EE16B
Designing Information Devices and Systems II

Lecture 5A
Control- state space representation

Announcements

- Last time:
  - Bode plots
  - Resonance systes and Q

- HW 4 extended to Friday
- No hw this week.
  - Study for the midterm!
  - Posted midterm practice

Today

- Start a new module: Control

- Describe dynamic systems as a state-space model
  - Extremely powerful model

- Show some concrete examples of how to contruct state space models

SELF-DRIVING CARS

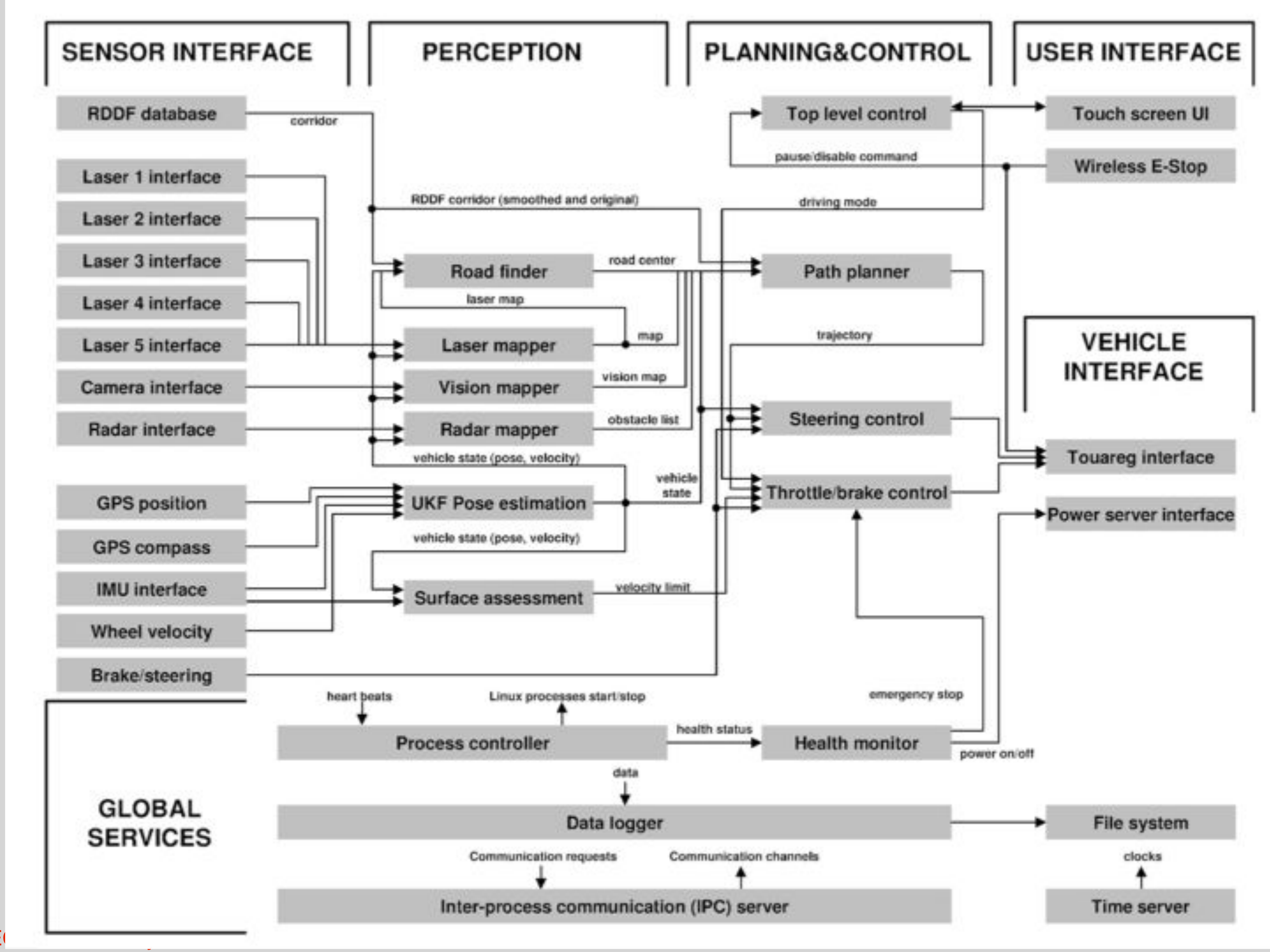
The idea has existed for a long time. Recent progress was enabled by sensors, radar, machine learning, and mapping.

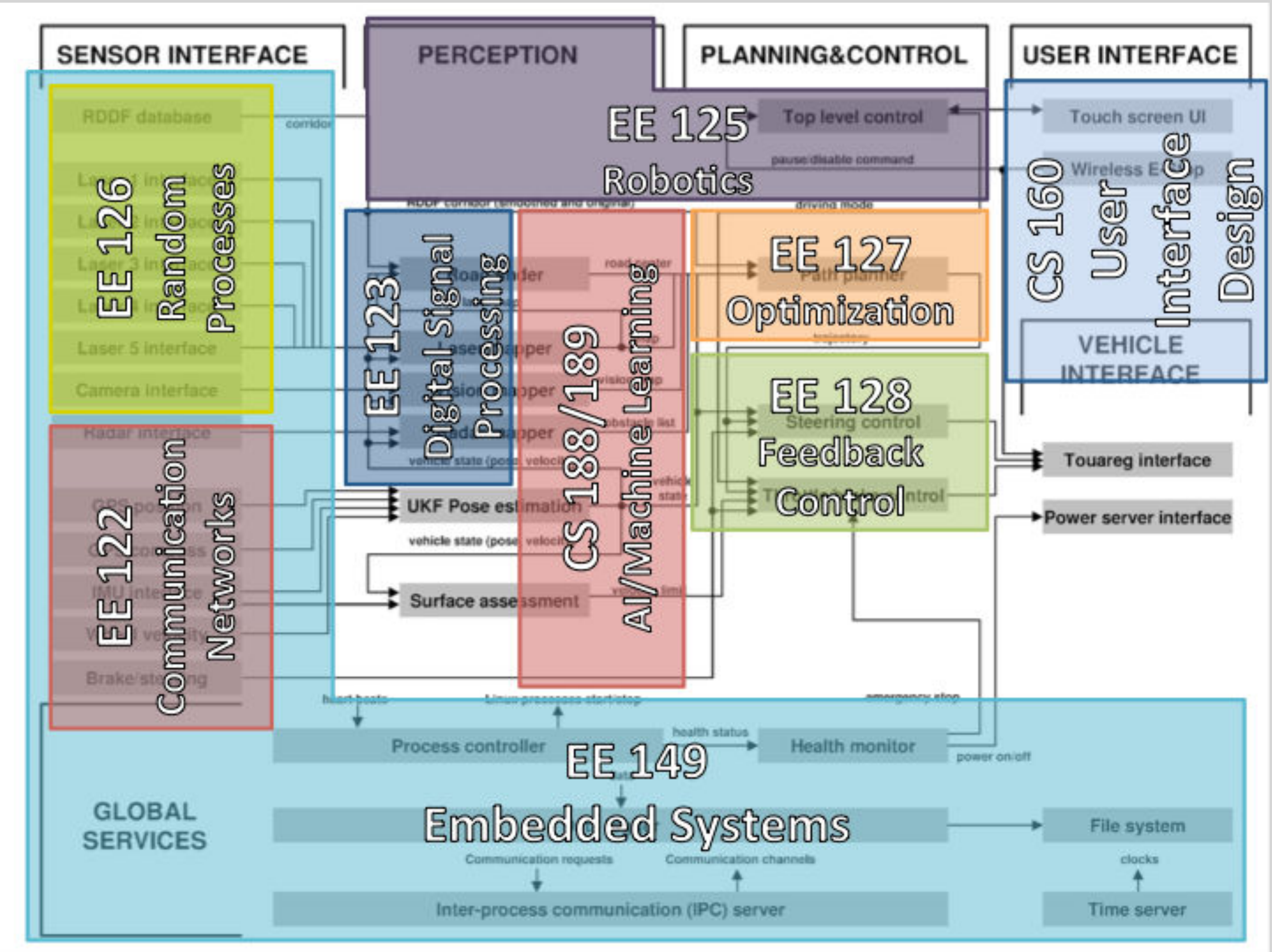
Google car
2016

ca. 1957

Stanley: winner of the DARPA Grand Challenge

Credit: Thrun, Journal of Field Robotics, 2006. DOI: 10.1002/rob.20147


SENSOR INTERFACE
PERCEPTION
PLANNING&CONTROL
USER INTERFACE
RDDF database
corridor
Laser 1 interface
Laser 2 interface
Laser 3 interface
Laser 4 interface
Laser 5 interface
Camera interface
Radar interface
GPS position
GPS compass
IMU interface
Wheel velocity
Brake/steering
RDDF corridor (smoothed and original)
Road finder
road center
laser map
Laser mapper
map
Vision mapper
vision map
Radar mapper
obstacle list
vehicle state (pose, velocity)
vehicle state
UKF Pose estimation
vehicle state (pose, velocity)
Surface assessment
velocity limit
Top level control
pause/disable command
driving mode
Path planner
trajectory
Steering control
Throttle/brake control
Touch screen UI
Wireless E-Stop
VEHICLE INTERFACE
Touareg interface
Power server interface
GLOBAL SERVICES
heart beats
Linux processes start/stop
emergency stop
Process controller
health status
Health monitor
power on/off
data
Data logger
File system
Communication requests
Communication channels
clocks
Inter-process communication (IPC) server
Time server

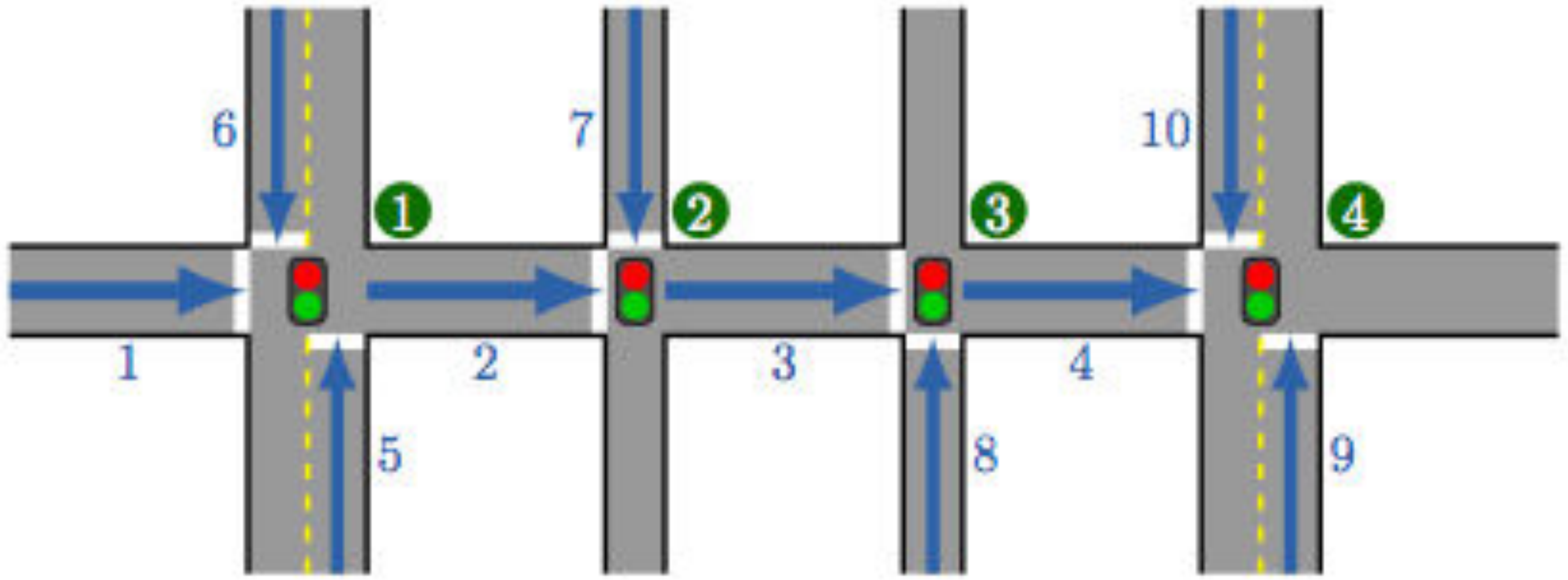


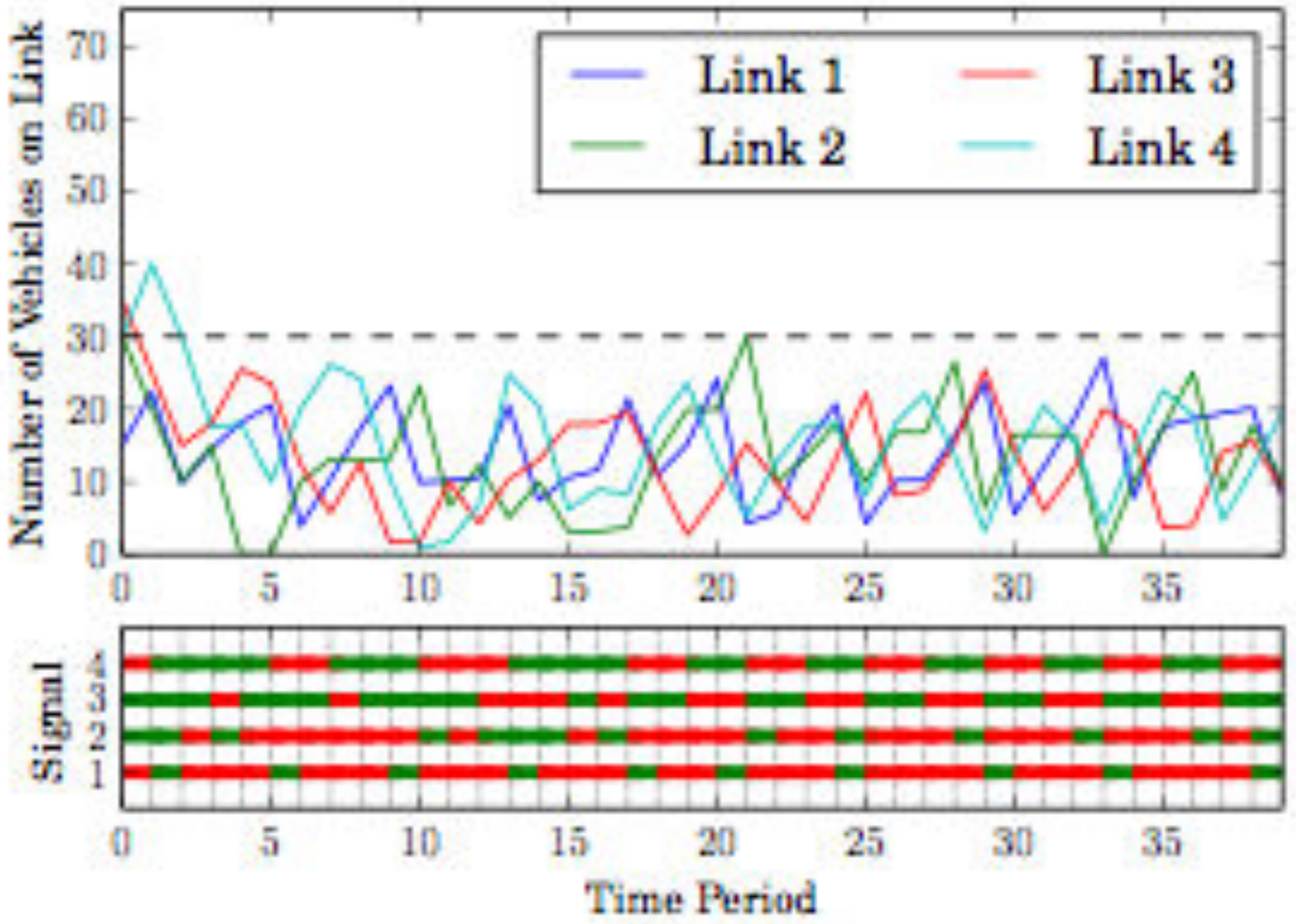
SENSOR INTERFACE
PERCEPTION
PLANNING&CONTROL
USER INTERFACE
EE 125
Robotics
Top level control
pause/disable command
Touch screen UI
CS 160
User
Interface
Design
RDDF database
corridor
EE 126
Random
Processes
Laser 5 interface
Camera interface
Radar interface
EE 123
Digital Signal
Processing
CS 188/189
AI/Machine Learning
EE 127
Optimization
driving mode
VEHICLE
INTERFACE
EE 128
Feedback
Control
Steering control
Touareg interface
Power server interface
UKF Pose estimation
vehicle state (pose, velocity)
Surface assessment
EE 122
Communication
Networks
heart beats
Linux processes start/stop
emergency stop
health status
Process controller
Health monitor
power on/off
EE 149
Embedded Systems
GLOBAL
SERVICES
File system
Communication requests
Communication channels
clocks
Inter-process communication (IPC) server
Time server

TRAFFIC CONTROL

Goal: increase throughput, reduce travel time, vehicle miles traveled, emissions.

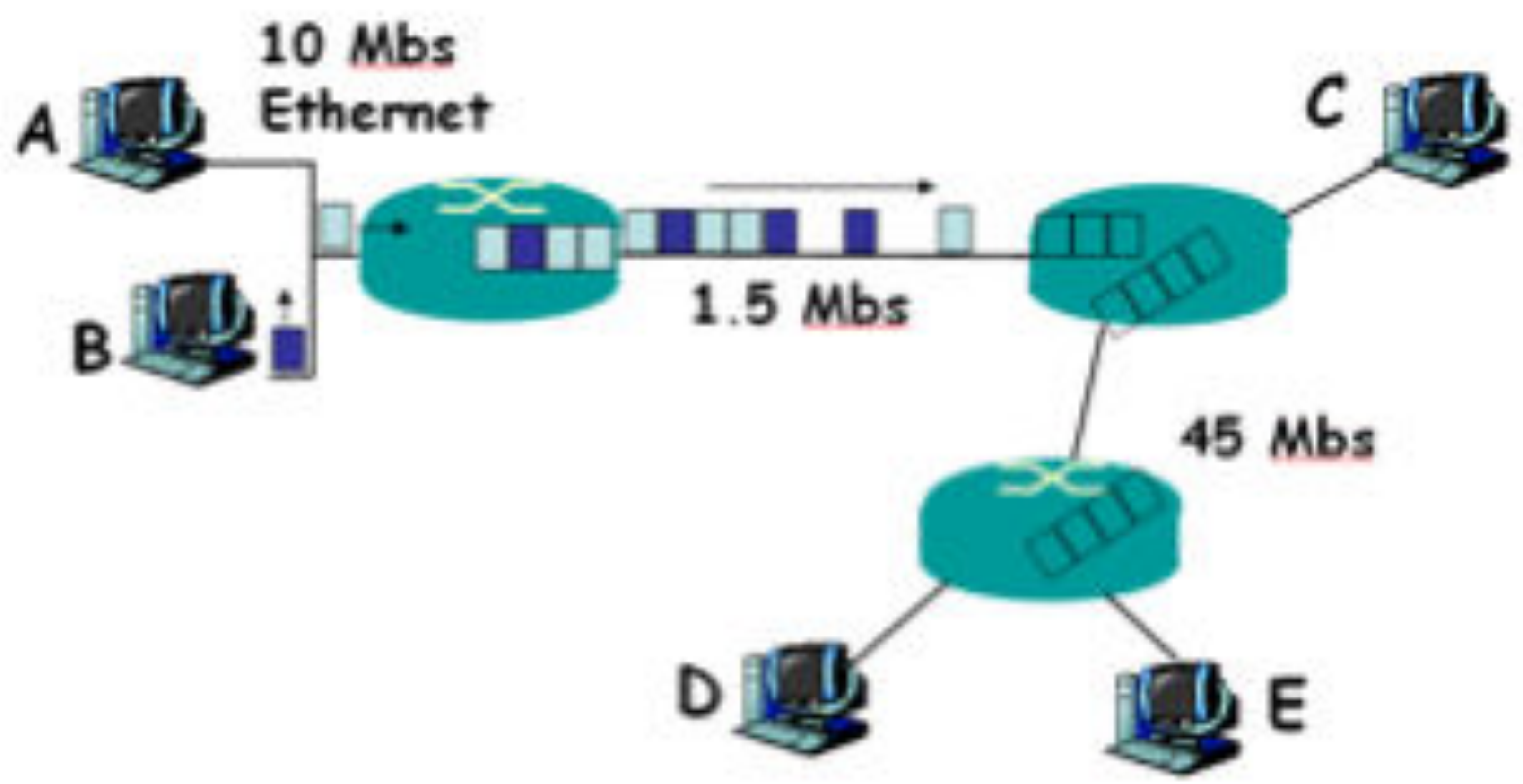
Control mechanisms: signal timing at intersections and onramps, speed advisory, lane management, etc.

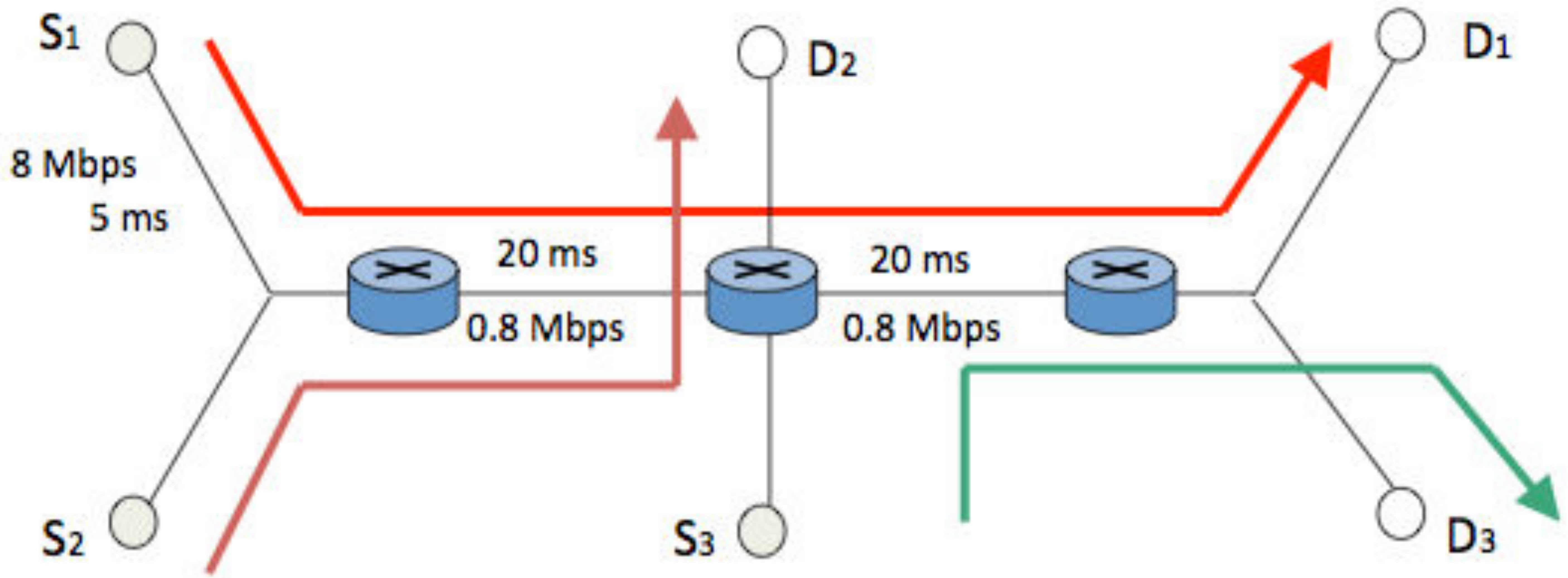




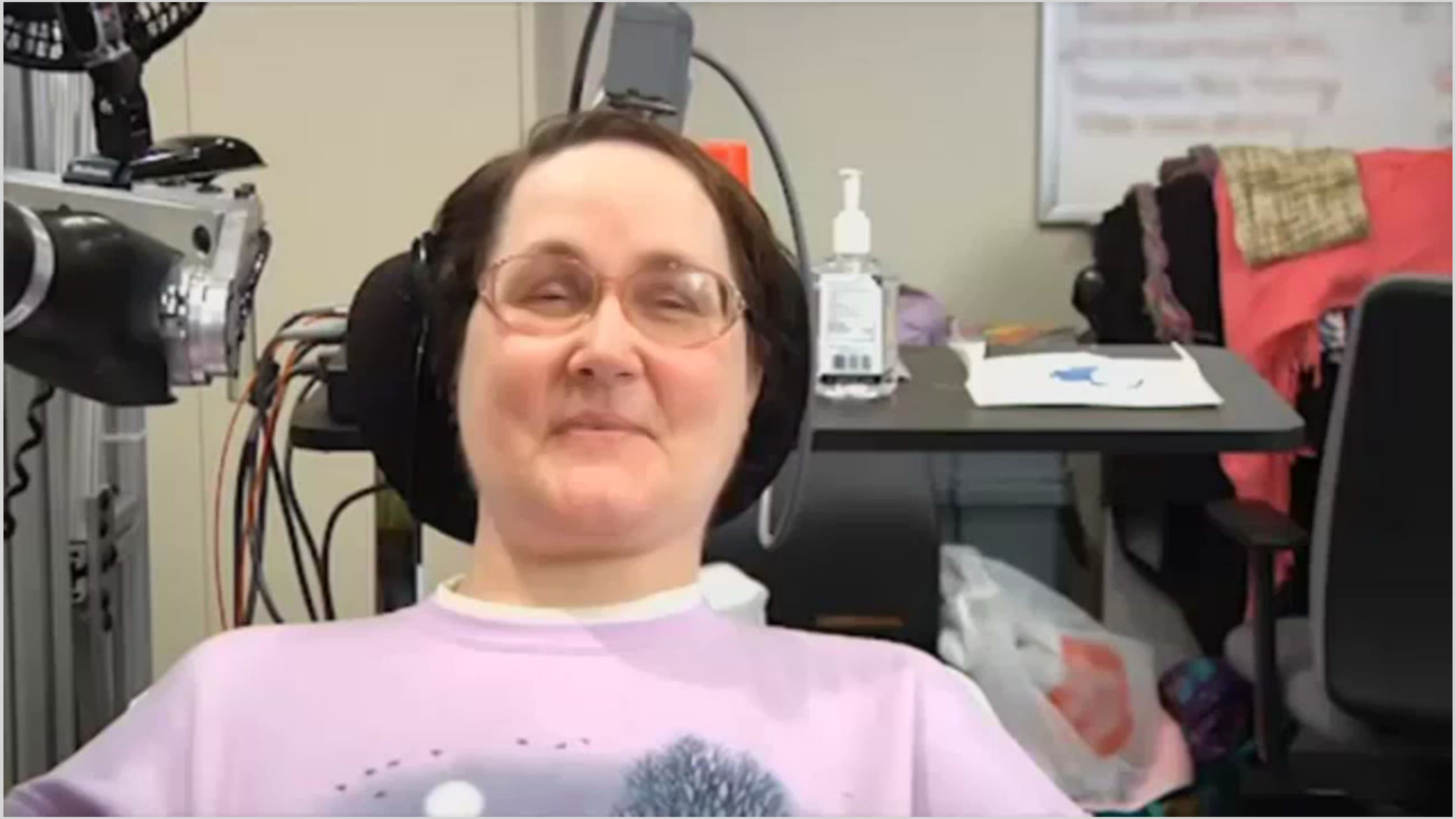
INTERNET CONGESTION CONTROL

TCP and variants increase sending rate when there is no congestion, and decrease when there is congestion, inferred from "ack" messages.



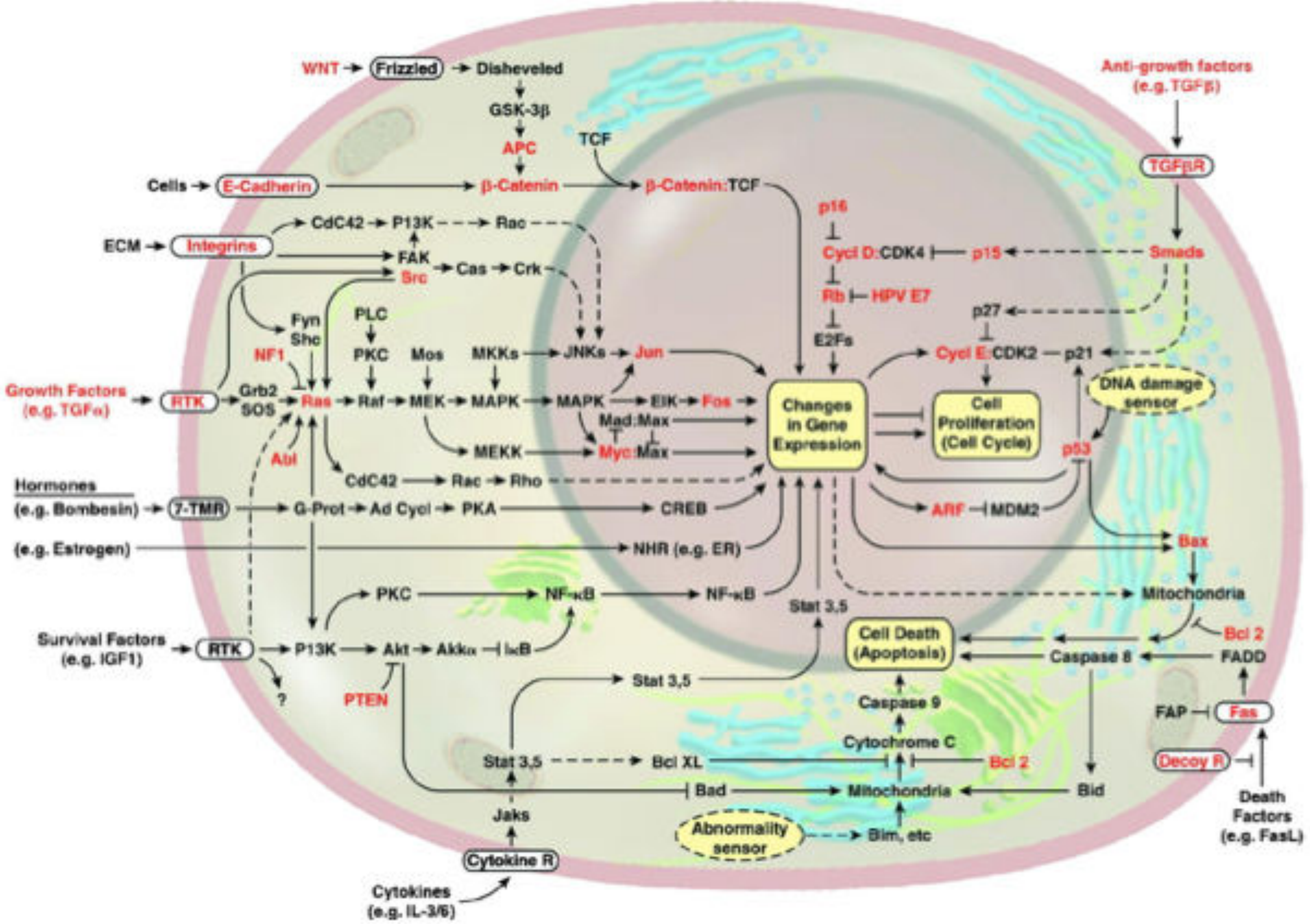


Brain Machine Interface



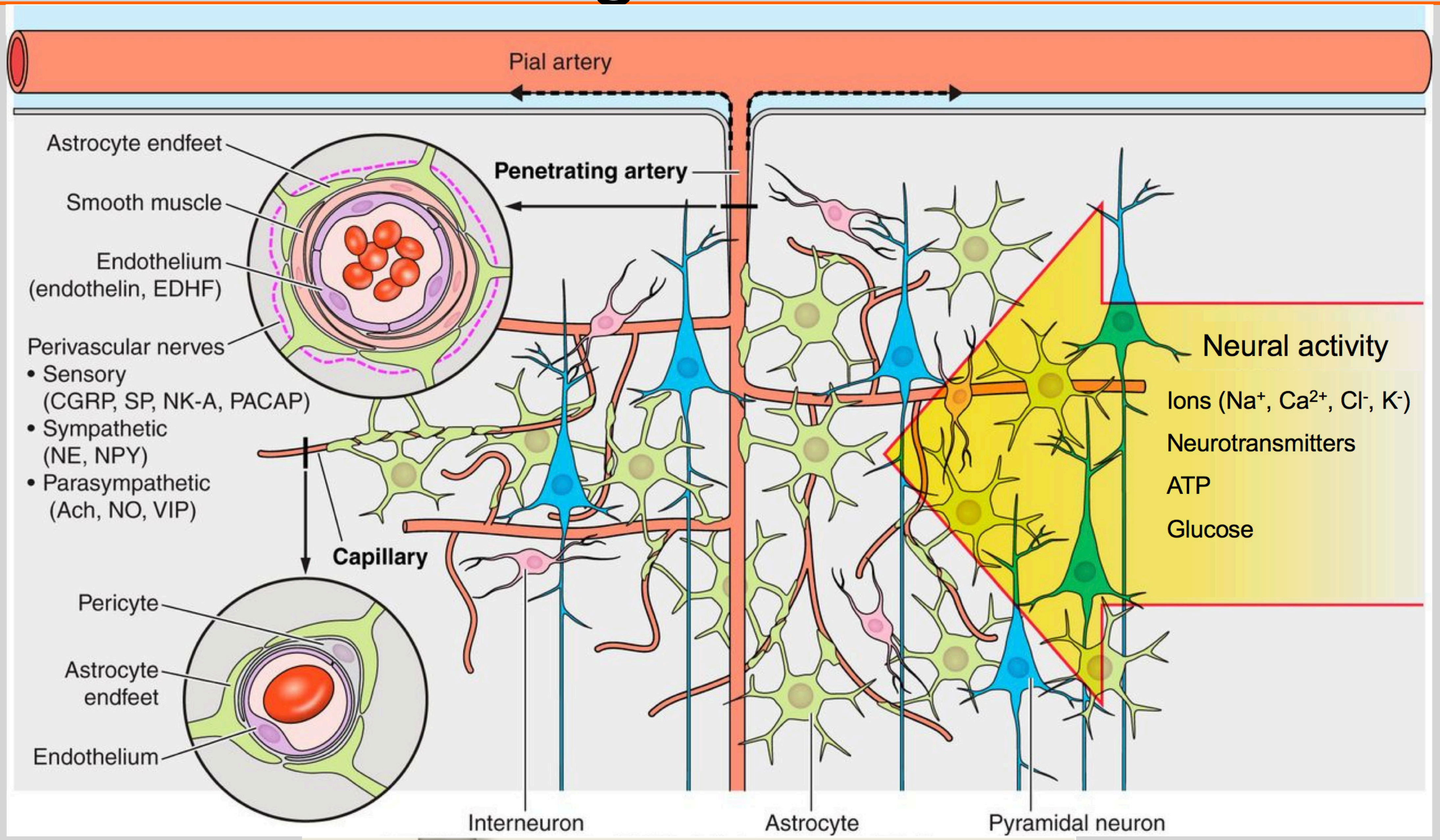
http://news.sky.com/story/woman-uses-her-mind-to-control-robotic-arm-10460512

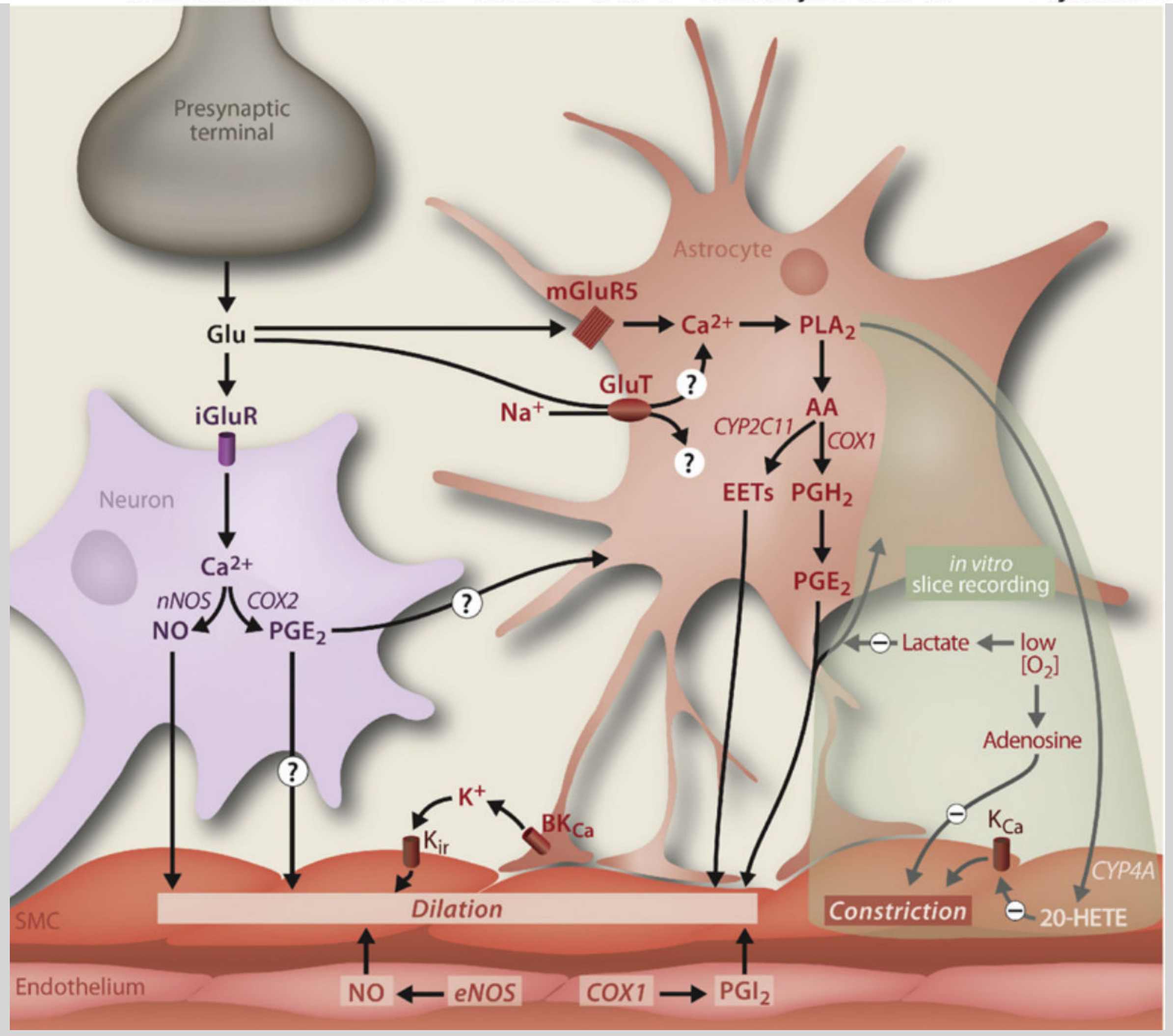
CONTROL NETWORKS IN BIOLOGY

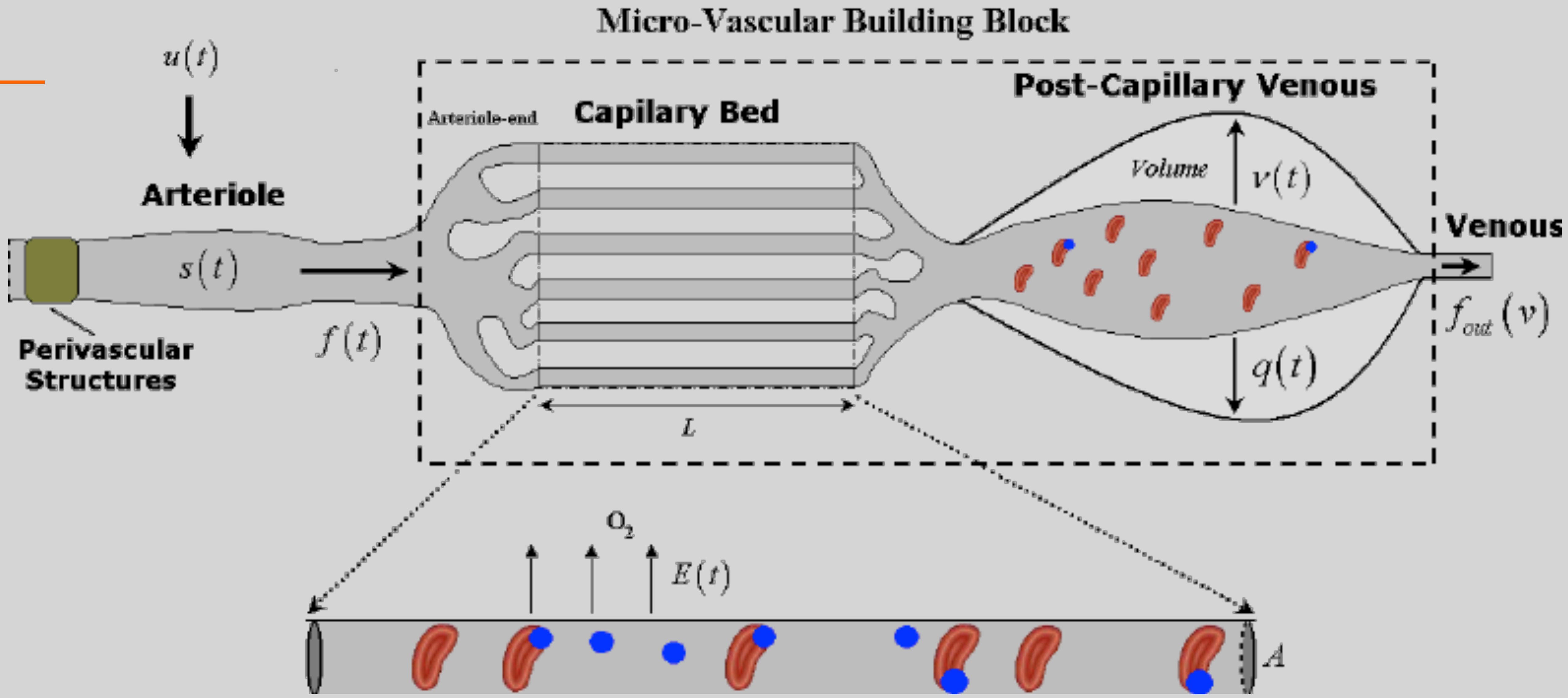


Credit: Hanahan and Weinberg, Cell, 2000

Blood Oxigenation in the Brain







Q(t): Total deoxyhemoglobin

V(t): Venous volume

F_{in}(t): Volume flow rate into tissue (ml/s)

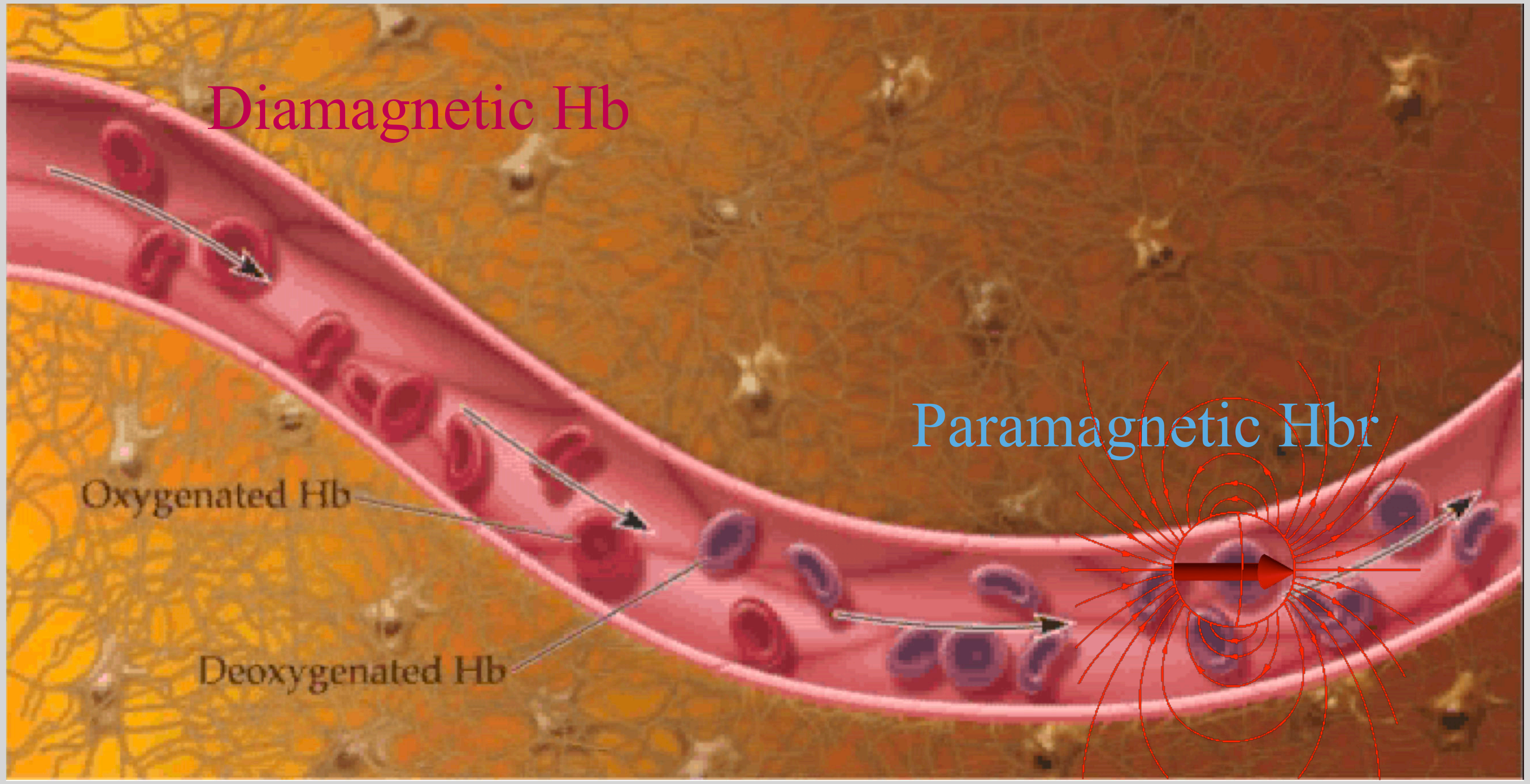
F_{out}(t): Volume flow rate out of tissue (ml/s)

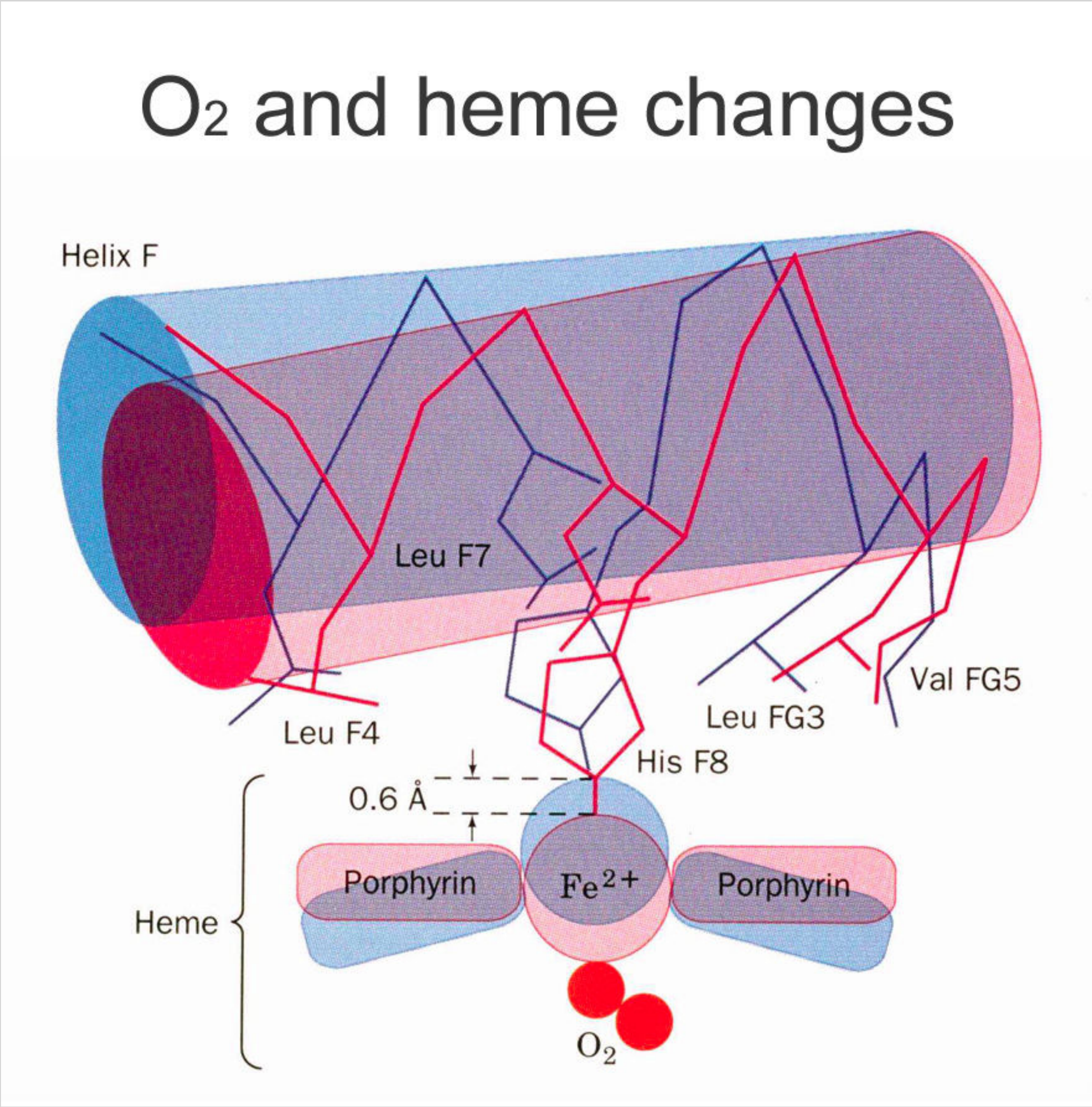
Ca: Arterial O2 concentration

E: Net extract of O2 from blood

Buxton, Richard B., Eric C. Wong, and Lawrence R. Frank. "Dynamics of blood flow and oxygenation changes during brain activation: the balloon model." Magnetic resonance in medicine 39.6 (1998): 855-864.

Blood-Oxygen-Level-Dependent Signal

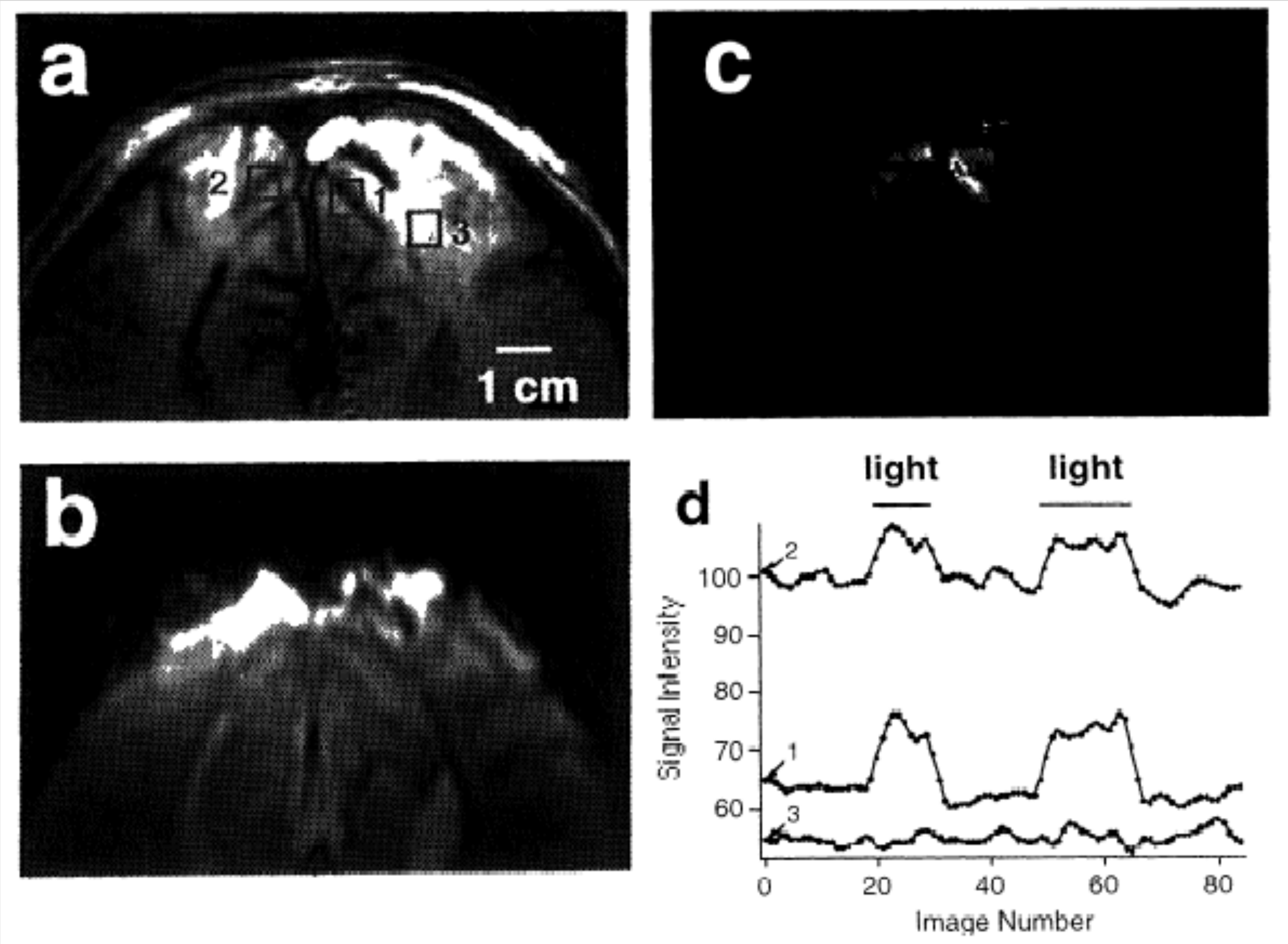




Brain tissue
(diamagnetic)

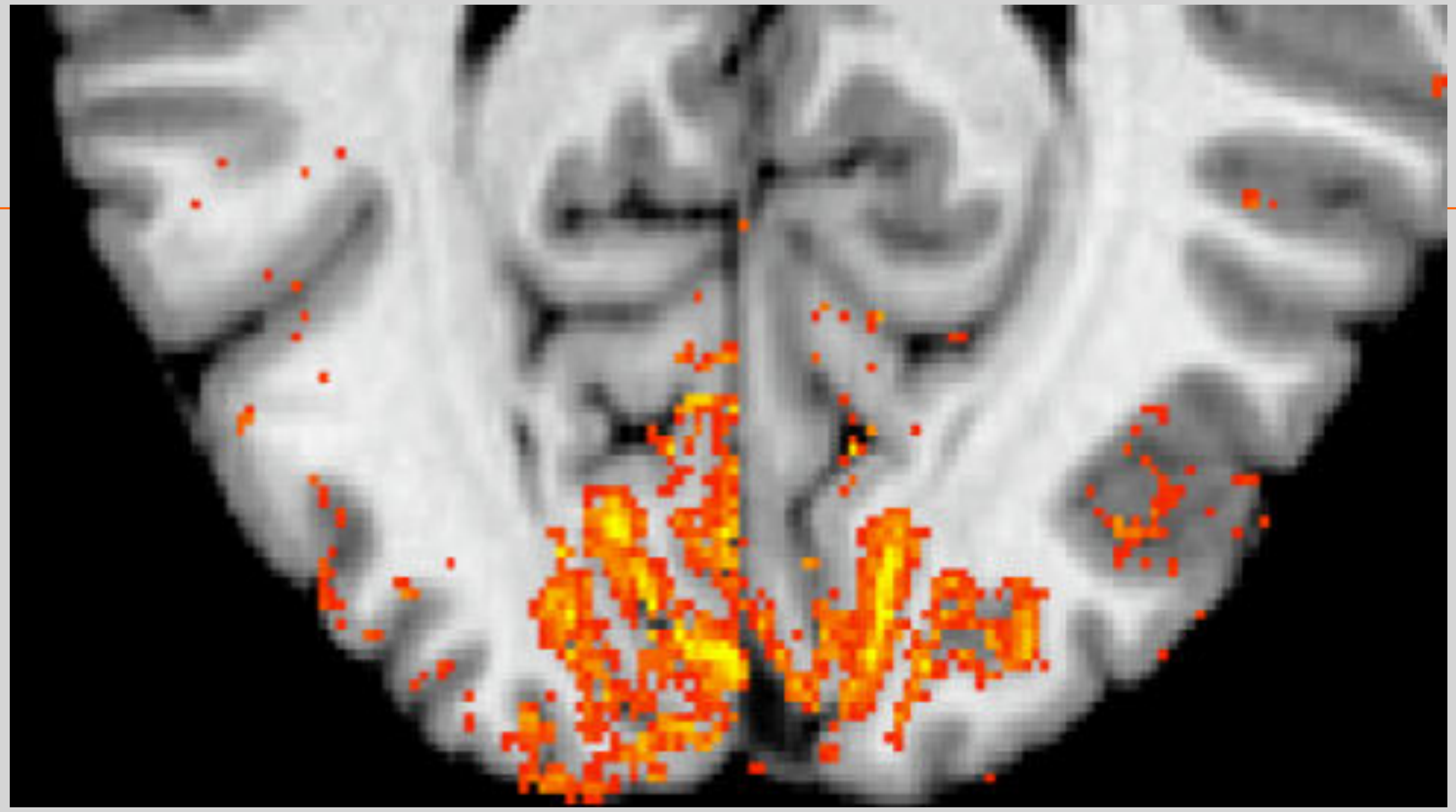
Oxyhemoglobin
(diamagnetic)

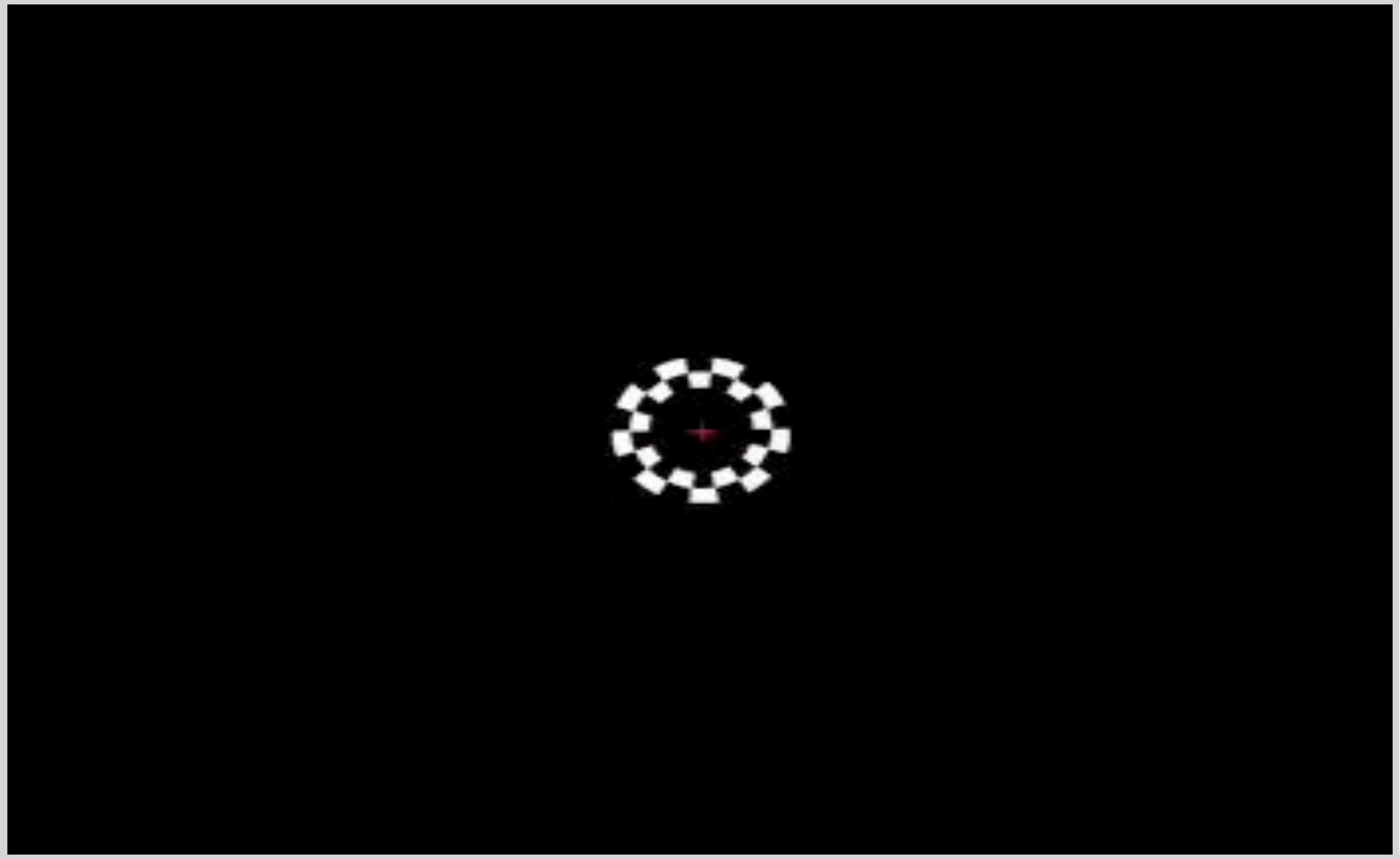
Deoxyhemoglobin
(paramagnetic)

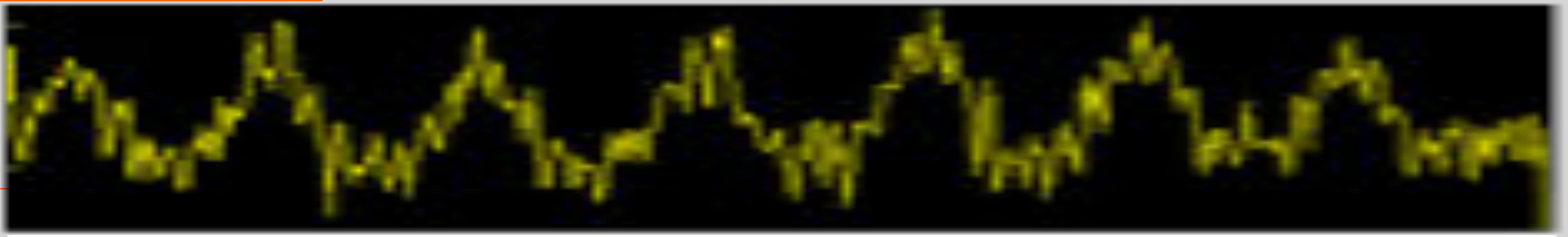


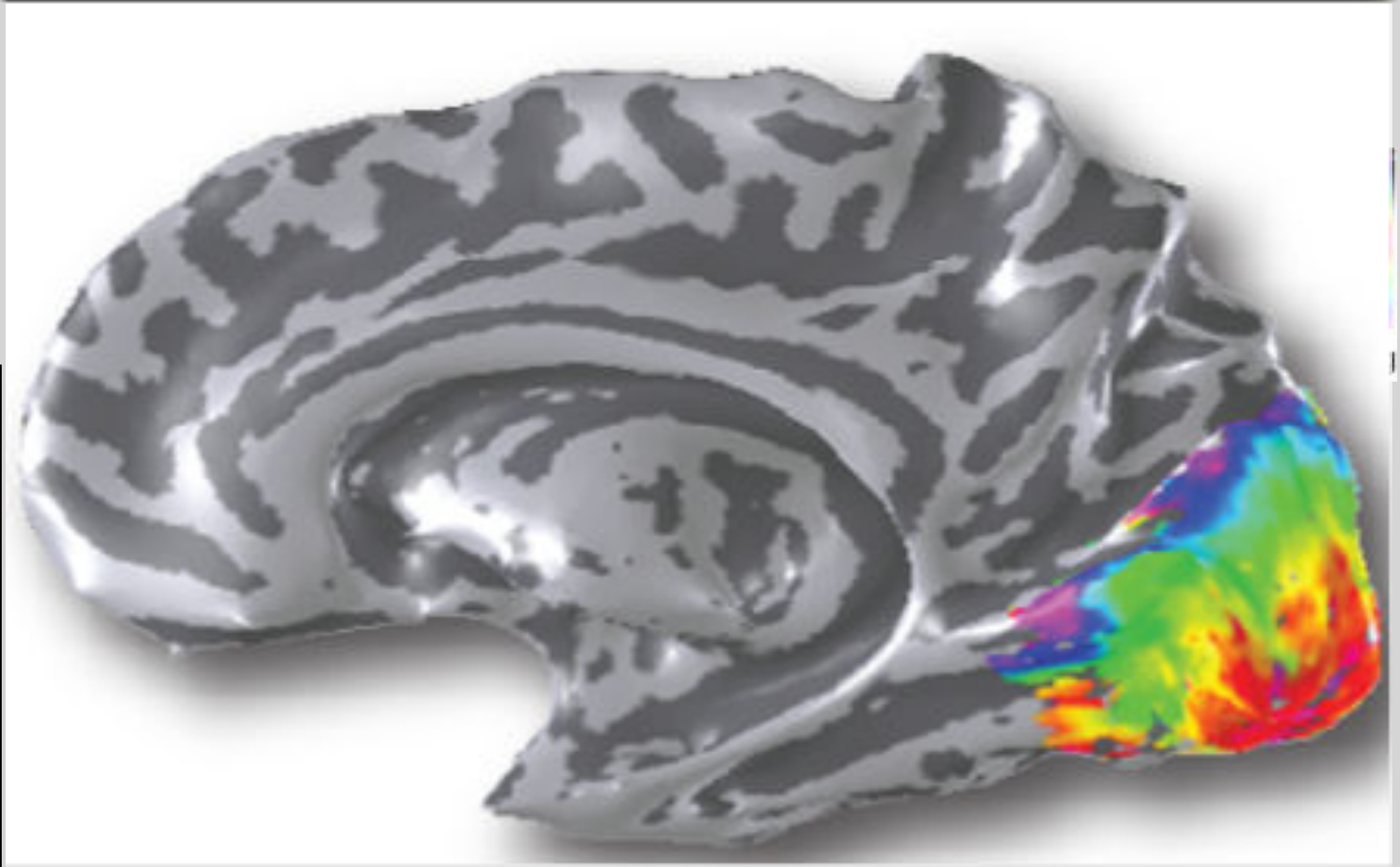
Ogawa S. et al, 1992

Brain Mapping with MRI









Control Design Steps

1. Describe physics with a differential or difference equations

⇒Continuous-time

⇒Discrete-time

Often where the "art" is

2. Design control algorithms that manipulate these equations for desired behavior

State Space Models

n first order (but typically coupled)
differential equations instead of a single nth order

$x_1, x_2, \cdots, x_n$: "state variables"

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

"state vector"

State eqn:

$$\frac{d}{dt}\vec{x}(t) = f\left(\vec{x}(t)\right)$$

control variable

$$\frac{d}{dt}\vec{x}(t) = f\left(\vec{x}(t), \vec{u}(t)\right)$$

State Space Models

Free running

$$\frac{d}{dt}\vec{x}(t) = f\left(\vec{x}(t)\right)$$

control variable

Controlled input

$$\frac{d}{dt}\vec{x}(t) = f\left(\vec{x}(t), \vec{u}(t)\right)$$

disturbance

with disturbance

$$\frac{d}{dt}\vec{x}(t) = f\left(\vec{x}(t), \vec{u}(t), \vec{w}(t)\right)$$

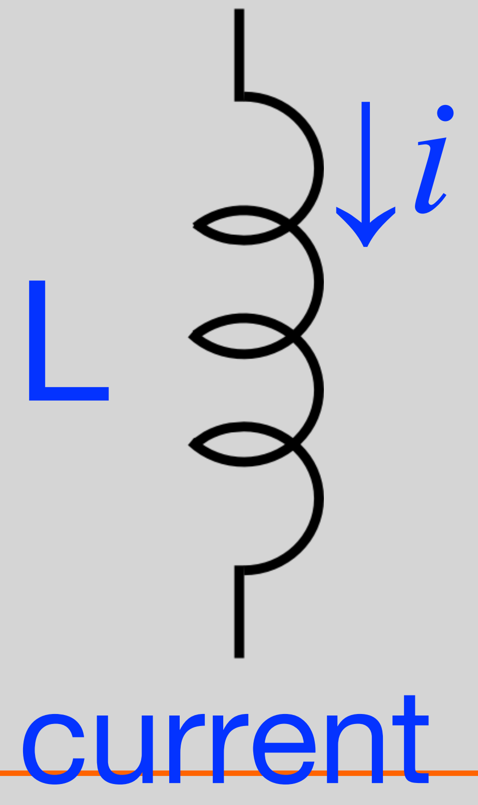
Reminder:

$$\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_0 y(t) = 0$$ 2nd order diff. eq.

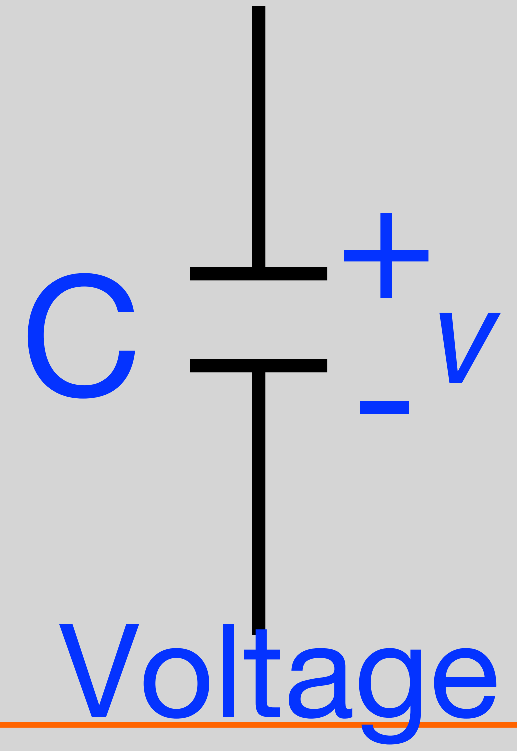
$$\vec{x} = \begin{bmatrix} y(t) \\ \frac{dy}{dt}(t) \end{bmatrix} \quad \Rightarrow \frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}\vec{x}(t)$$

Today: write state model directly.

example:



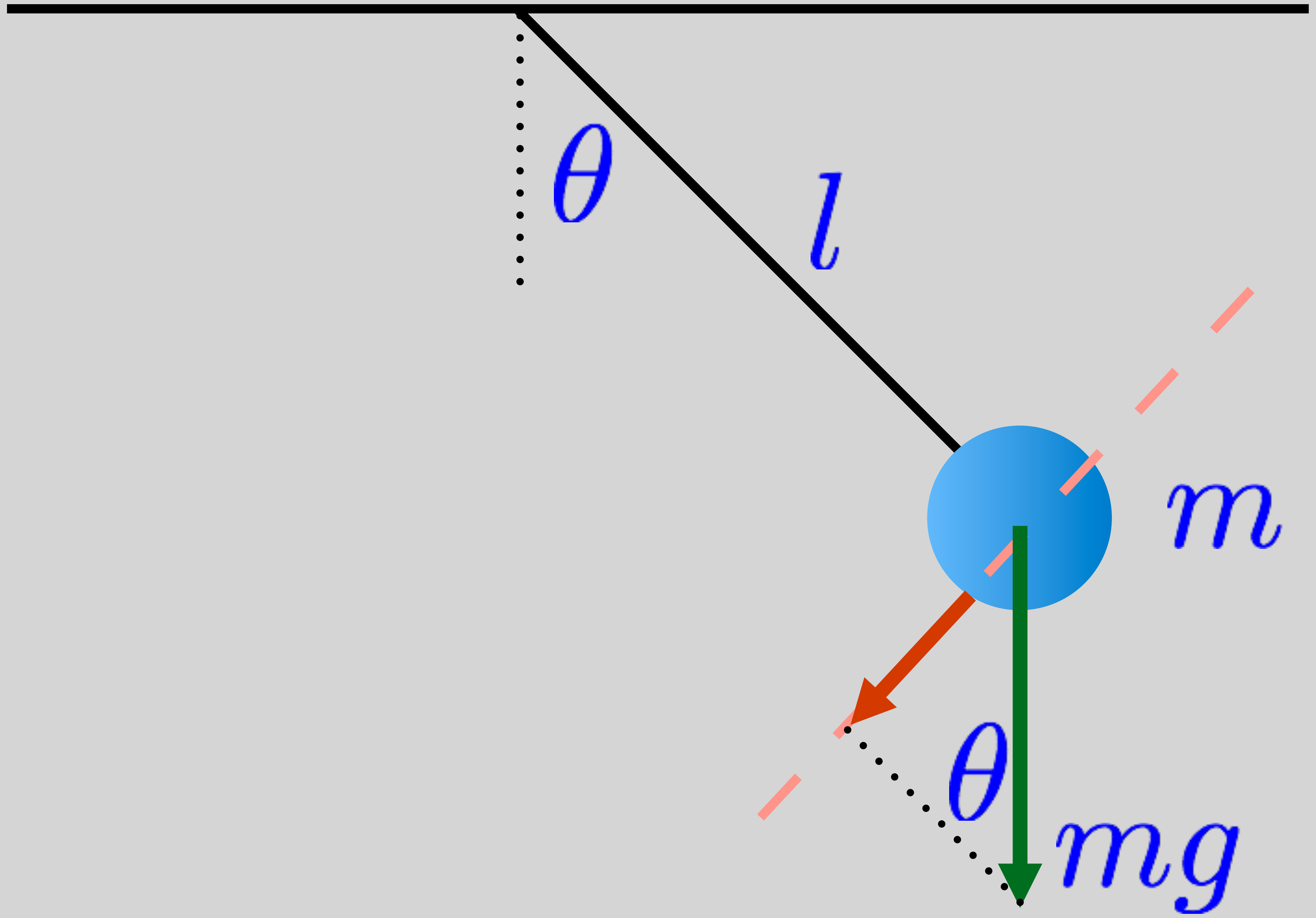
$$L\frac{di}{dt} = \text{voltage}$$



$$C\frac{dv}{dt} = \text{current}$$

Example 1: Pendulum

$$x_1(t) = \theta(t)$$
$$x_2(t) = \dot{\theta}(t) = \frac{d\theta(t)}{dt}$$



$$ma = F$$

$$m\underbrace{\left(l\frac{d^2\theta}{dt^2}\right)}_{\Rightarrow\text{acceleration}} = -mg\sin(\theta) - k(\underbrace{l\dot{\theta}(t)}_{\Rightarrow\text{tangent velocity}})$$

$$\frac{dx_1(t)}{dt} = x_2(t)$$

$$\frac{dx_2(t)}{dt} = \ddot{\theta}(t) = -\frac{g}{l}\sin(\underbrace{\theta(t)}_{x_1(t)}) - \frac{k}{m}\underbrace{\dot{\theta}(t)}_{x_2(t)}$$

Example 1 Cont.

Two 1st order diff. Eq. instead of 1 2nd order

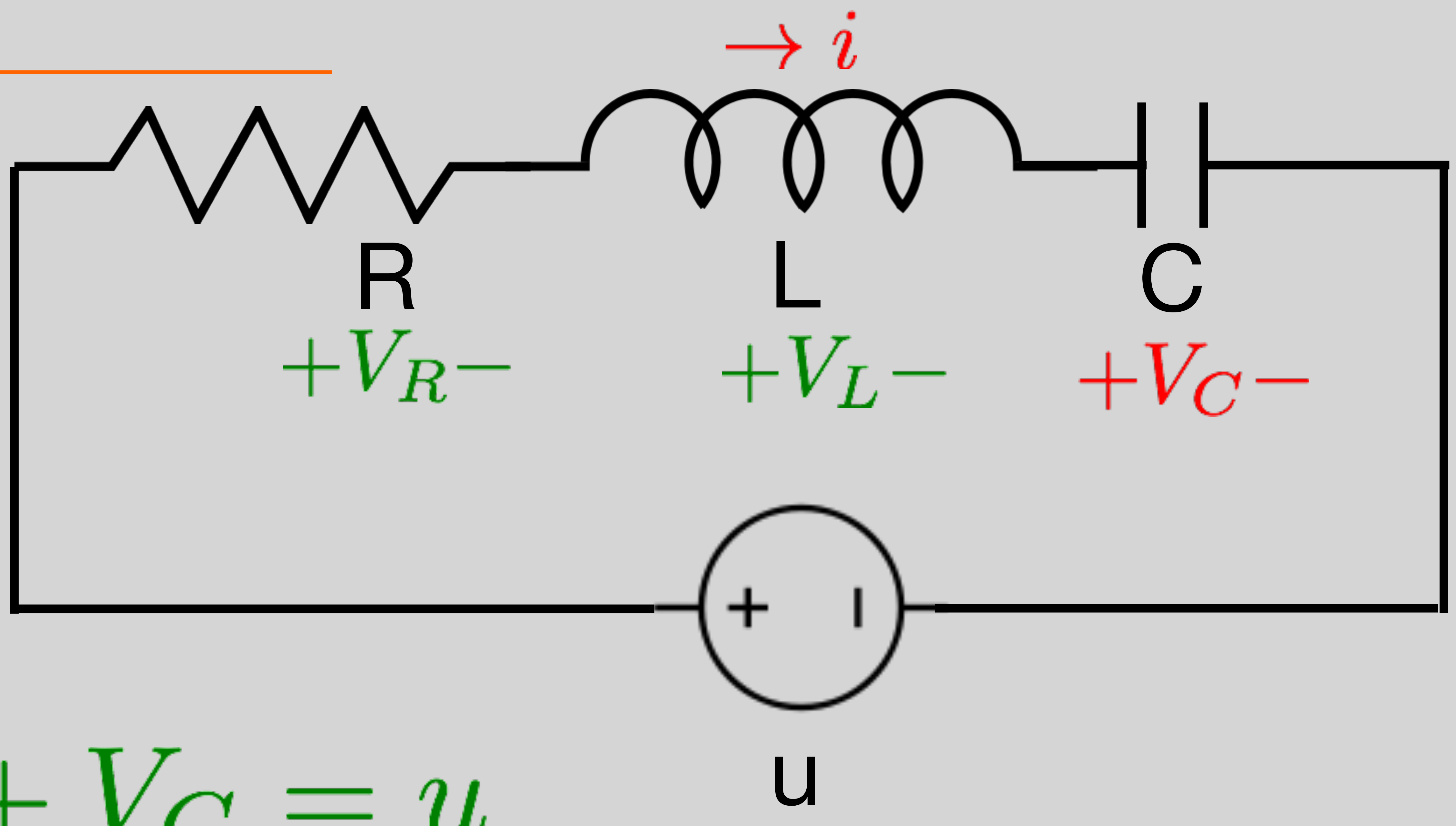
$$\frac{dx_1(t)}{dt} = x_2(t)$$

$$\frac{dx_2(t)}{dt} = \underbrace{-\frac{g}{l}\sin(x_1(t)) - \frac{k}{m}x_2(t)}_{f(\vec{x}(t))}$$

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\Rightarrow \frac{d\vec{x}(t)}{dt} = f(\vec{x}(t)) = \begin{bmatrix} x_2(t) \\ -\frac{g}{l}\sin(x_1(t)) - \frac{k}{m}x_2(t) \end{bmatrix}$$

Example 2: RLC Circuits



$$x_1 = V_c$$
$$x_2 = i$$

From KVL: $V_R + V_L + V_C = u$

$$L\frac{di(t)}{dt} = V_L = u - V_c - V_R = u - V_c - Ri$$

$$C\frac{dV_C(t)}{dt} = i$$

$$\boxed{\begin{aligned} \frac{dx_1(t)}{dt} &= \frac{1}{C}x_2(t) \\ \frac{dx_2(t)}{dt} &= \frac{1}{L}u(t) - \frac{1}{L}x_1(t) - \frac{R}{L}x_2(t) \end{aligned}}$$

Discrete Time Systems

In discrete-time systems, $\vec{x}(t)$,evolves according to a *difference* equation

$$\vec{x}(t+1) = f\left(\vec{x}(t), \vec{u}(t), \vec{w}(t)\right) \qquad t = 0,\ 1,\ 2, \cdots$$

Example 3: Manufacturing

State $\left\{\begin{array}{l} \text{s(t): inventory at the start of day t} \\ \text{g(t): goods manufactured on day t (tomorrow inventory)} \\ \text{r(t): raw material available in the morning (becomes goods)} \end{array}\right.$

control { u(t): raw materials ordered today (arrives next AM)

Disturbance { w(t): amount sold

$$s(t+1) =$$
$$g(t+1) = r(t)$$
$$r(t+1) = u(t)$$

Example 3: Manufacturing

State
- s(t): inventory at the start of day t
- g(t): goods manufactured on day t (tomorrow inventory)
- r(t): raw material available in the morning (becomes goods)

control
- u(t): raw materials ordered today (arrives next AM)

Disturbance
- w(t): amount sold

$$s(t+1) = s(t) + g(t) - w(t)$$
$$g(t+1) = r(t)$$
$$r(t+1) = u(t)$$

Example 4: EECS Professors

p(t): EECS professors in year t

r(t): # of industry researchers

δ < 1 : fraction that leave the profession

$$\left.\begin{aligned} p(t+1) &= p(t) - \delta p(t) \\ r(t+1) &= r(t) - \delta r(t) \end{aligned}\right\} \text{without input will diminish to 0}$$

u(t): average # of PhD/prof/year

ɣ: fraction of new PhD that become professors

Example 4: EECS Professors

u(t): average # of PhD/prof/year

ɣ: fraction of new PhD that become professors

# of new PhDs = p(t)u(t)

$$p(t+1) = p(t) - \delta p(t) \quad + \gamma p(t) u(t)$$

$$r(t+1) = r(t) - \delta r(t) \quad + (1-\gamma) p(t) u(t)$$

Linear Systems

When the state equation is linear

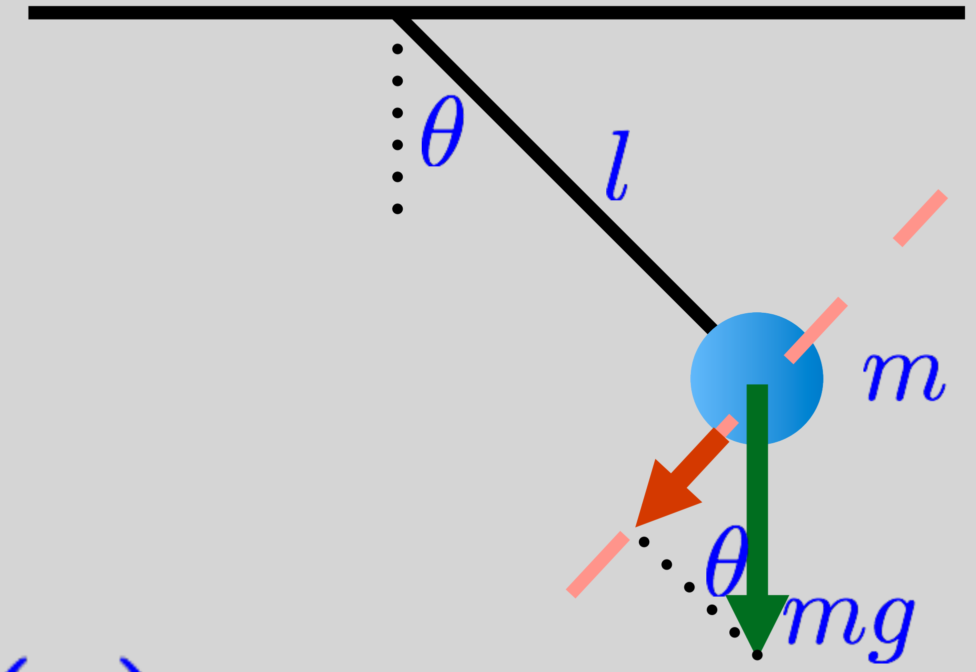
$$\frac{d\vec{x}(t)}{dt} = f(\vec{x}(t), \vec{u}(t), \vec{w}(t)) = A\vec{x}(t) + B_u\vec{u}(t) + B_w\vec{w}(t)$$

n×n

n×1 for scalar u
n×m for m inputs

$$\vec{x}(t+1) = A\vec{x}(t) + B_u\vec{u}(t) + B_w\vec{w}(t)$$

Revisiting Examples

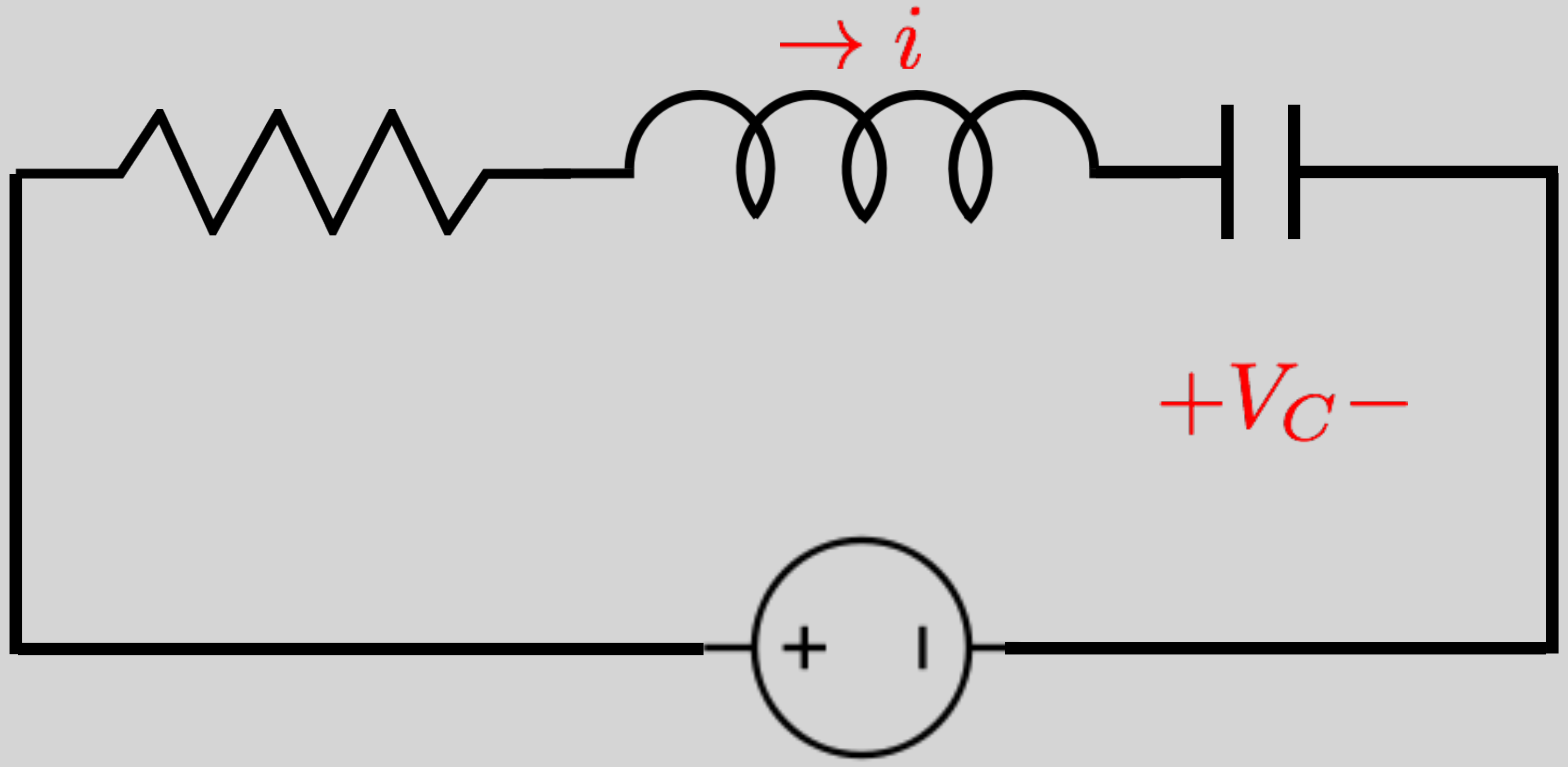


Q: Is example 1 linear?

$$\frac{d\vec{x}(t)}{dt} = f(\vec{x}(t)) = \begin{bmatrix} x_2(t) \\ -\frac{g}{l}\sin(x_1(t)) - \frac{k}{m}x_2(t) \end{bmatrix}$$

A: Non Linear

Q: example 2 linear?



$x_1 = V_c$
$x_2 = i$

$$\frac{dx_1(t)}{dt} = \frac{1}{C}x_1(t)$$

$$\frac{dx_2(t)}{dt} = \frac{1}{L}u(t) - \frac{1}{L}x_1(t) - \frac{R}{L}x_2(t)$$

A: Linear

Revisiting Example 2:

$$\frac{dx_2(t)}{dt} = \frac{1}{C}x_2(t)$$

$$\frac{dx_2(t)}{dt} = \frac{1}{L}u(t) - \frac{1}{L}x_1(t) - \frac{R}{L}x_2(t)$$

Linear relationship:

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{C} & 0 \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_{B} u(t)$$

Revisiting Examples

Q: Is example 3 linear?

$$s(t+1) = s(t) + g(t) - w(t)$$
$$g(t+1) = r(t)$$
$$r(t+1) = u(t)$$

A: Linear

$$\underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{\vec{x}(t+1)} = \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{\vec{x}(t)} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{B_u} u(t) + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{W_u} w(t)$$

Revisiting Examples

Q: Is example 3 linear?

$$s(t+1) = s(t) + g(t) - w(t)$$
$$g(t+1) = r(t)$$
$$r(t+1) = u(t)$$

A: Linear

$$\underbrace{\begin{bmatrix} s(t+1) \\ g(t+1) \\ r(t+1) \end{bmatrix}}_{\vec{x}(t+1)} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} s(t) \\ g(t) \\ r(t) \end{bmatrix}}_{\vec{x}(t)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B_u} u(t) + \underbrace{\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}}_{W_u} w(t)$$

Revisiting Examples

Q: Is example 4 linear?

$$p(t+1) = p(t) - \delta p(t) \quad + \gamma p(t)u(t)$$

$$r(t+1) = r(t) - \delta r(t) \quad + (1-\gamma) p(t)u(t)$$

A: non Linear

Changing State Variable

State variables are not unique!

$$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$$

Let T be an invertible matrix:

$$\vec{z} = T\vec{x}$$

Then,

$$\vec{z}(t+1) = T\vec{x}(t+1) = TA\vec{x}(t) + TB\vec{u}(t)$$

$$= TAT^{-1}\vec{z}(t) + TB\vec{u}(t)$$

Changing State Variables

$$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$$

Define:

$$\vec{z} = T\vec{x} \qquad A_{\text{new}} = TAT^{-1} \qquad B_{\text{new}} = TB$$

Can be written as,

$$\vec{z}(t+1) = A_{\text{new}}\vec{z}(t) + B_{\text{new}}\vec{u}(t)$$

Similarly for continuous systems!

Next: We will see how a special choice of T will make it easy to analyze system properties like *stability*, and *controllability*

Summary

- Learned to describe systems in a state-space model
  - Extremely powerful model!!!
- State space model leads to coupled
  - 1st order (coupled) differential equations (Cont. time)
  - 1st order (coupled) difference equations (Disc. time)
- Talked about linear systems
  - Described state evolution in matrix form
- Showed how to change state variables
- Next: Linearization of non-linear systems