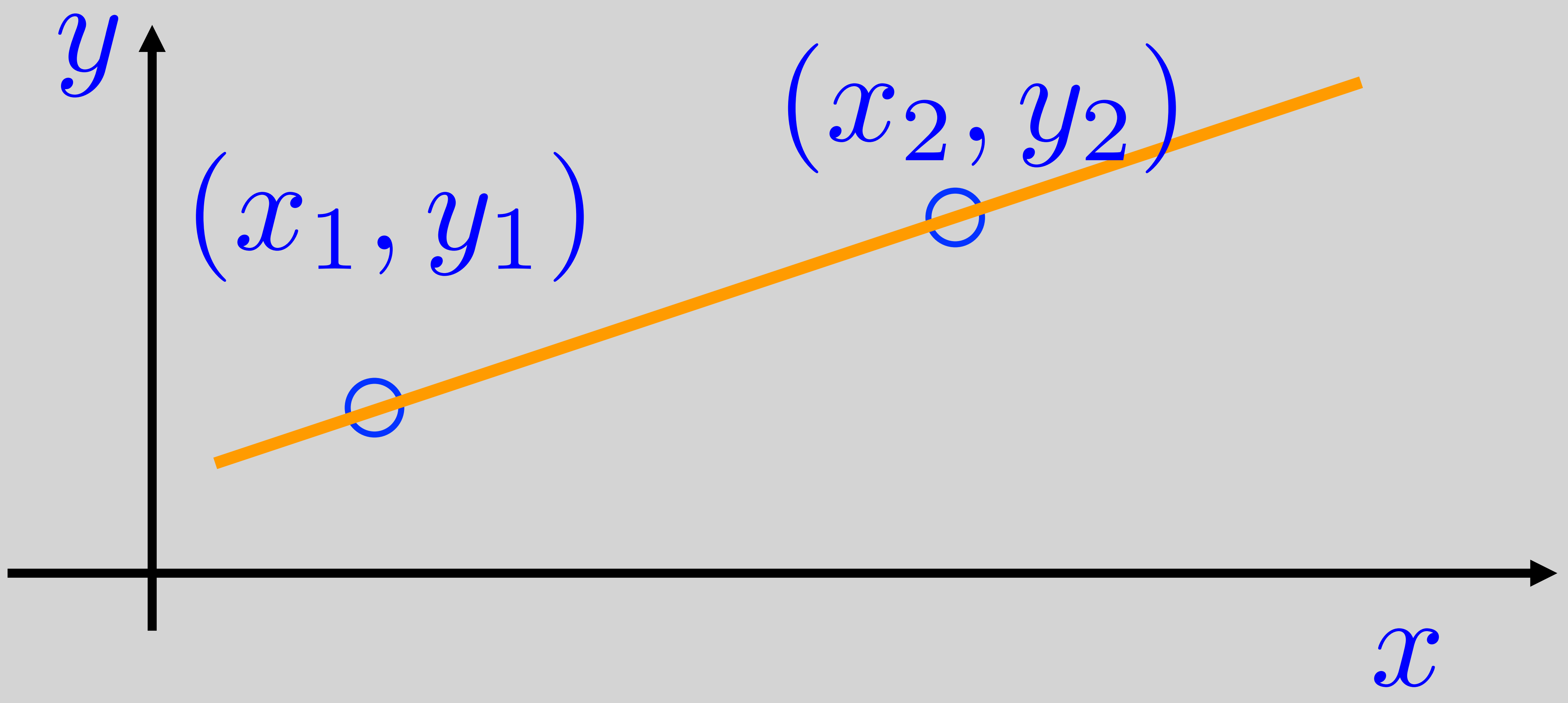


Interpolation Example

- Smile

Polynomial Interpolation

- Assumption:
  - Data are samples of a polynomial function (smooth)
  - Lowest order polynomial that exactly fits points



$$y = a_0 + a_1 x$$

$$a_0 + a_1 x_1 = y_1$$

$$a_0 + a_1 x_2 = y_2$$

$$\rightarrow \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Polynomial Interpolation

- Assumption:
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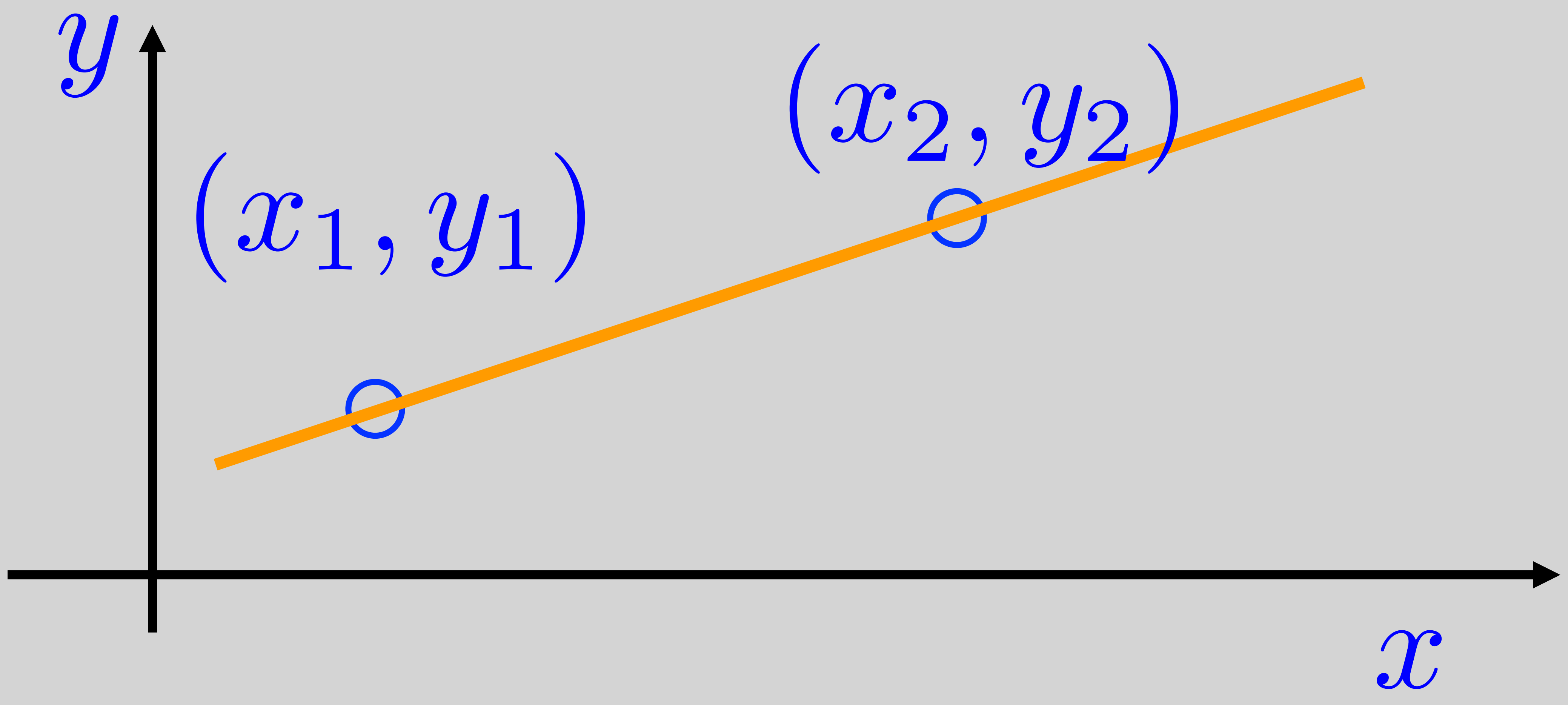


$$y = a_0 + a_1 x$$

$$\begin{aligned} a_0 + a_1 x_1 &= y_1 \\ a_0 + a_1 x_2 &= y_2 \end{aligned} \rightarrow \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

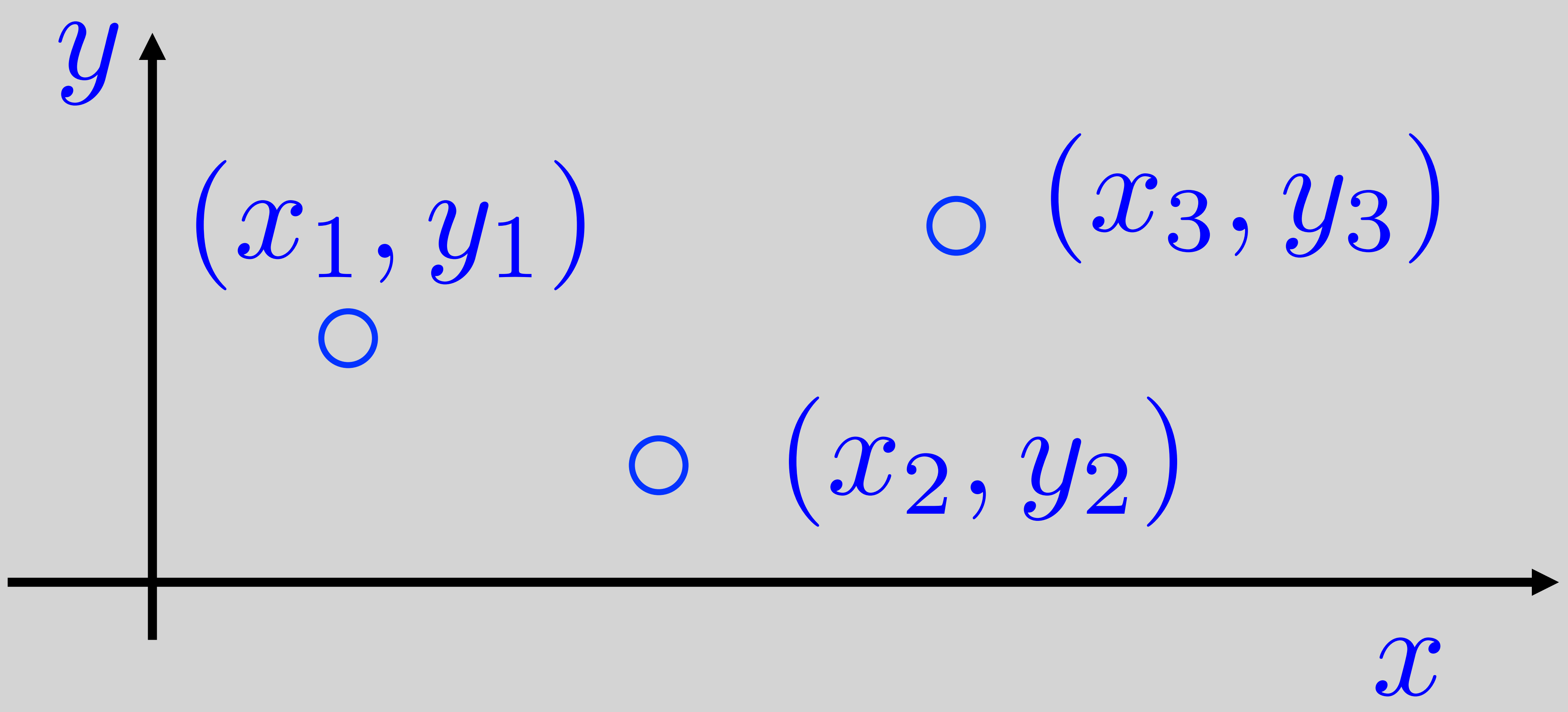
Invertible if $x_1 \neq x_2$

Polynomial Interpolation

$$(x_1, y_1)\ (x_2, y_2)\ (x_3, y_3)$$

$$y = a_0 + a_1 x + a_2 x^2 \rightarrow \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$x_1 \neq x_j$$



Polynomial Interpolation

- Given n distinct points, then there's exist a unique (n-1) order polynomial passing through them

$$y = a_0 + a_1 x + a_2 x^2 + \cdots a_{n-1} x^{n-1}$$

$$\rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Polynomial Interpolation

- Given n distinct points, then there's exist a unique (n-1) order polynomial passing through them

$$y = a_0 + a_1 x + a_2 x^2 + \cdots a_{n-1} x^{n-1}$$

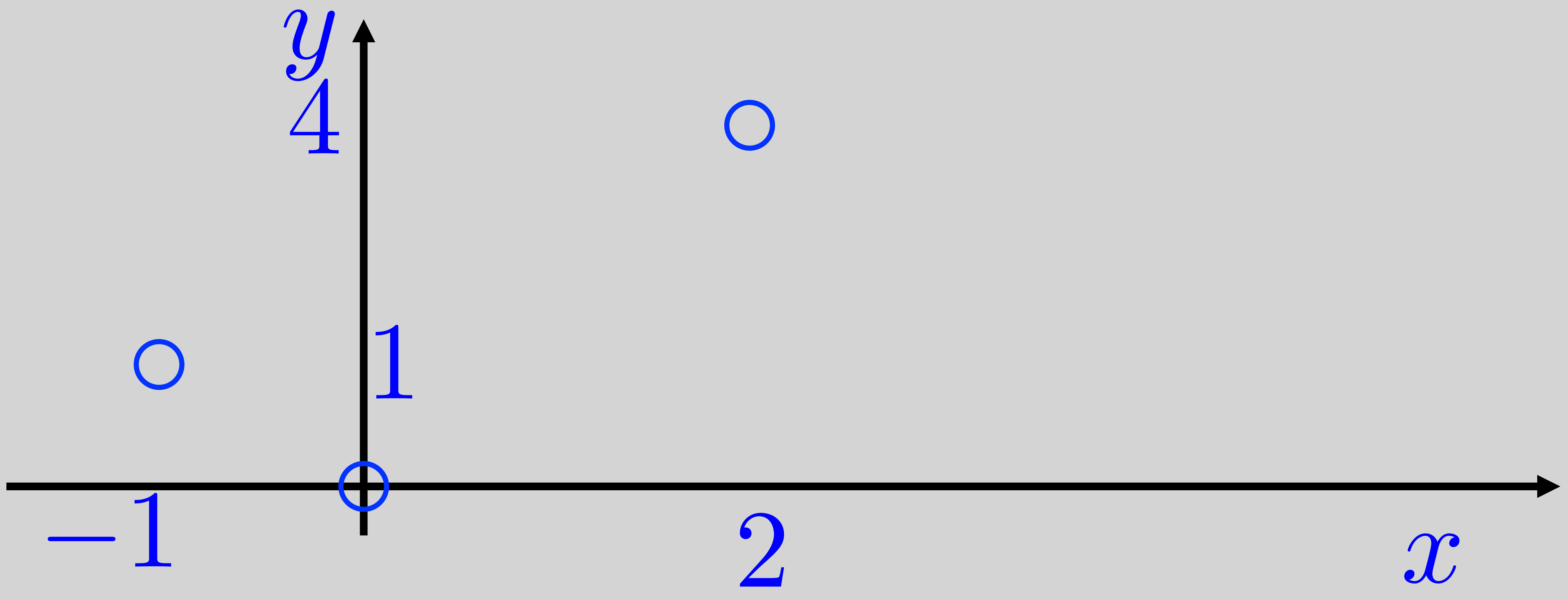
$$\rightarrow \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ & \vdots & & & \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

"Vandermonde" Matrix $\det(v) = \Pi_{1 \leq i < j \leq n}(x_j - x_i)$

Quiz

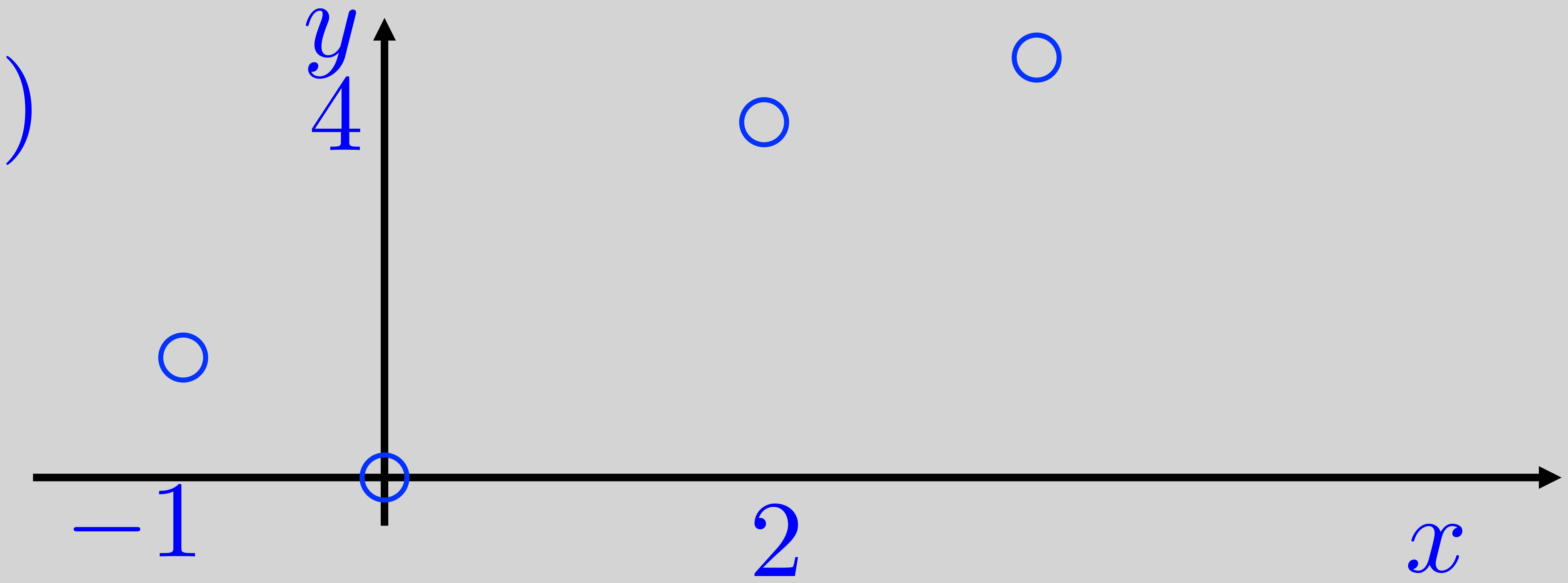
- What's the polynomial that passes through these points:

$$(-1, 1), (0, 0), (2, 4)$$



Polynomial Regression

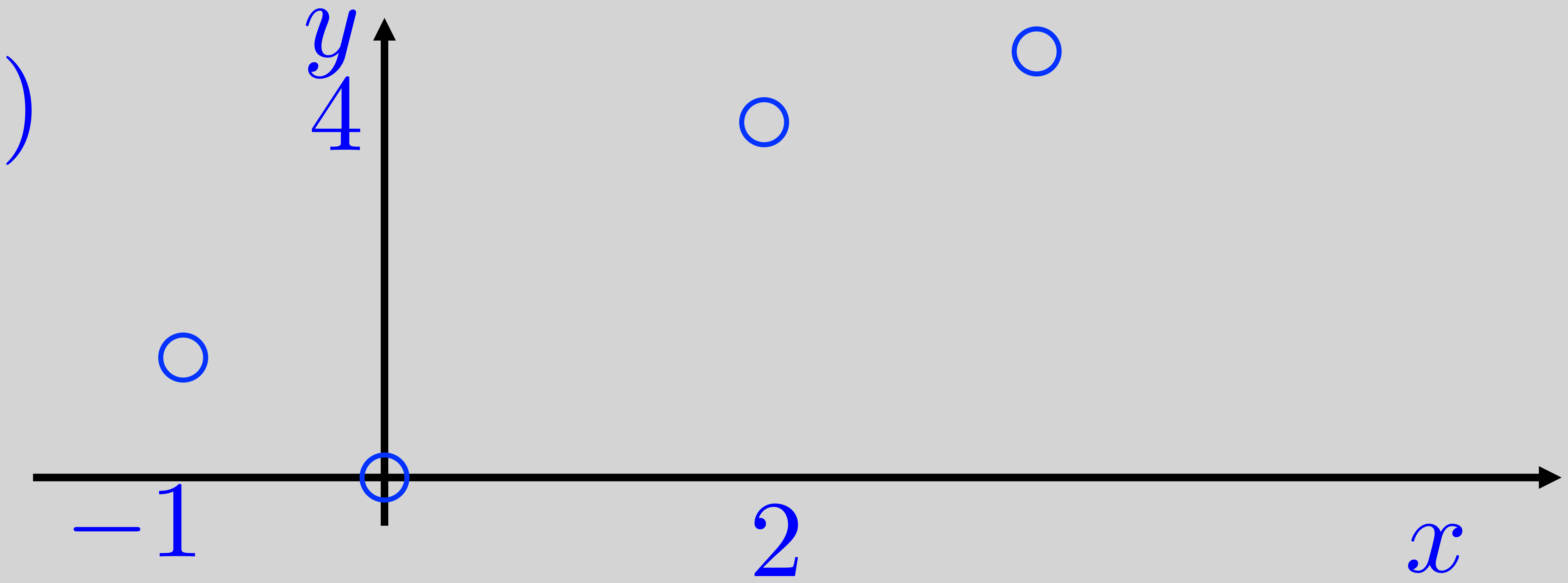
$(-1,1),(0,0),(2,4),(3,5)$



- What is the "best" cubic polynomial that passes through the points?

Polynomial Regression

$$(-1,1),(0,0),(2,4),(3,5)$$

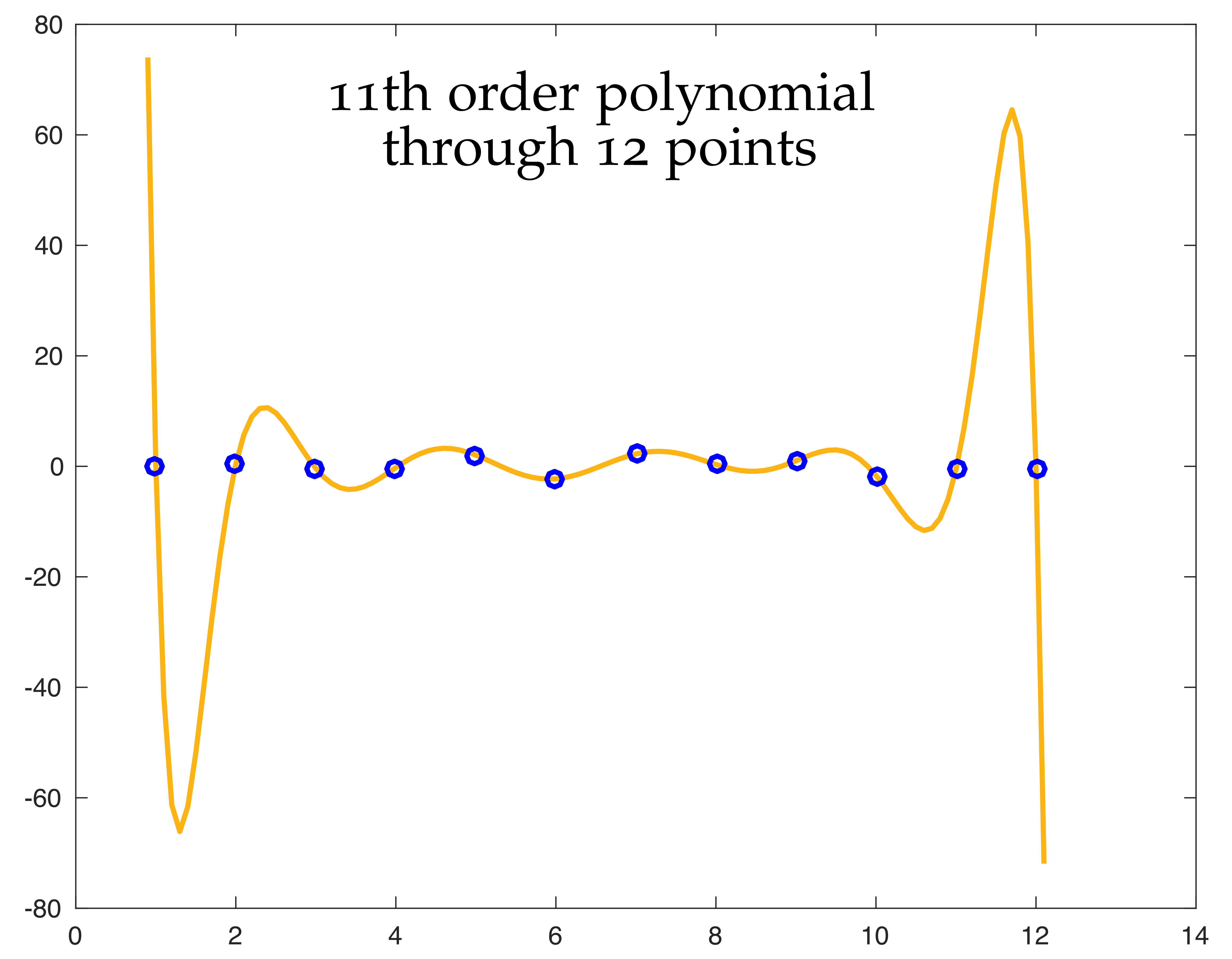


- What is the “best” cubic polynomial that passes through the points? $y = a_0 + a_1 x + a_2^2 + \cdots a_{n-1} x^{n-1}$

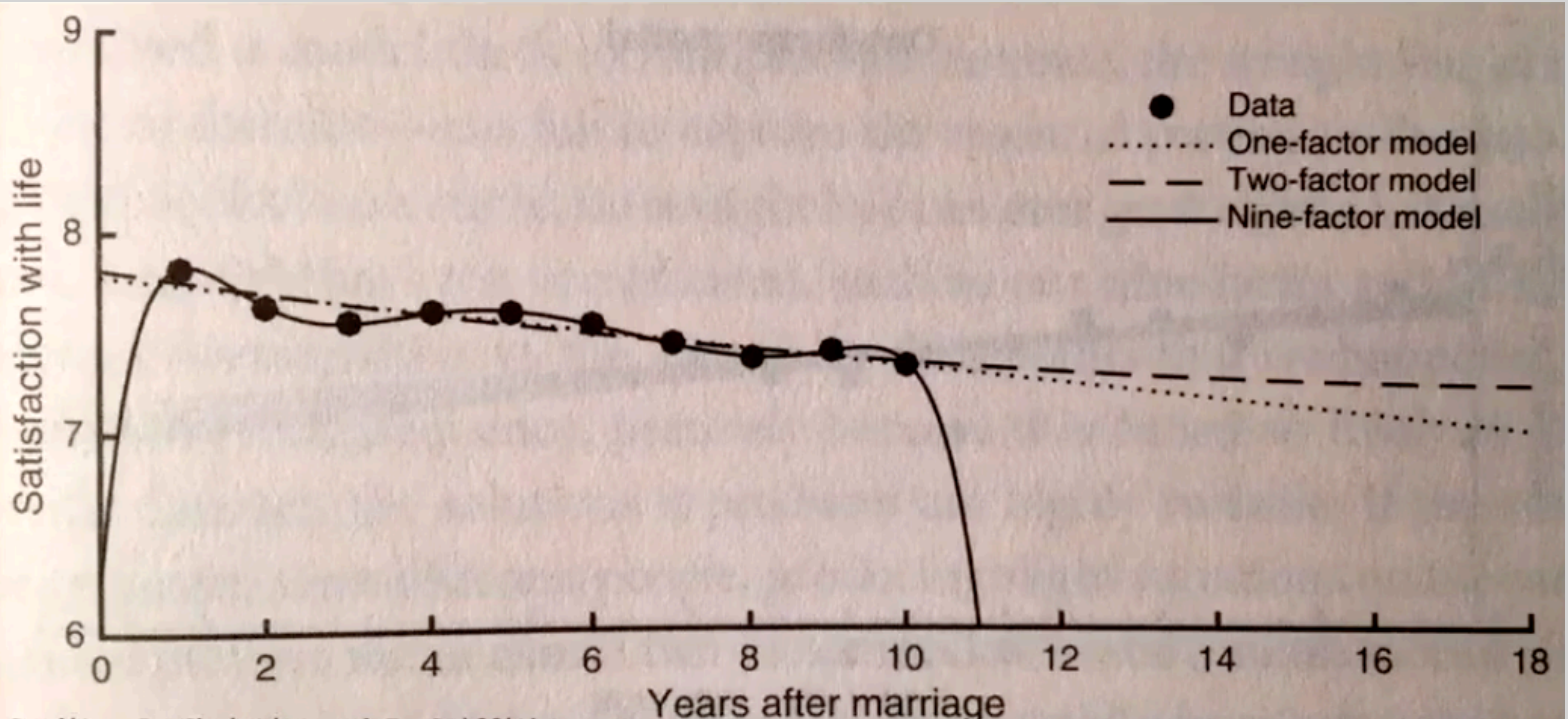
$$\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ 1 & x_4 & x_4^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Issue with Polynomial Interpolation

- Tend to be oscilatory in high order interp/regression
- Not numerically stable! x^{n-1}



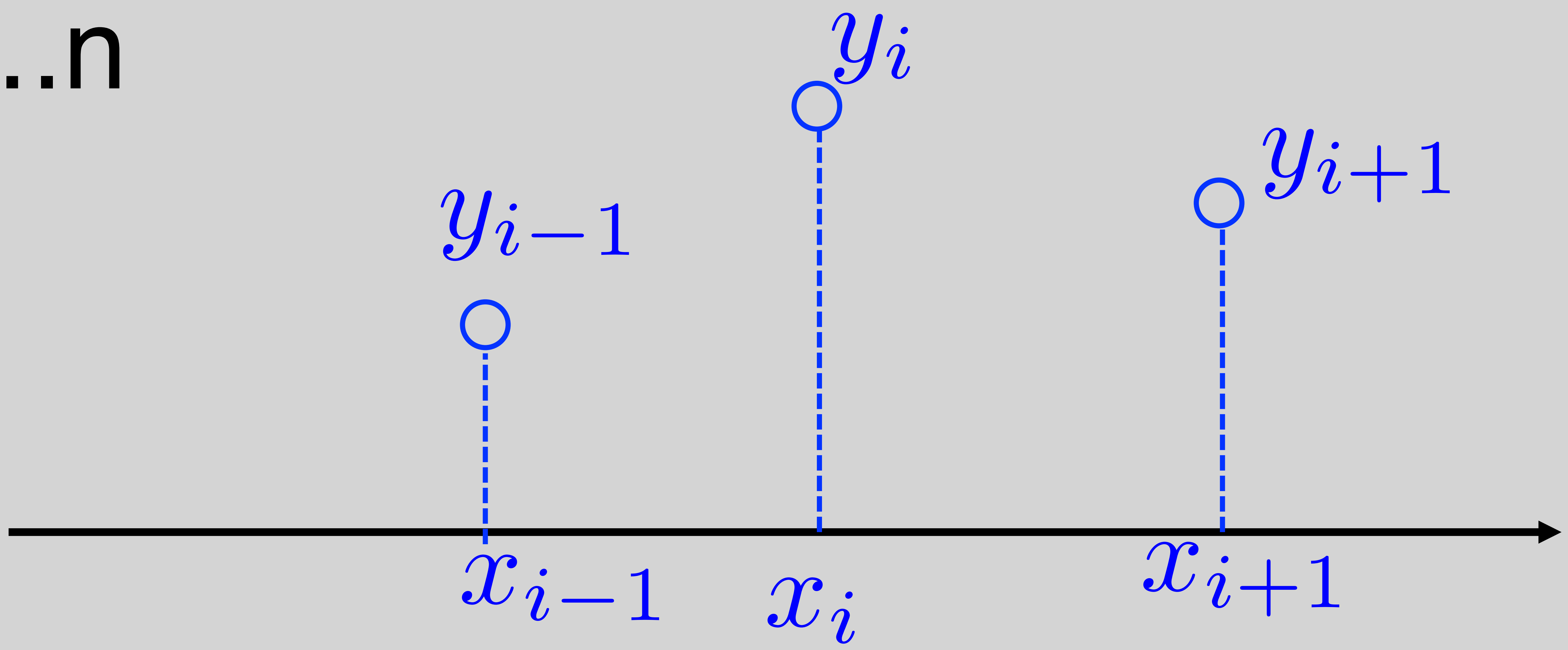
Example



Credit: B. Christian and T. Griffiths,

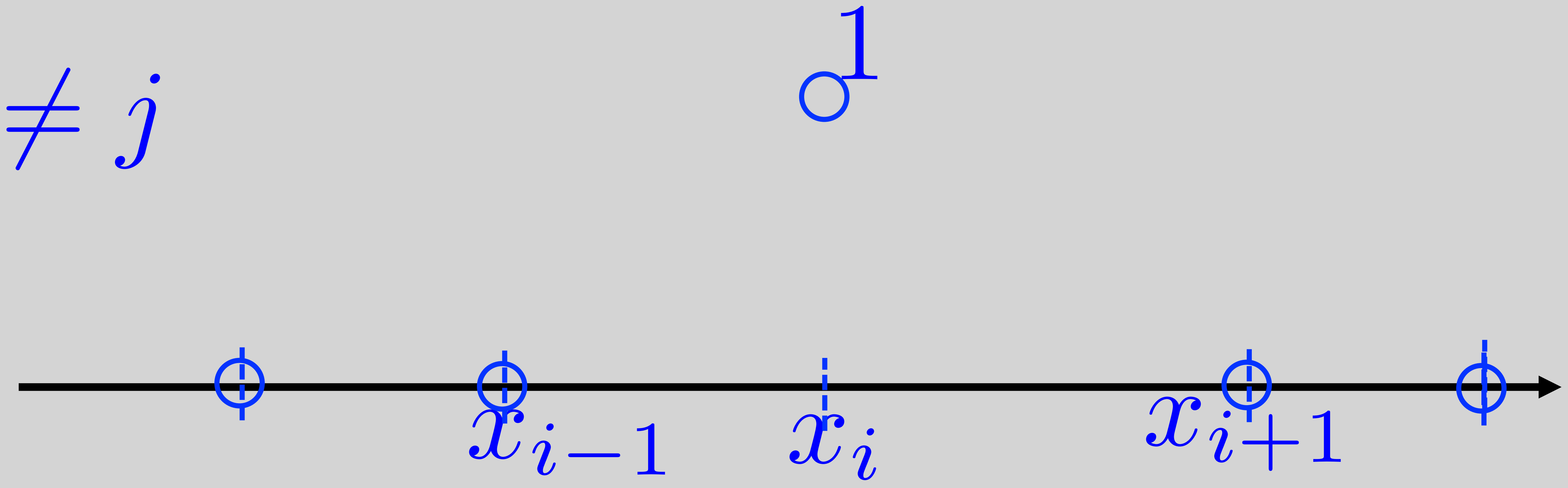
Interpolation with Basis Functions

- Given (x_i, y_i), i=1,….n

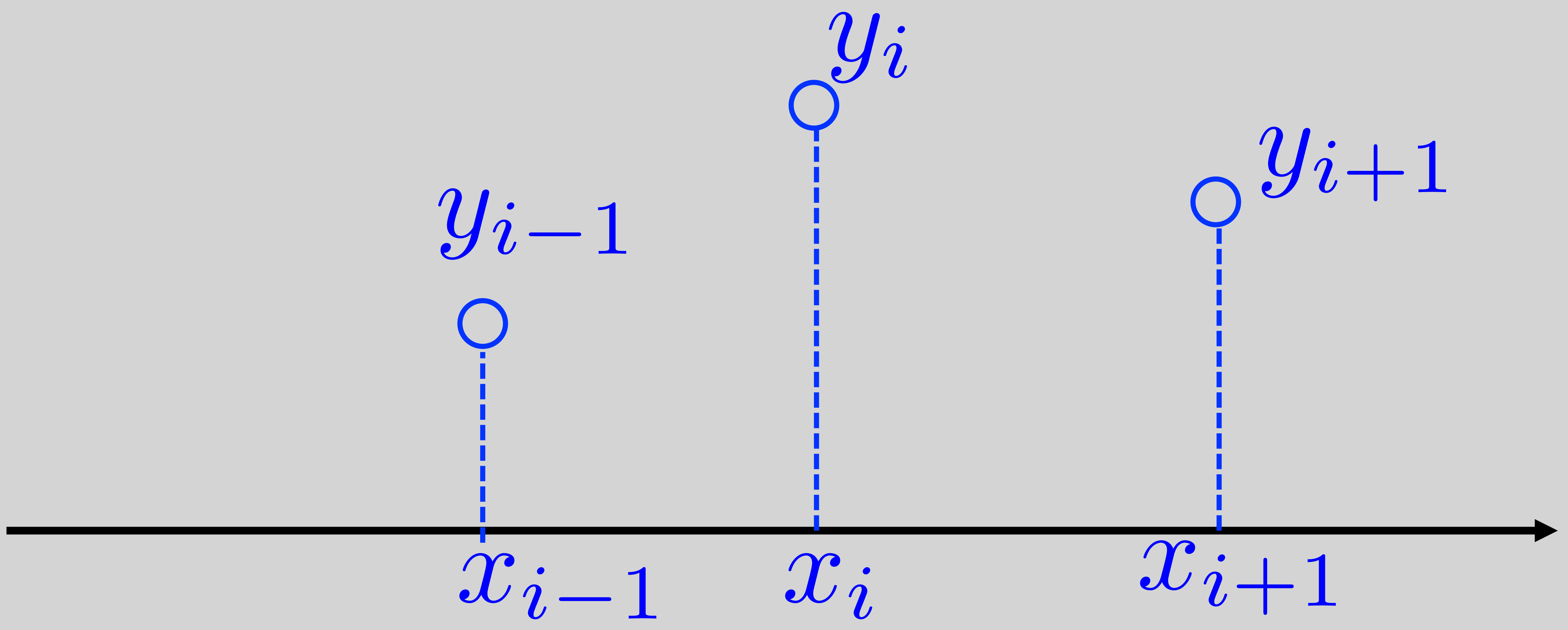


- Assign to each point a function $\Phi_i(x)$ s.t.

$$\Phi_i(x_i) = 1$$
$$\Phi_i(x_j) = 0 \quad i \neq j$$



Interpolation with Basis Functions



- Interpolation:

$$y(x) = \sum_{i=1}^{n} y_i \Phi_i(x) \quad \Rightarrow y(x_i) = \sum_{i=1}^{n} y_i \Phi_i(x_i) = y_i$$

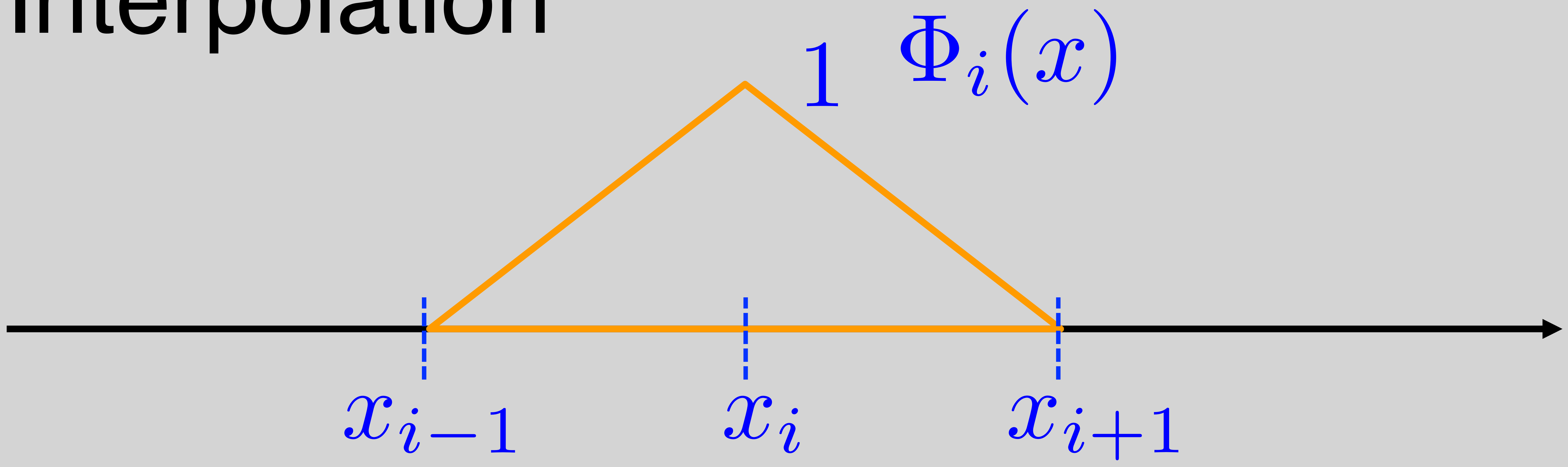
Interpolation with Basis Functions

- Guarenteed value of known points
- Control of continuous function behaviour between known points

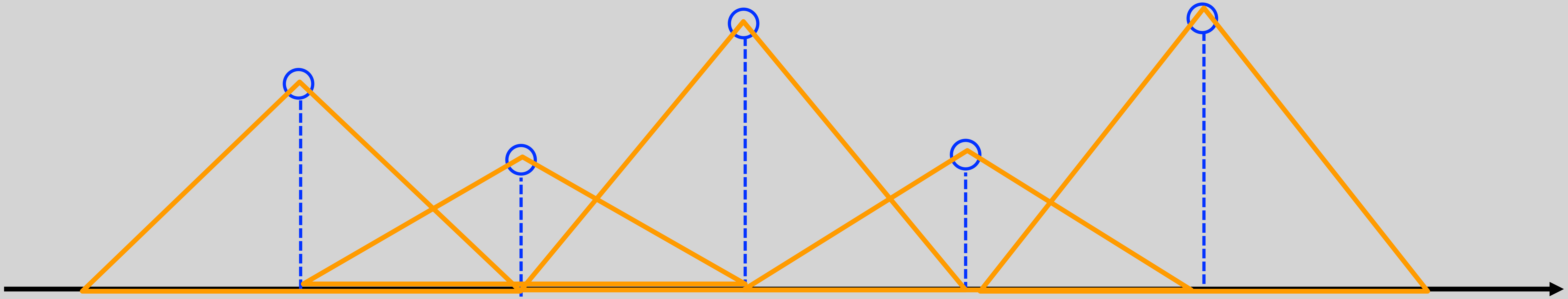
- Often used for equispaced points

Example:

- Linear Interpolation

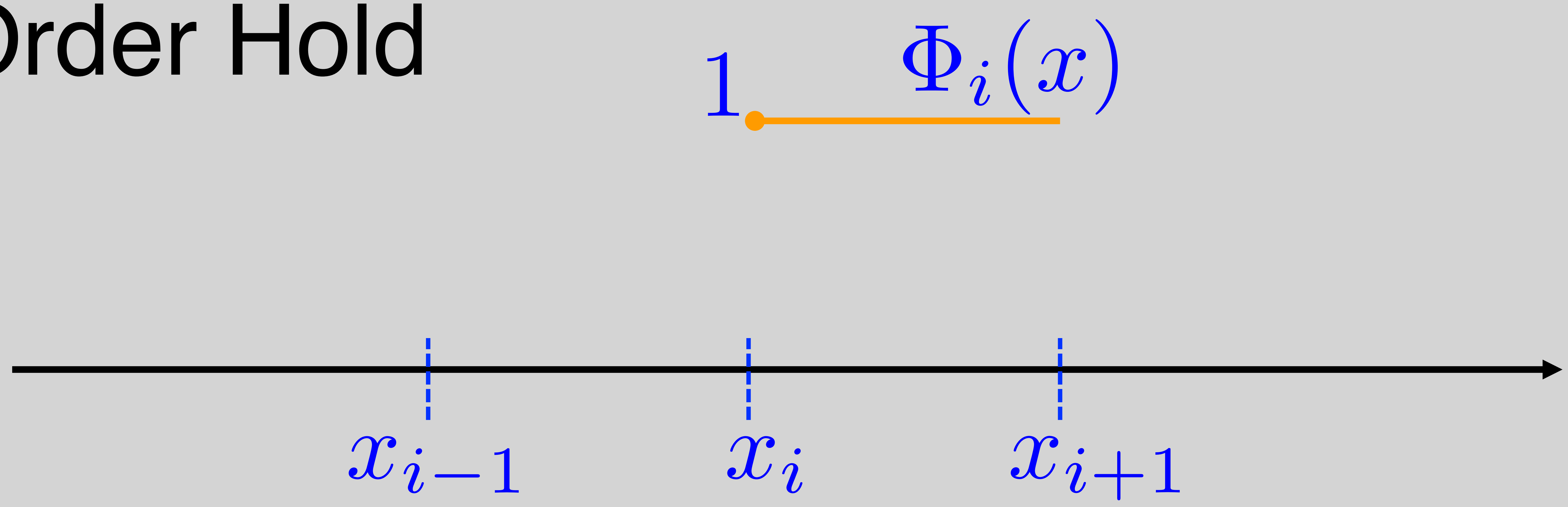


$$y(x) = \sum_{i=1}^{n} y_i \Phi_i(x)$$

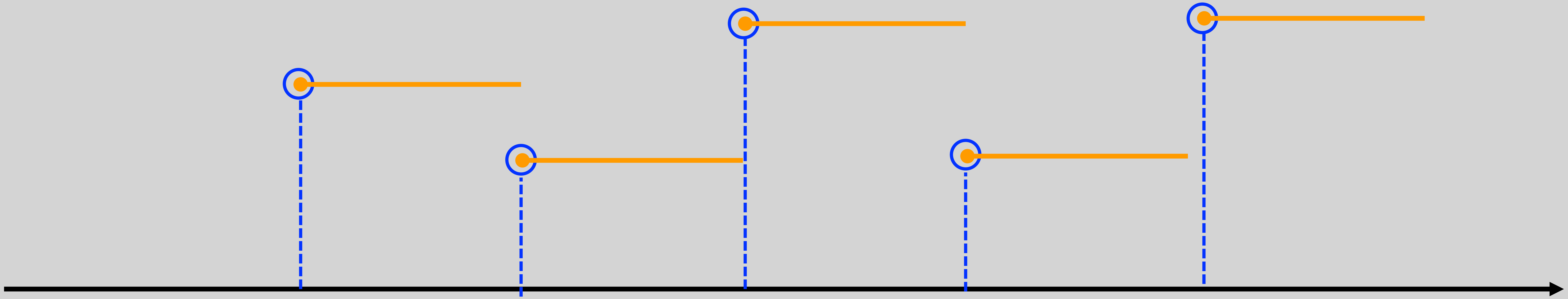


Example:

- Zero-Order Hold



$$y(x) = \sum_{i=1}^{n} y_i \Phi_i(x)$$



Interpolating Michel

Zero-order-hold

linear