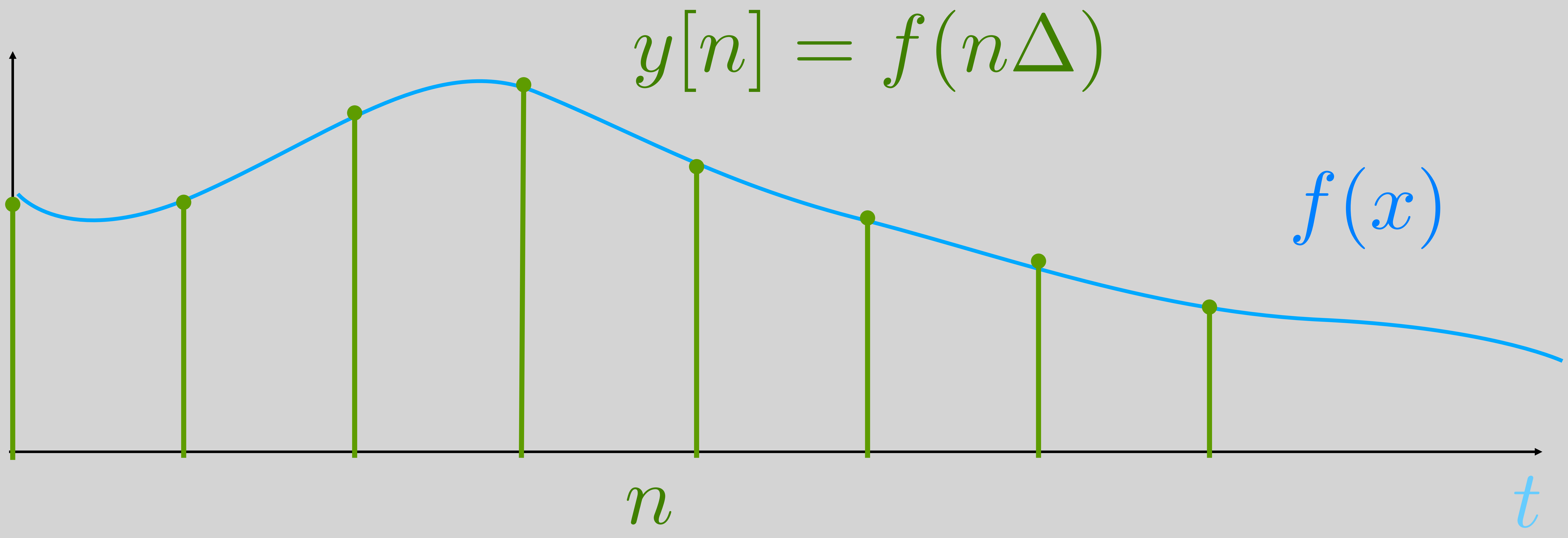
EE16B
Designing Information Devices and Systems II

Lecture 12A
Discrete Signals and Systems

Discrete Time Signals

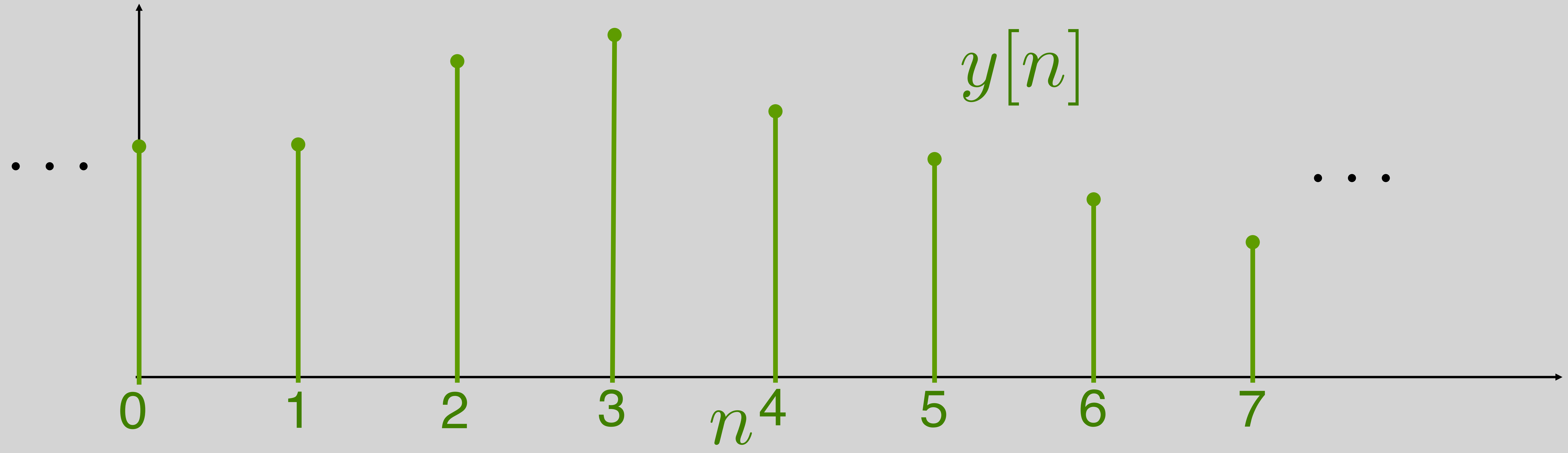
- Samples of a CT signal:



- Or, inherently discrete (Examples?)

Discrete Time Signals

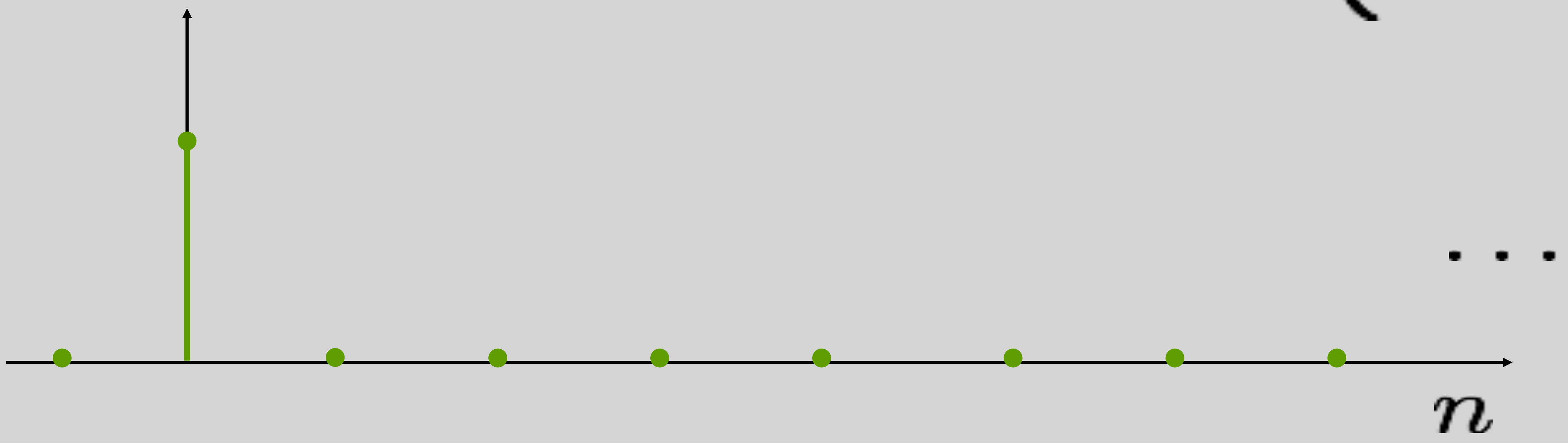
- At their core are "just samples"!



Basic Sequences

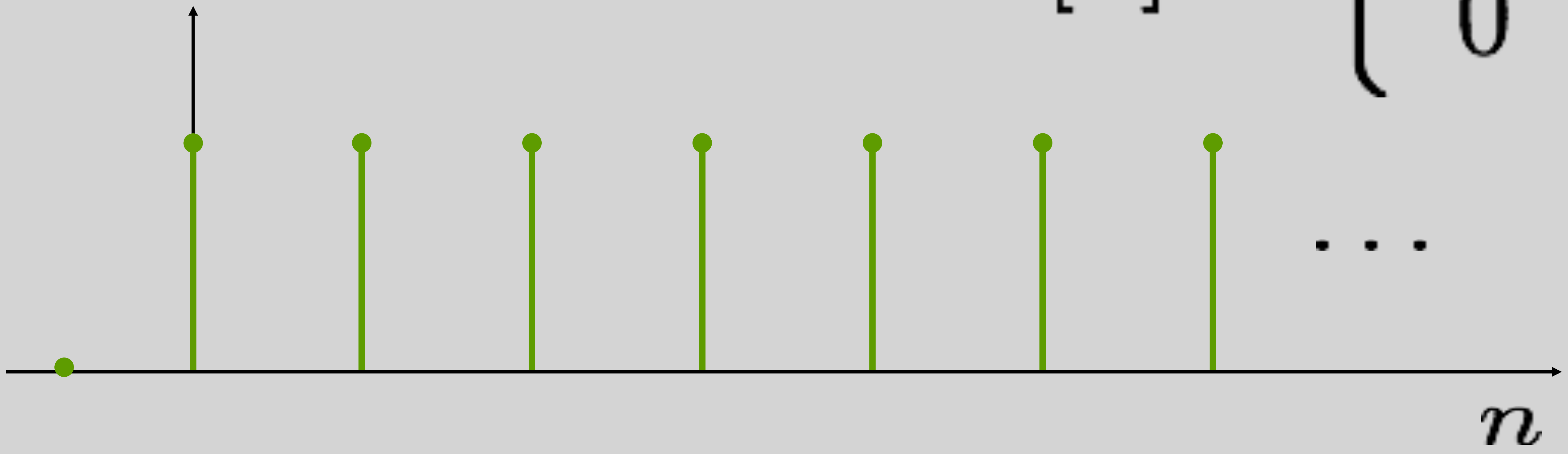
- Unit Impulse

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



- Unit Step

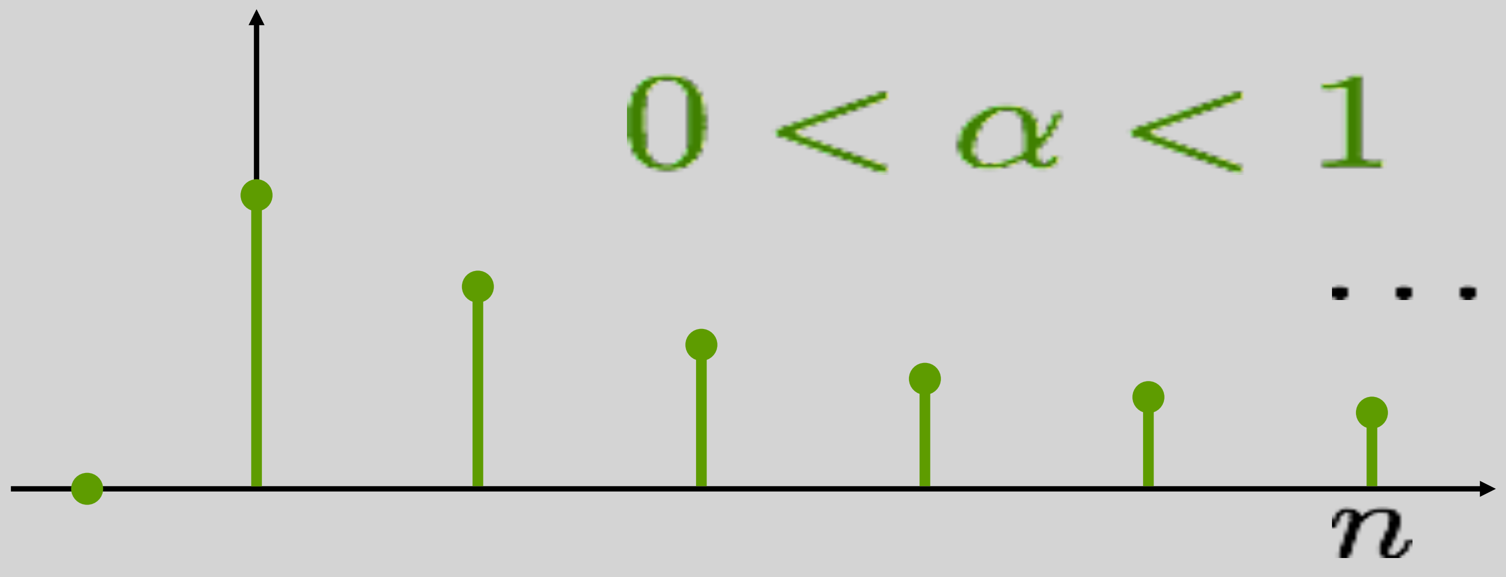
$$U[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

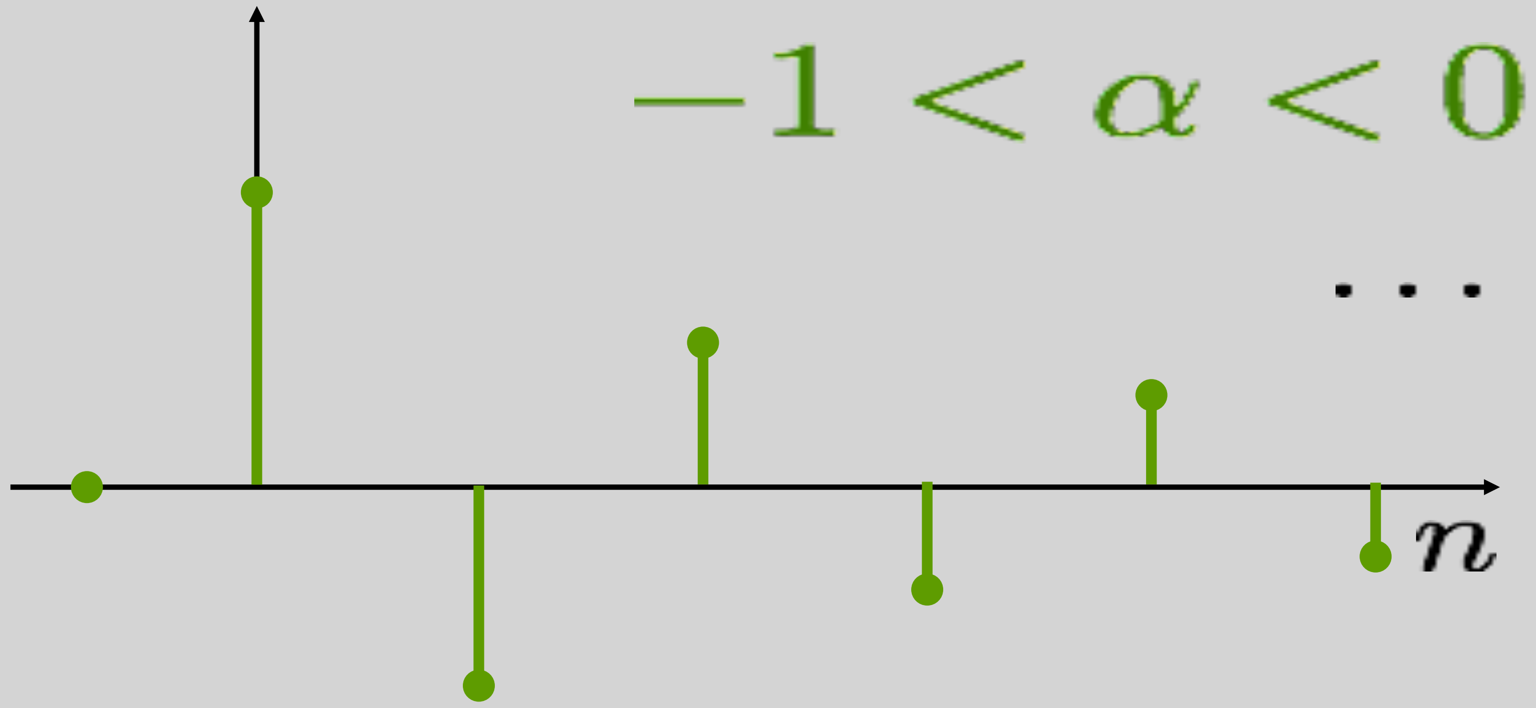


Basic Sequences

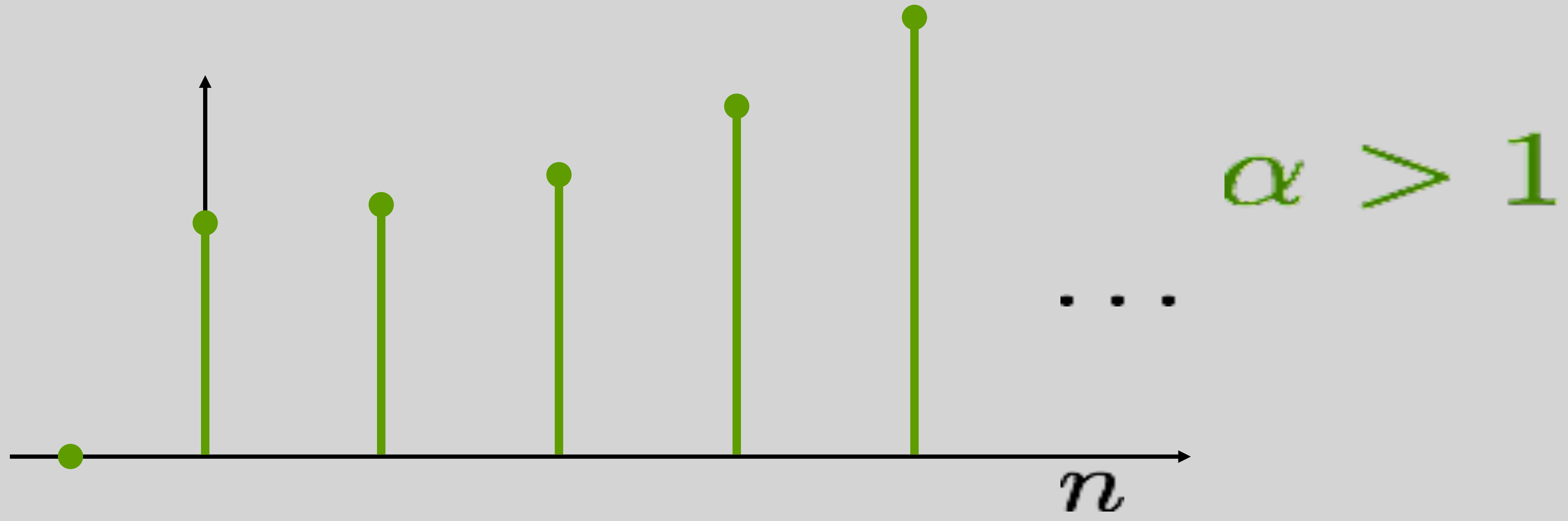
- Exponential

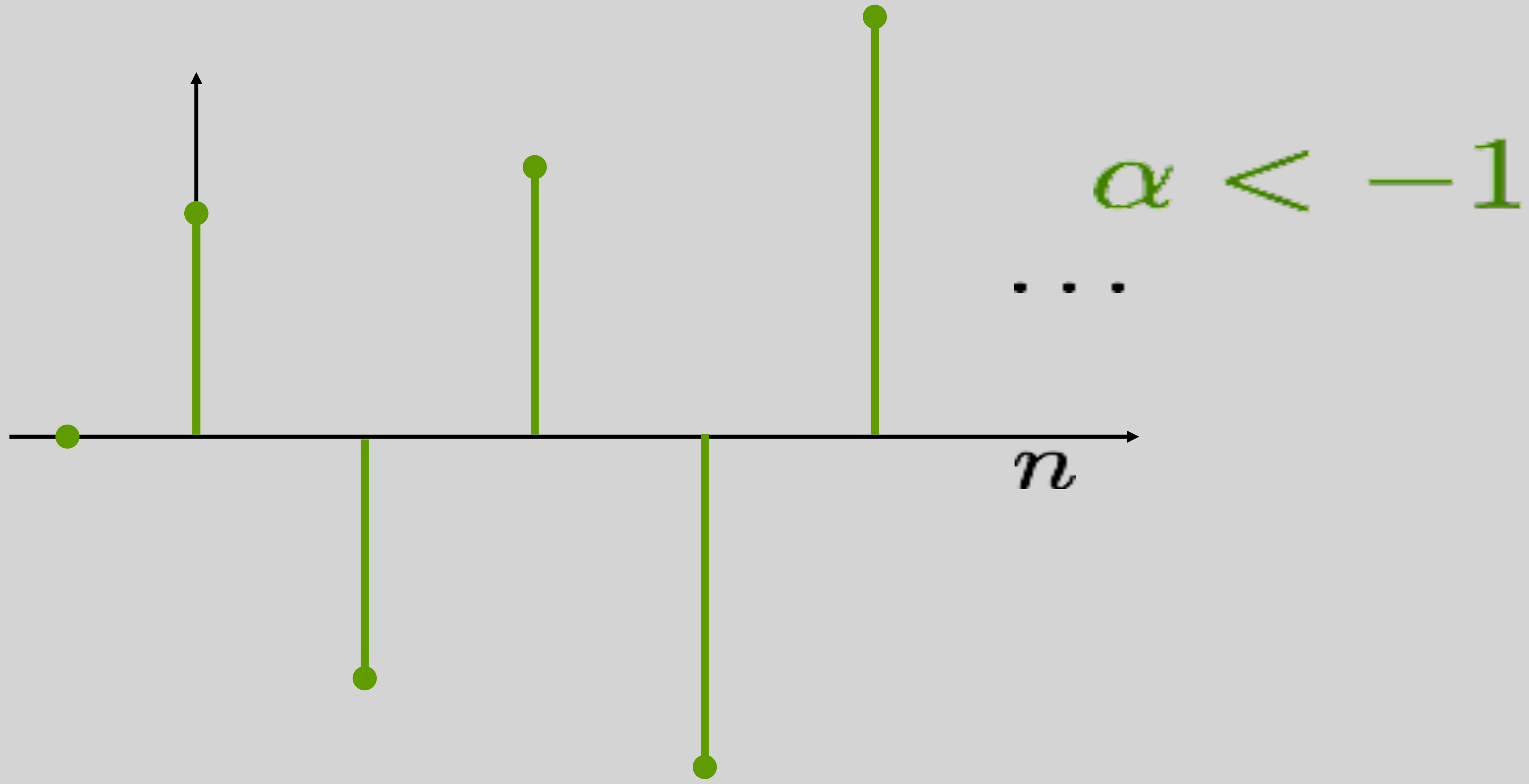
$$y[n] = \begin{cases} A\alpha^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$





Bounded





unBounded

Discrete Sinusoids

$$y[n] = A\cos(\omega_0 n + \phi)$$

or, $y[n] = A\mathrm{e}^{j\omega_0 n + j\phi}$

Q: Is y[n] periodic? $y[n+N] = y[n] \quad | N \in \text{Integer}$

Q: Only if ω_0/π is rational

- To find fundamental period, N
  - Find the smallest integers N,K:

$$\omega_0 N = 2\pi K$$

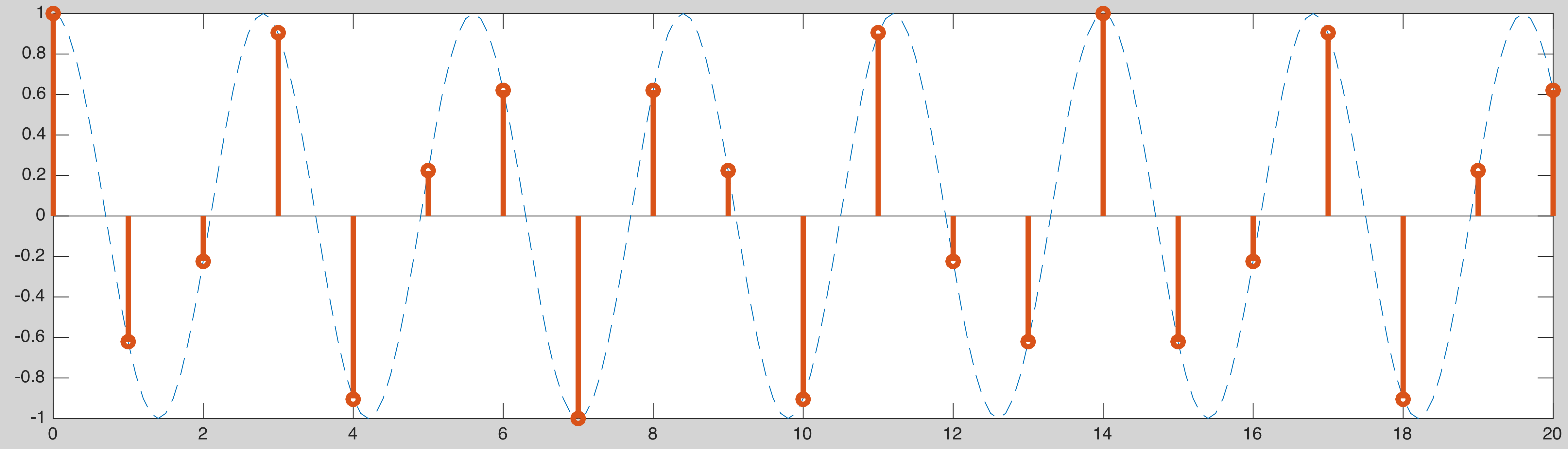
Discrete Sinusoids

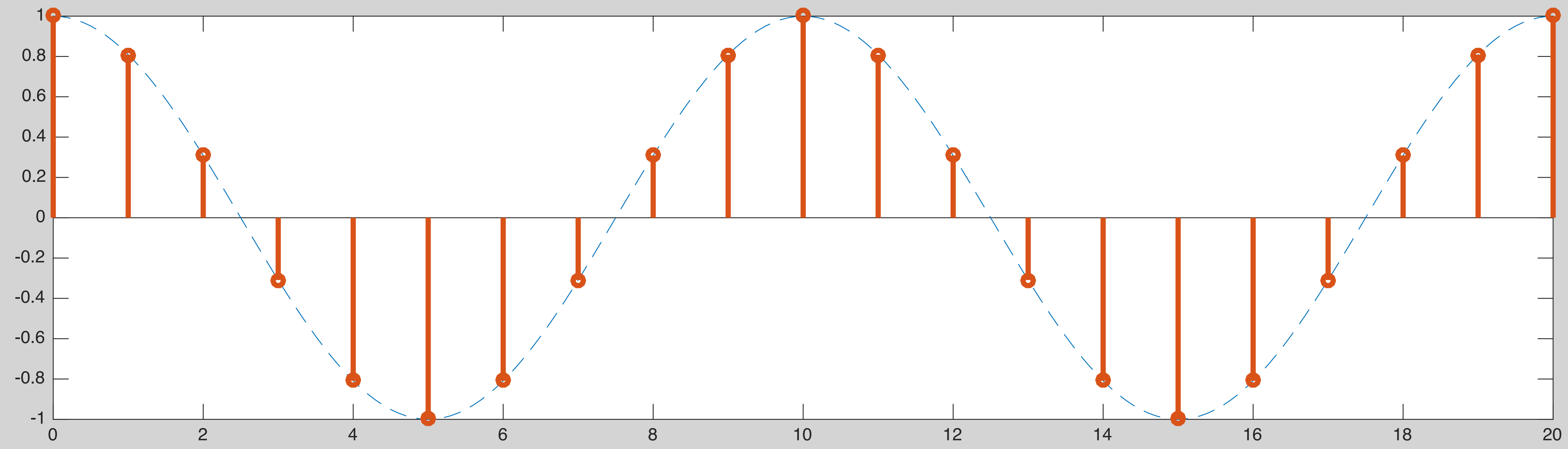
$$\omega_0 N = 2\pi K$$

- Examples:

$\cos(\pi/5n)$ $\quad N = 10 \quad K = 1$

$\cos(\frac{5\pi}{7}n)$ $\quad N = 14 \quad K = 5$



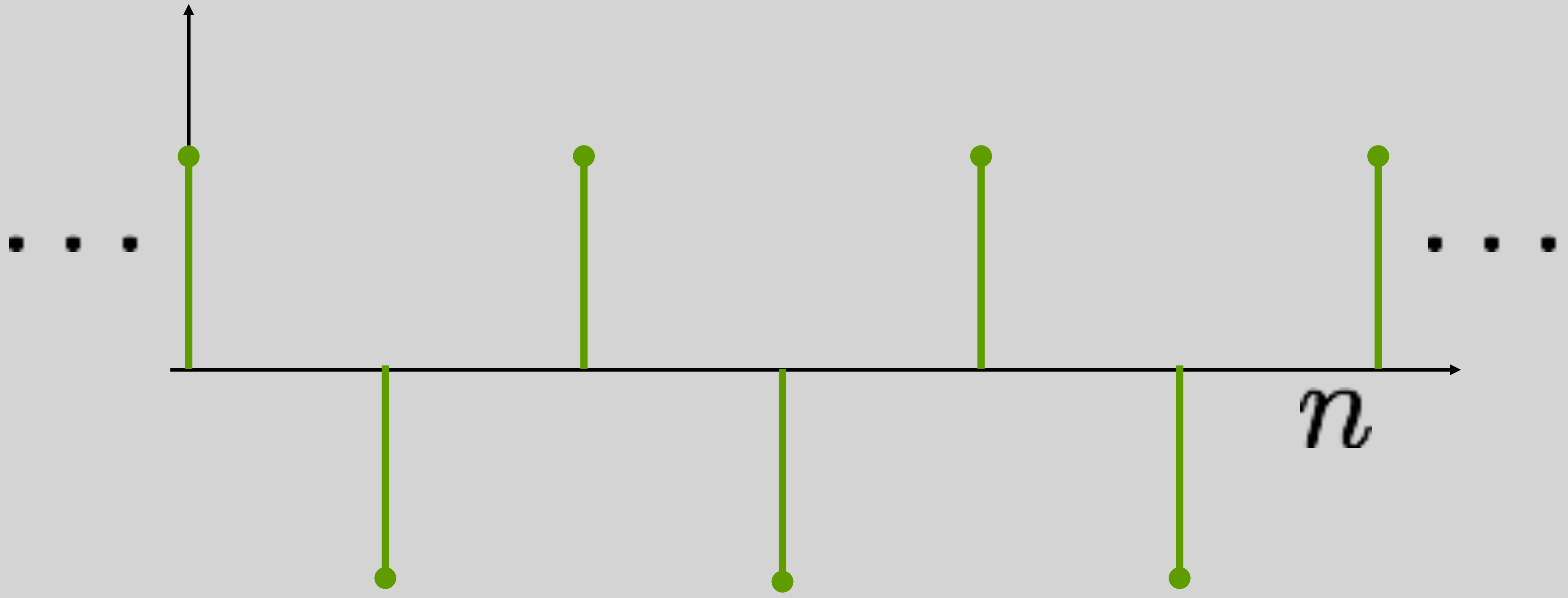


$$\cos(\frac{5\pi}{7}n) + \cos(\pi/5n) \quad \Rightarrow N = \text{S.C.M}\{10, 14\} = 70$$

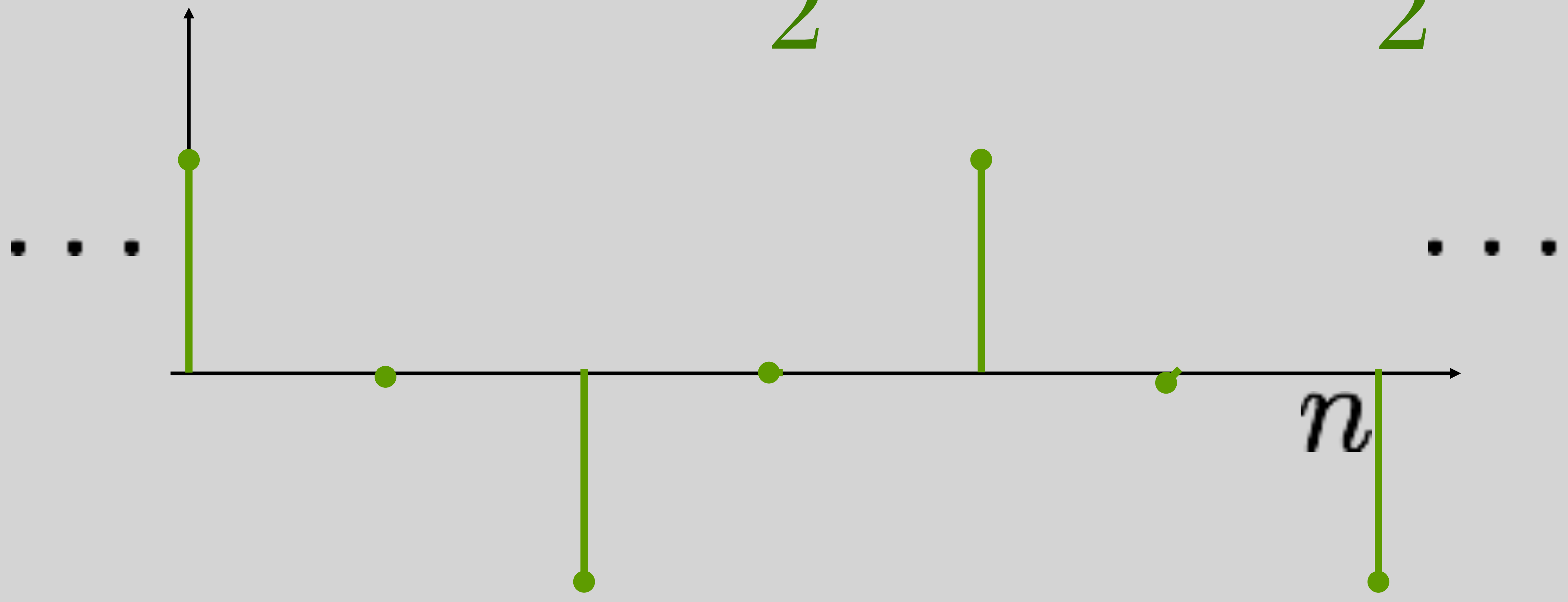
Discrete Sinusoids

Q: Which signal has a higher frequency?

$$\cos(\pi n)$$



$$\cos(\frac{3\pi}{2}n) = \cos(\frac{\pi}{2}n)$$



Discrete Sinusoids

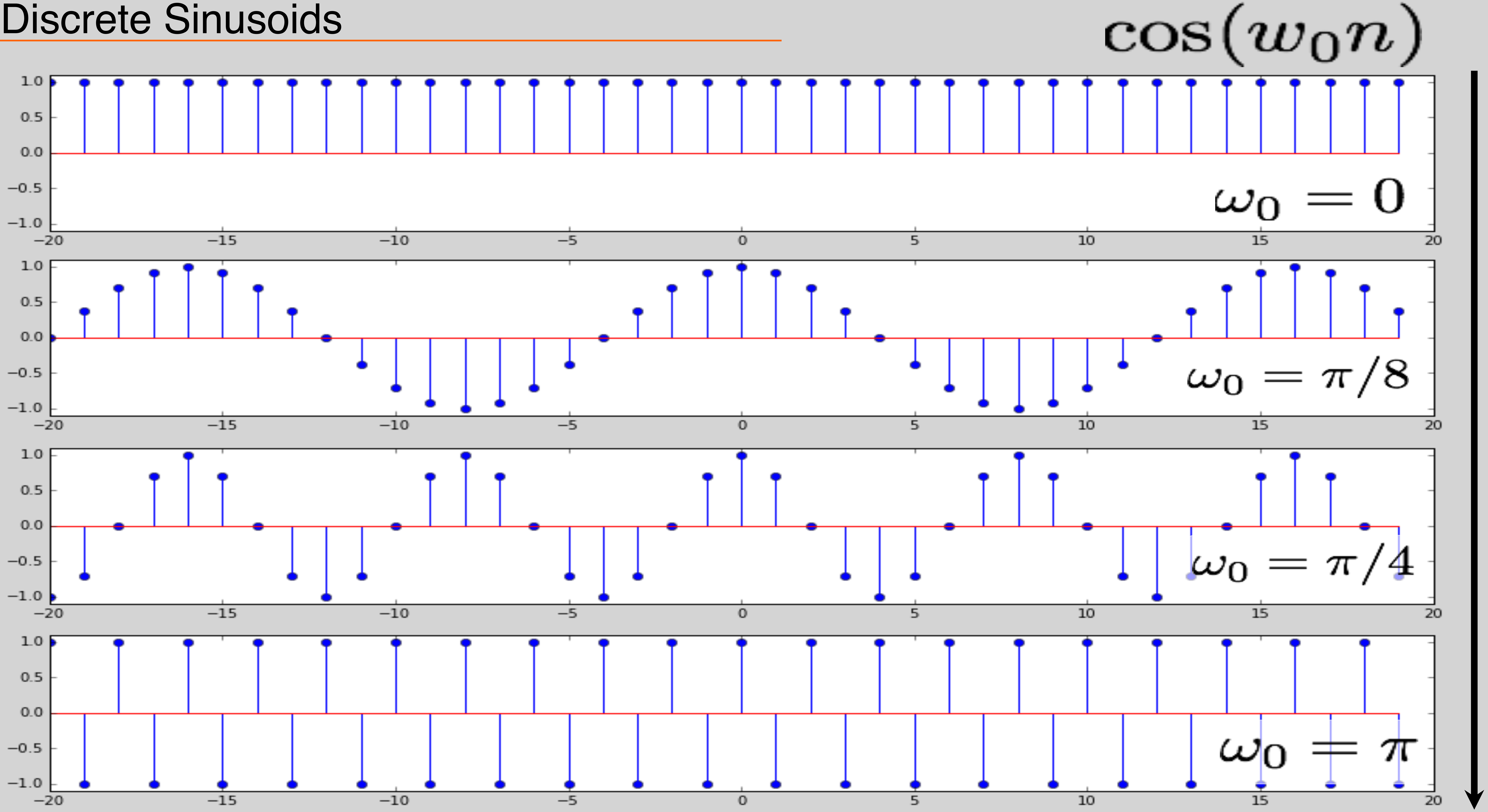
- What's the lowest discrete frequency?

$$y[n] = \cos(0n) = 1$$

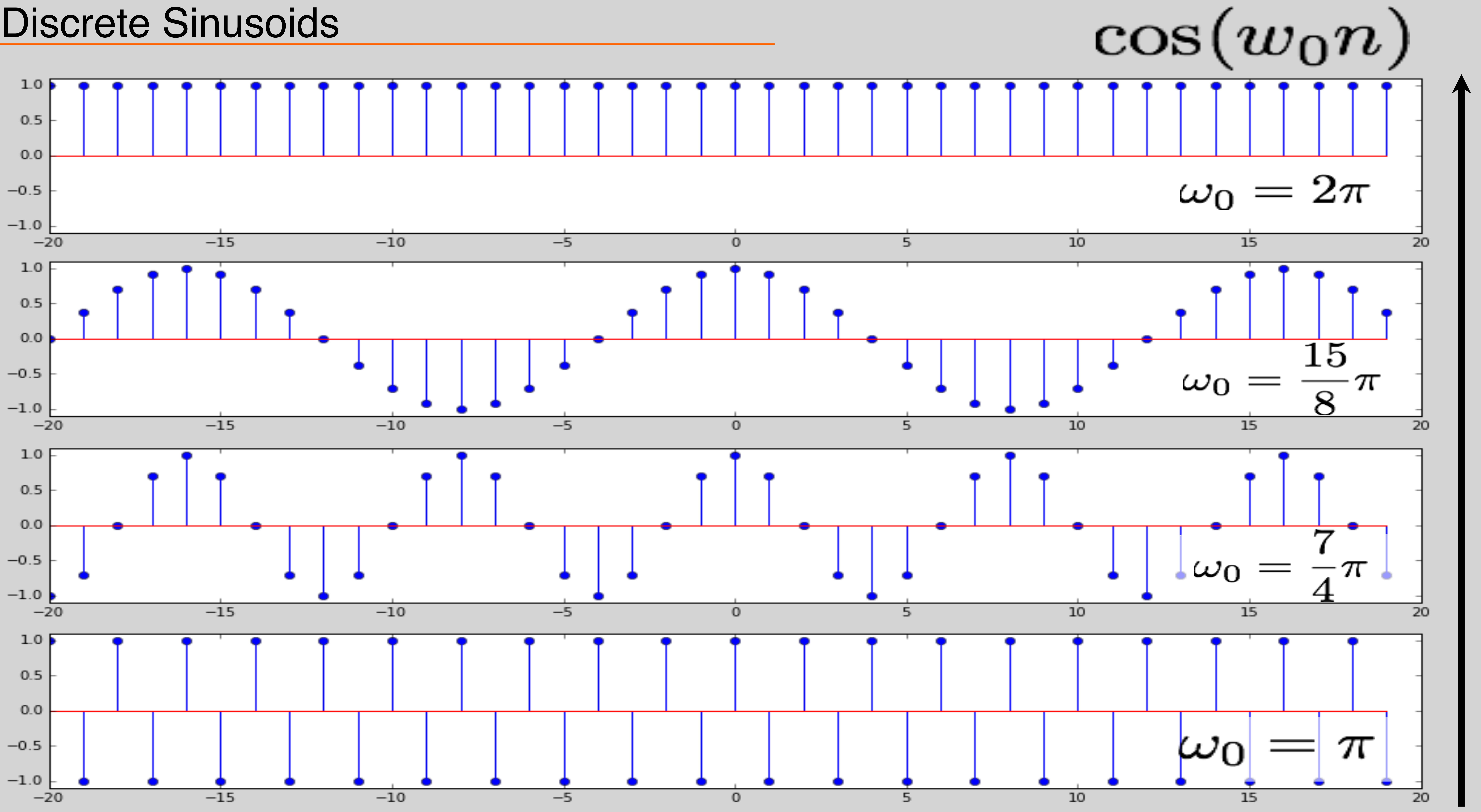
- What's the highest discrete frequency?

$$y[n] = \cos(\pi n) = (-1)^n$$

Discrete Sinusoids



Discrete Sinusoids



Complex Frequencies

- Sinusoids are sums of left and right rotating complex exponentials

$$2\cos(\omega t) = e^{j\omega t} + e^{-j\omega t}$$

"Positive" and "Negative" frequencies

Discrete frequencies with period N:

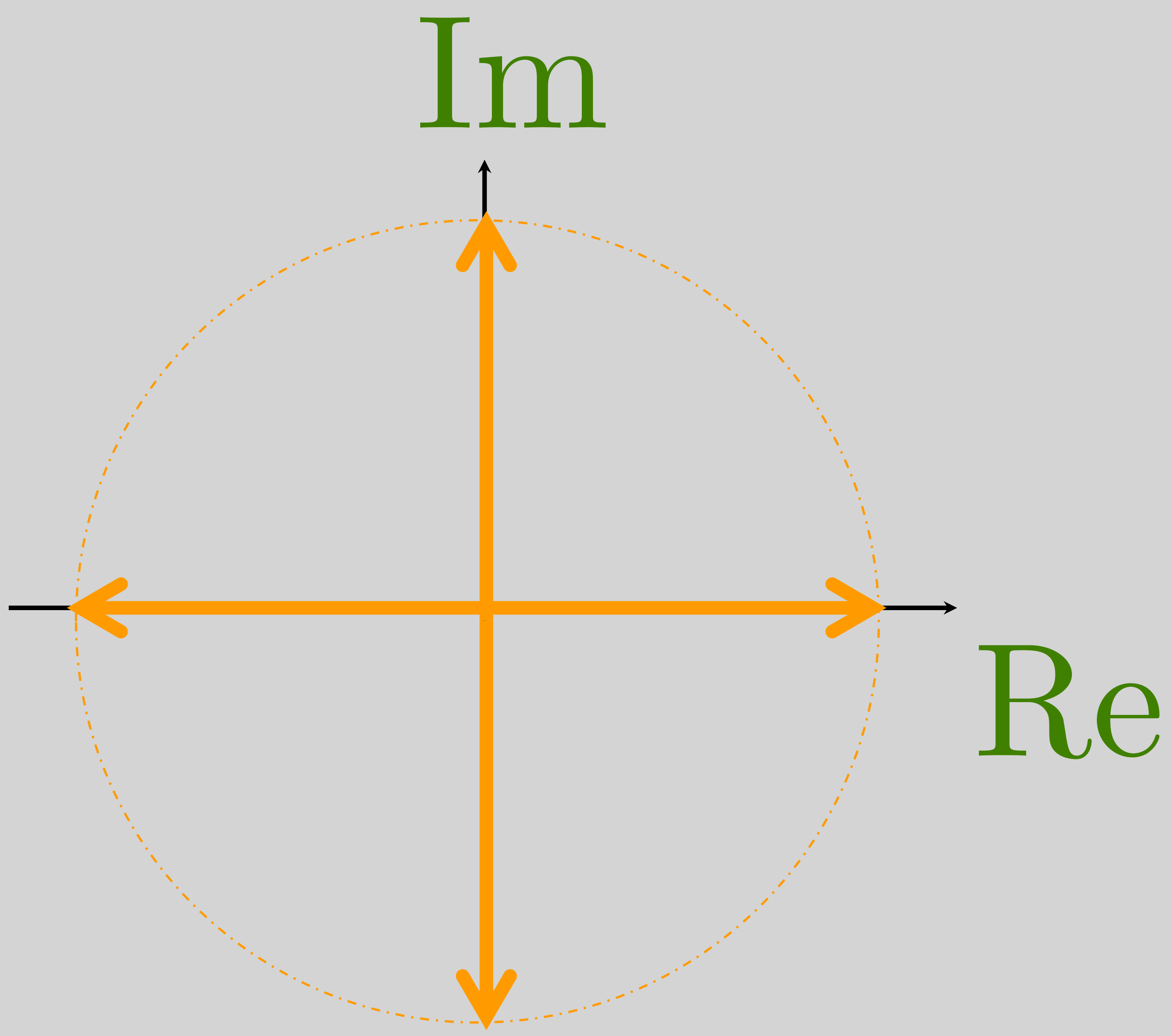
$$y[n] = e^{j2\pi n/N}$$

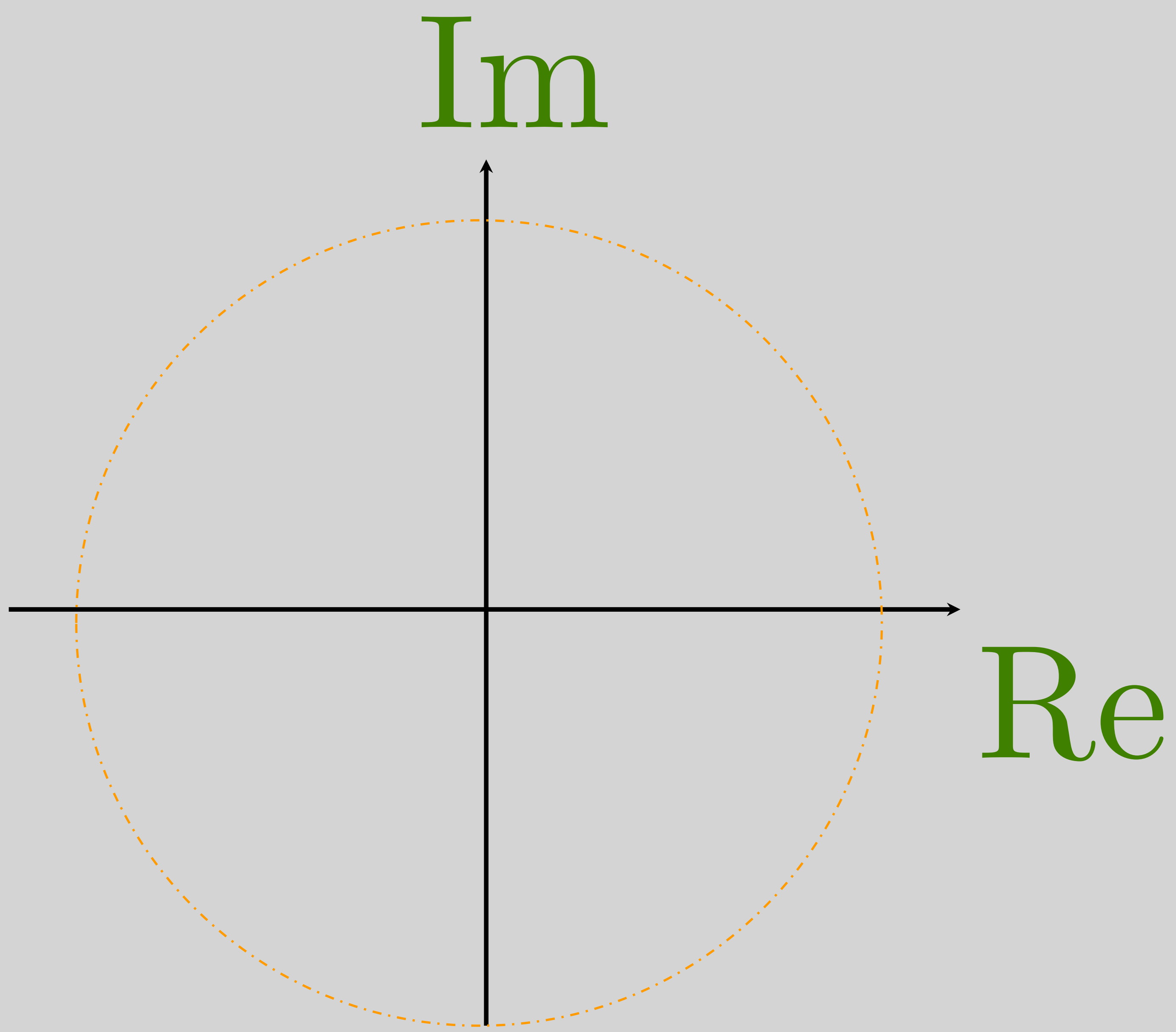
$$W_N \triangleq e^{j2\pi/N} \quad \Rightarrow y[n] = W_N^n$$

Complex Frequencies

$$W_N \triangleq e^{j2\pi/N} \quad \Rightarrow y[n] = W_N^n$$

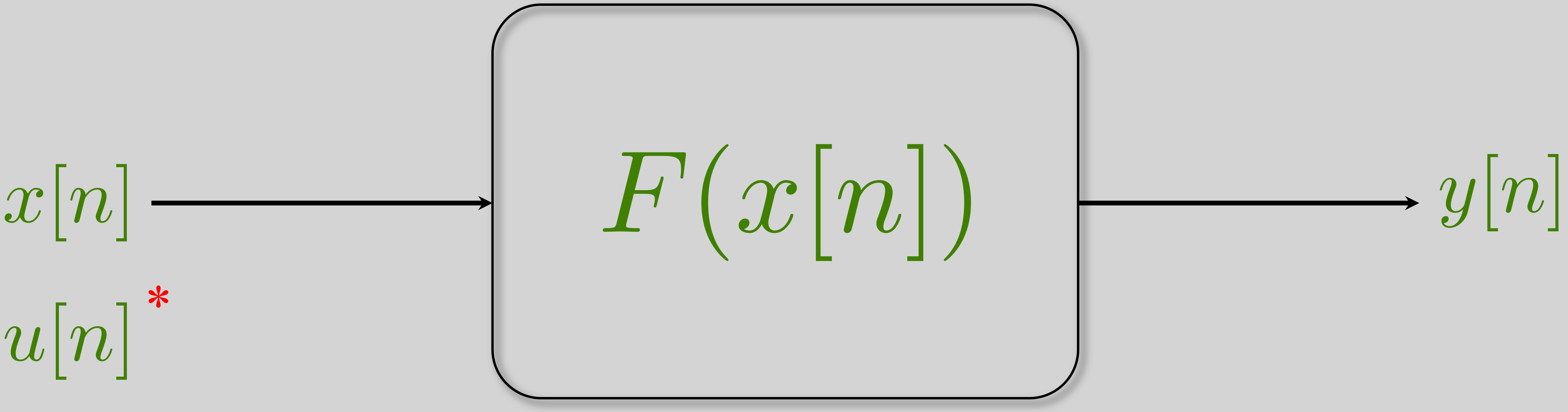
- N = 4 $\quad y[n] = W_4^n$





- N = 6, neg. freq. $\quad y[n] = W_6^{-n}$

Discrete Time Systems



- What Properties?
  - Causality
  - Linearity
  - Stability
  - Time/shift invariance

*WARNING: Going to interchange x[n] and u[n] as inputs

$\vec{x}[n]$ will be a state, not input

U[n] is unit step, not to be confused with u[n]

Properties of D.T. Systems

- Causality:
  - $y[n_0]$ depends only on $x[n]$ for $\infty \leq n \leq n_0$

Causal?

$$\vec{x}[n+1] = A\vec{x}[n] + Bu[n]$$
$$y[n] = C\vec{x}[n]$$

$$\vec{x}[n] = A^n\vec{x}[0] + \sum_{k=0}^{n-1} A^{n-1-k}Bu[k]$$

$$y[n] = CA^n\vec{x}[0] + \sum_{k=0}^{n-1} CA^{n-1-k}Bu[k]$$

Properties of D.T. Systems

$$y[n] = F\{x[n]\}$$

- Linearity
  - Homogeneity: scaling the input, scales the output

$$F\{ax[n]\} = aF\{x[n]\} = ay[n]$$

Properties of D.T. Systems

$$y[n] = F\{x[n]\}$$

- Linearity
  - Homogeneity: scaling the input, scales the output

$$F\{ax[n]\} = aF\{x[n]\} = ay[n]$$

  - Superposition: sum of inputs ⇒ sum of outputs

$$F\{x_1[n] + x_2[n]\} = F\{x_1[n]\} + F\{x_2[n]\} = y_1[n] + y_2[n]$$

Example:

$$\vec{x}[n+1] = A\vec{x}[n] + Bu[n]$$
$$y[n] = C\vec{x}[n]$$

Linear?

$$y[n] = CA^n\vec{x}[0] + \sum_{k=0}^{n-1} CA^{n-1-k}Bu[k]$$

Properties of D.T. Systems

$$y[n] = F\{x[n]\}$$

- BIBO Stability
  - If x[n] is bounded, then y[n] is bounded

$$|x[n]| < M < \infty \quad \forall n \Rightarrow \quad |y[n]| < P < \infty \quad \forall n$$

BIBO stable?

$$y[n] = CA^n\vec{x}[0] + \sum_{k=0}^{n-1} CA^{n-1-k}Bu[k]$$

Properties of D.T. Systems

$$y[n] = F\{x[n]\}$$

- Time Invariance: Shifted input ⇒ shifted output

$$y[n-n_0] = F\{x[n-n_0]\}$$

Time Invariant?

$$\vec{x}[n+1] = A\vec{x}[n] + Bu[n]$$

$$y[n] = C\vec{x}[n]$$

$$y[n] = CA^n\vec{x}[0] + CBu[n-1] + CABu[n-2] + \cdots + CA^{n-1}Bu[0]$$

Linear Time Invariant Systems

- Linear Time Invariant (LTI) systems are completely characterized by their impulse response h[n]

h[n] is the "DNA" of an LTI system

Knowing h[n] is enough to find y[n] for <u>ANY</u> x[n]!

Linear Time Invariant Systems

- Decompose x[n]:

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$= \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

- Compute output:

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \quad = x[n] * h[n]$$

Convolution sum

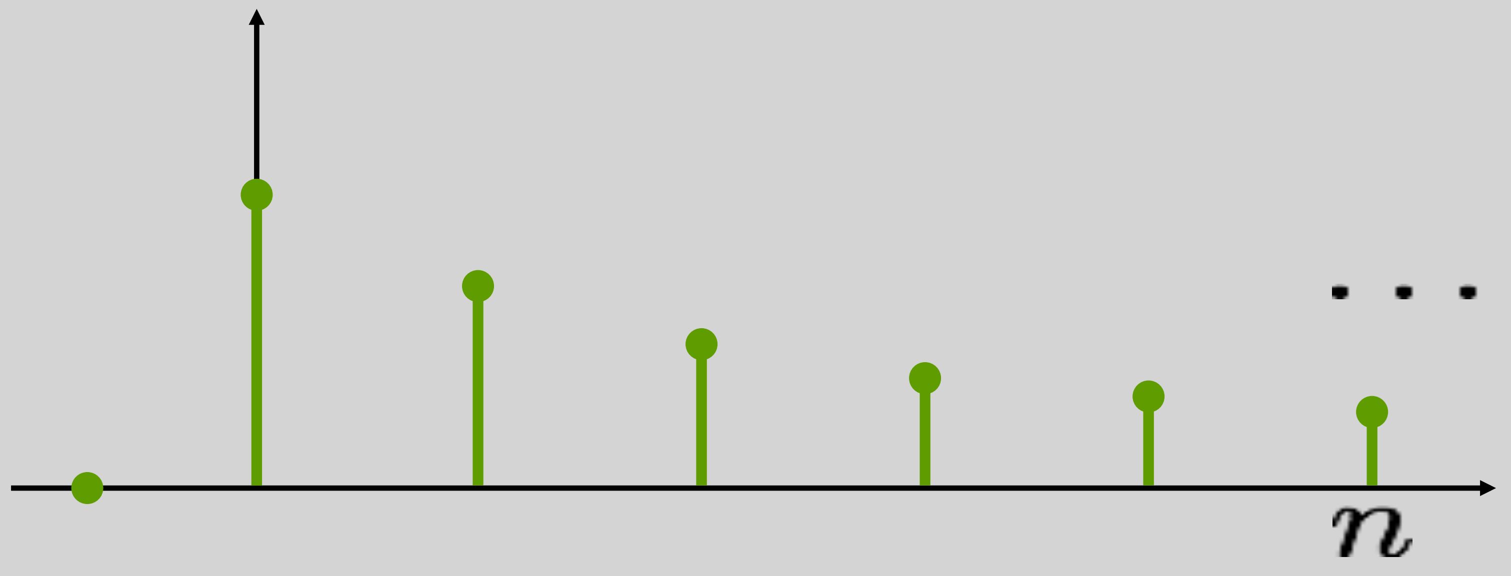
Sum of weighted, delayed impulse responses!

Example:

$$y[n] = ay[n-1] + u[n]$$

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

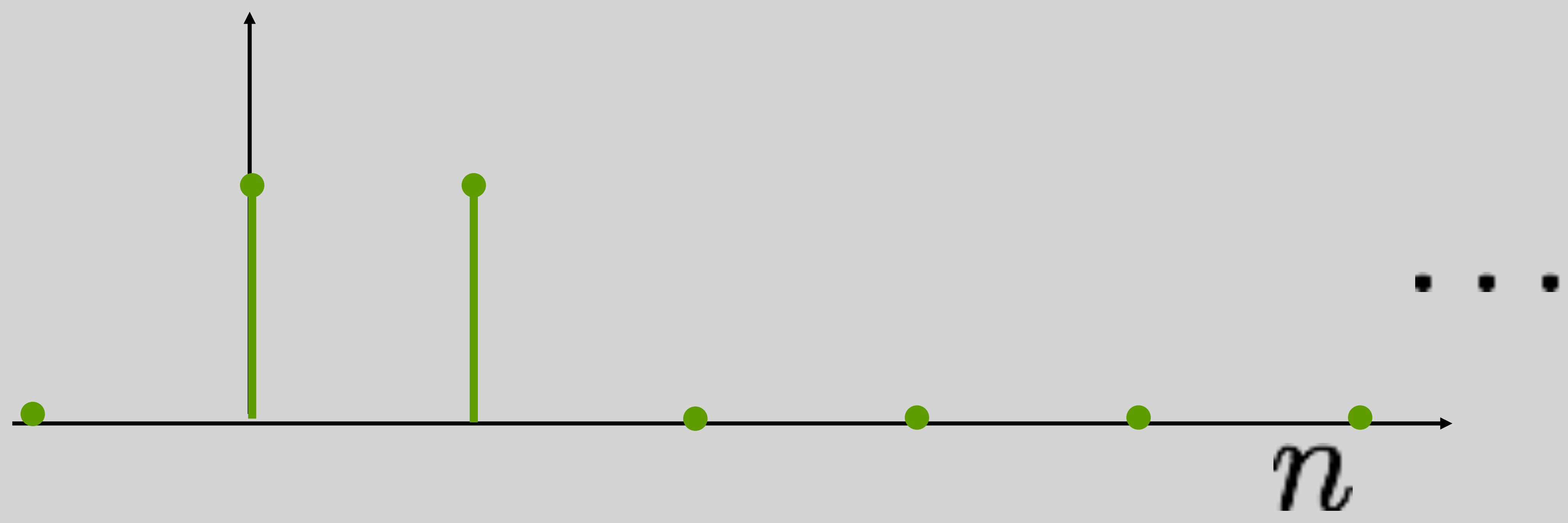
Infinite impulse response (IIR)



$$y[n] = x[n] + x[n-1]$$

$$h[n] = \delta[n] + \delta[n-1]$$

finite impulse response (FIR)



FIR Example:

$$x[n] = \begin{cases} n + (-1)^n & 0 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

$$y[n] = x[n]/2 + x[n-1]/2 \qquad y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$