Problem 9.19 Generate Bode magnitude and phase plots (straight-line approximation) for the following voltage transfer functions:

(a) $\mathbf{H}(\omega) = \dfrac{4\times 10^4(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}$

(b) $\mathbf{H}(\omega) = \dfrac{(1+j0.2\omega)^2(100+j2\omega)^2}{(j\omega)^3(500+j\omega)}$

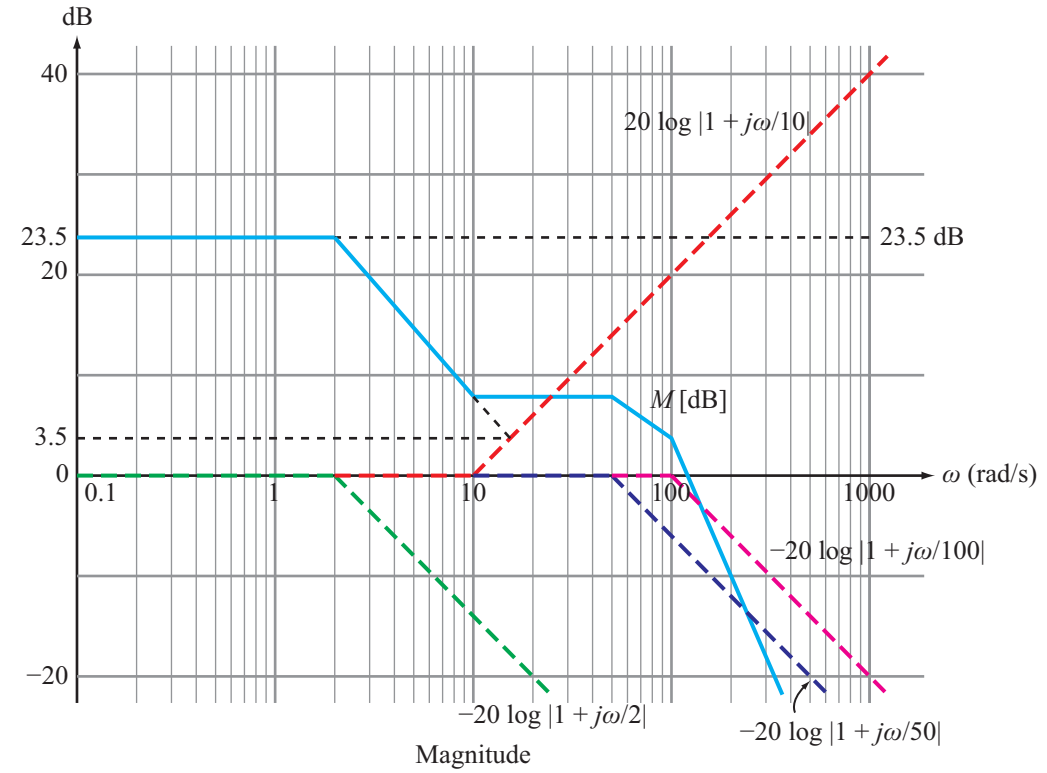
(c) $\mathbf{H}(\omega) = \dfrac{8\times 10^{-2}(10+j10\omega)}{j\omega(16-\omega^2+j4\omega)}$

(d) $\mathbf{H}(\omega) = \dfrac{4\times 10^4\omega^2(100-\omega^2+j50\omega)}{(5+j5\omega)(200+j2\omega)^3}$

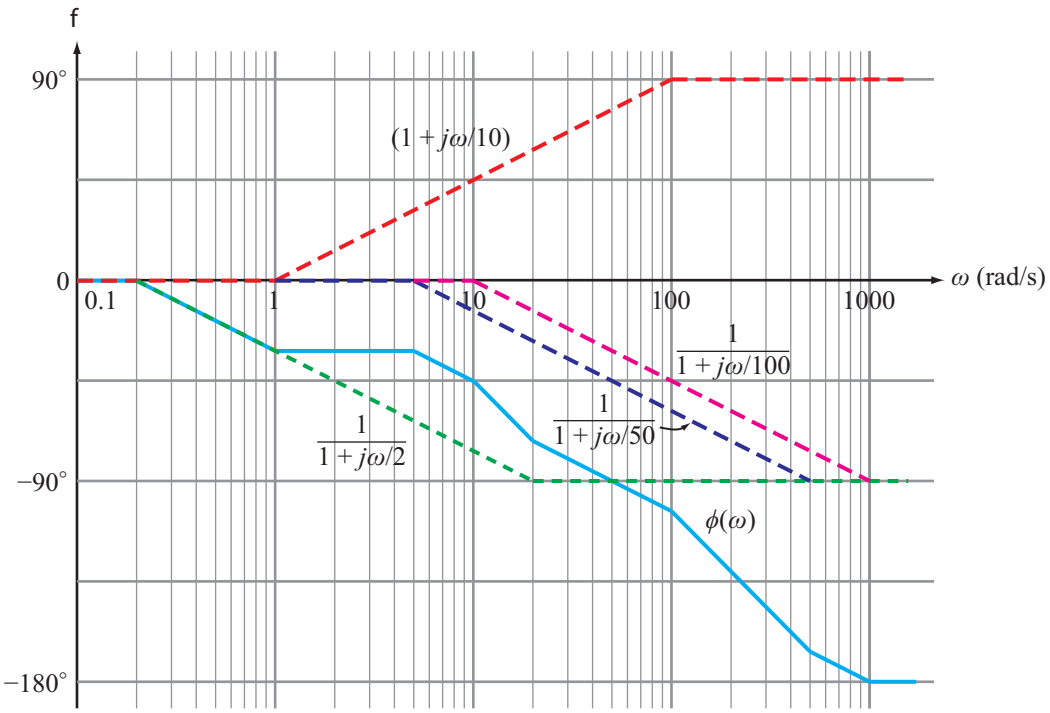
Solution:
(a)

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{4\times 10^4(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}\\ &= \frac{4\times 10^4\times 60(1+j\omega/10)}{4\times 100\times 400(1+j\omega/2)(1+j\omega/50)(1+j\omega/100)}\\ &= \frac{15(1+j\omega/10)}{(1+j\omega/2)(1+j\omega/50)(1+j\omega/100)}\ .\end{aligned}$$

- Constant term 15 $\Longrightarrow$ 23.5 dB
- Simple pole with $\omega_c = 2$ rad/s
- Simple zero with $\omega_c = 10$ rad/s
- Simple pole with $\omega_c = 50$ rad/s
- Simple pole with $\omega_c = 100$ rad/s



Magnitude



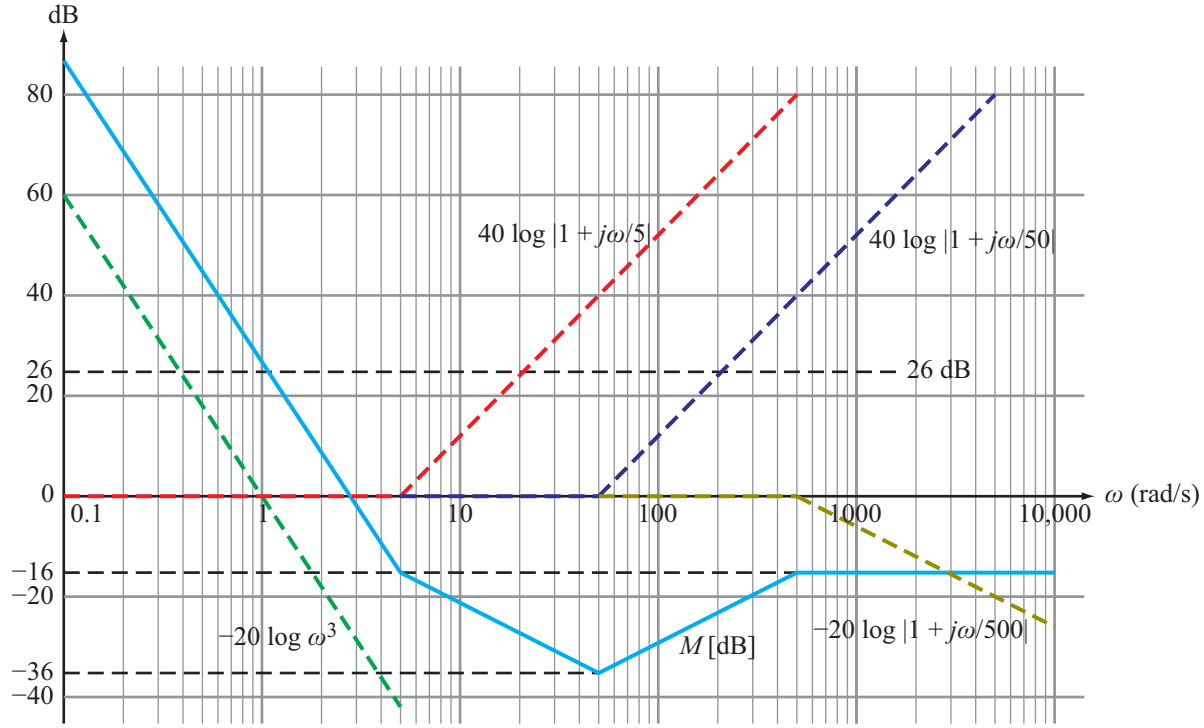
Phase

(b)

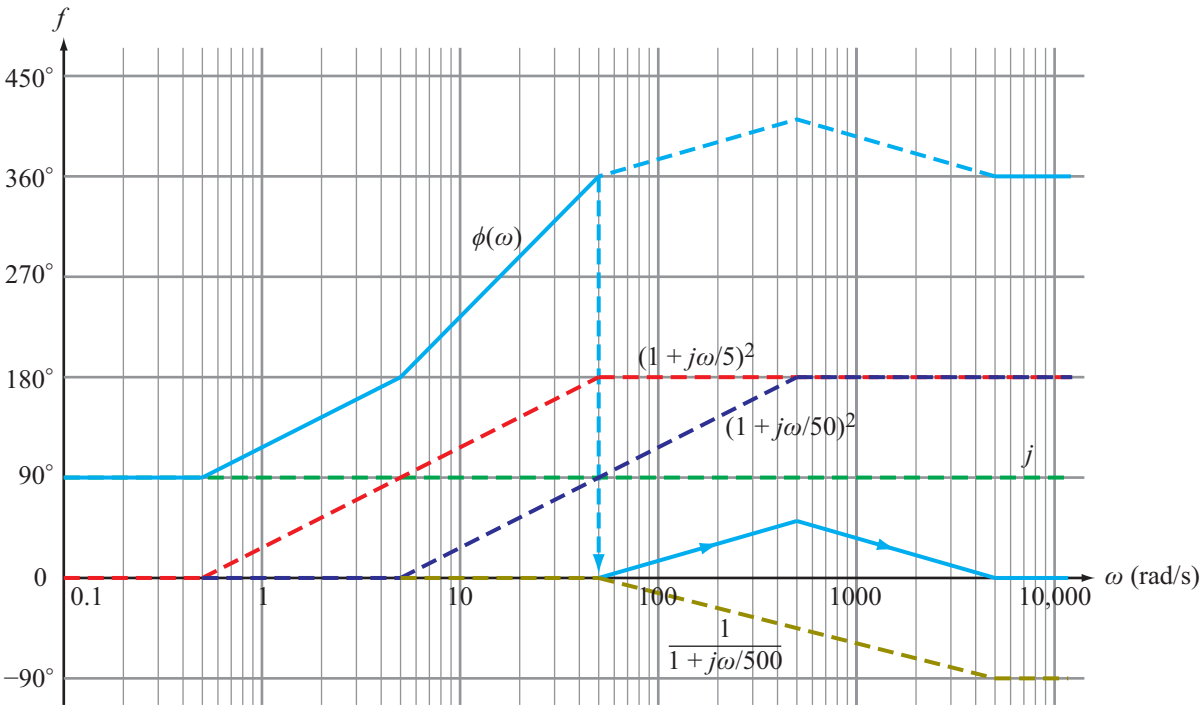
$$\mathbf{H}(\omega) = \frac{(1 + j0.2\omega)^2(100 + j2\omega)^2}{(j\omega)^3(500 + j\omega)}$$

$$= \frac{j10^4(1+j\omega/5)^2(1+j\omega/50)^2}{500\omega^3(1+j\omega/500)}$$
$$= \frac{j20(1+j\omega/5)^2(1+j\omega/50)^2}{\omega^3(1+j\omega/500)}$$

- Constant term 20 $\implies$ 26 dB
- Pole of order 3 @ origin
- Simple zero with $\omega_c = 5$ rad/s, of order 2
- Simple zero of order 2 with $\omega_c = 50$ rad/s
- Simple pole with $\omega_c = 500$ rad/s



Magnitude



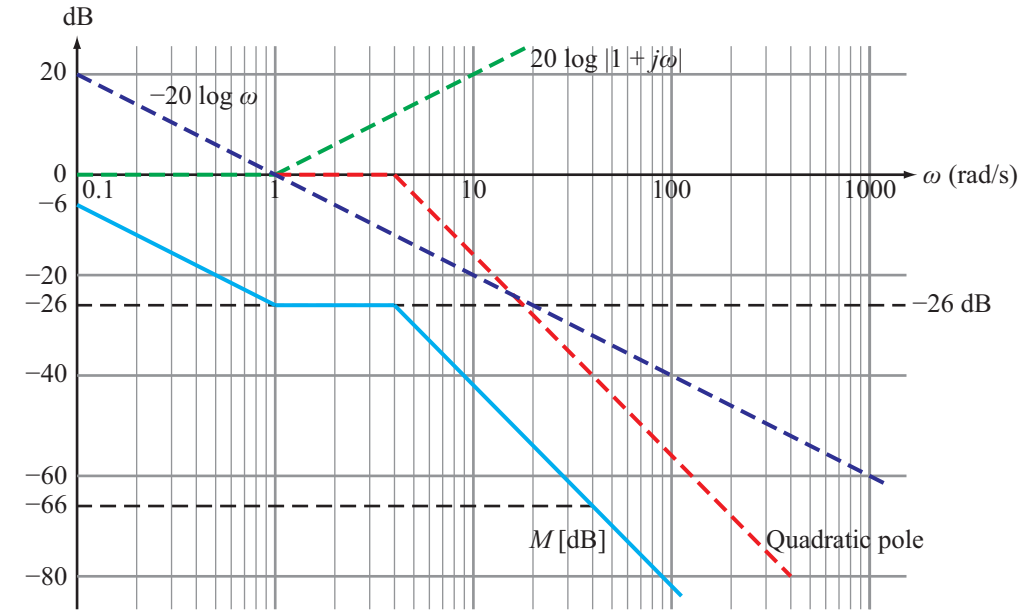
Phase

(c)

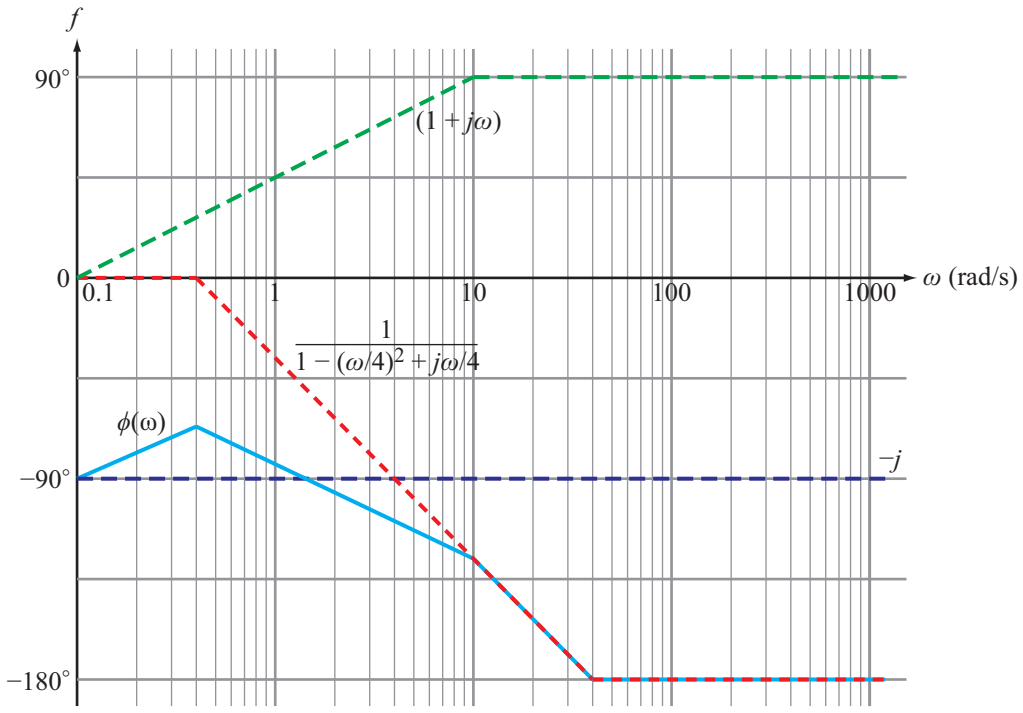
$$\mathbf{H}(\omega) = \frac{8 \times 10^{-2}(10 + j10\omega)}{j\omega(16 - \omega^2 + j4\omega)}$$

$$= \frac{-j8 \times 10^{-2} \times 10(1+j\omega)}{16\omega[1-(\omega/4)^2+j\omega/4]}$$
$$= \frac{-j5 \times 10^{-2}(1+j\omega)}{\omega[1-(\omega/4)^2+j\omega/4]}$$

- Constant factor $5 \times 10^{-2} \quad \Longrightarrow \quad -26$ dB
- Zero factor with $\omega_c = 1$ rad/s
- Quadratic pole with $\omega_c = 4$ rad/s
- Pole @ origin



Magnitude



Phase

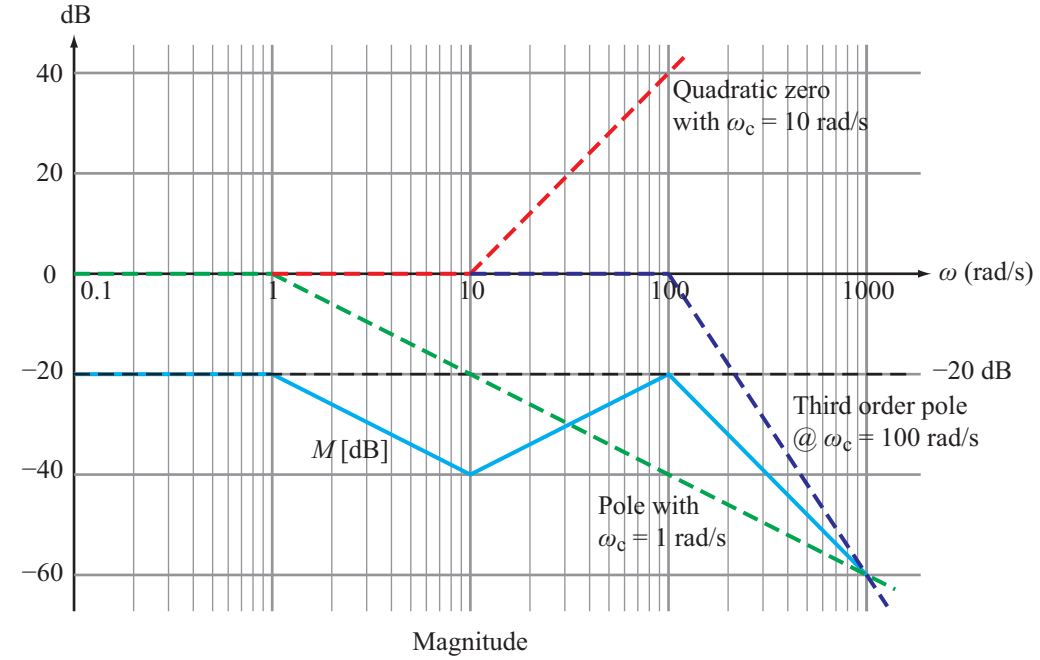
(d)

$$\mathbf{H}(\omega) = \frac{4\times 10^4\omega^2(100-\omega^2+j50\omega)}{(5+j5\omega)(200+j2\omega)^3}$$

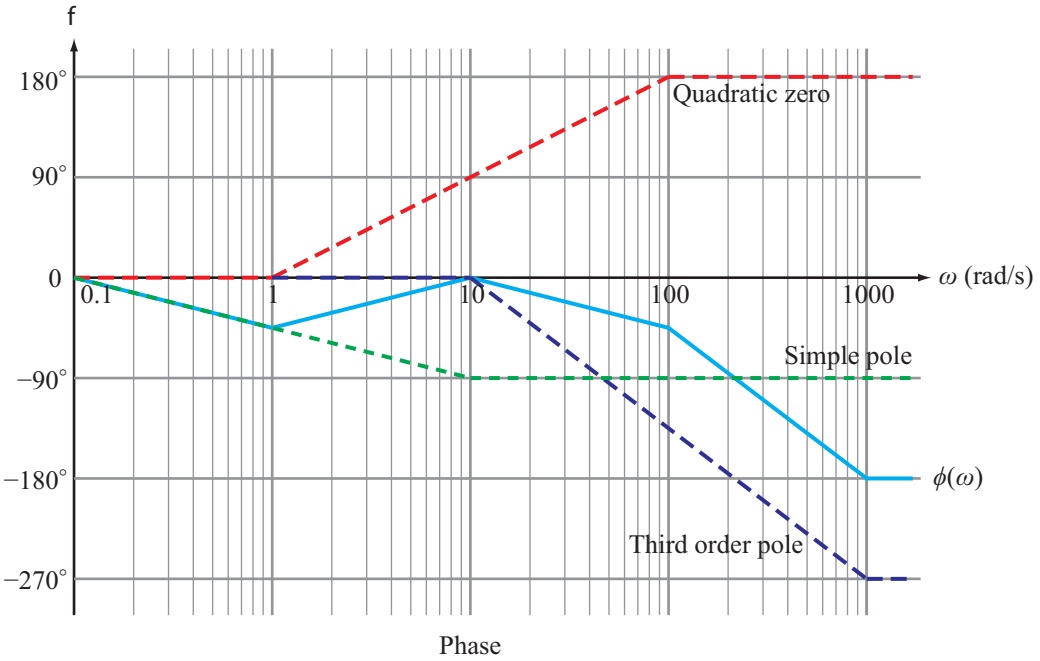
$$= \frac{4\times 10^4\times 100\omega^2[1-(\omega/10)^2+j\omega/2]}{5\times(200)^3(1+j\omega)(1+j\omega/100)^3} = \frac{0.1\omega^2[1-(\omega/10)^2+j\omega/2]}{(1+j\omega)(1+j\omega/100)^3}$$

- Constant factor 0.1 $\Longrightarrow$ -20 dB

- Zero at origin of order 2
- Simple pole with $\omega_c = 1$ rad/s
- Quadratic zero with $\omega_c = 10$ rad/s
- Third order pole with $\omega_c = 100$ rad/s



Magnitude



Phase