Problem 9.31 For the circuit shown in Fig. P9.31:

(a) Obtain an expression for $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_i$ in standard form.

(b) Generate spectral plots for the magnitude and phase of $\mathbf{H}(\omega)$, given that $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 1\ \mu$F, and $C_2 = 2\ \mu$F.

(c) Determine the cutoff frequency ω_c and the slope of the magnitude (in dB) when $\omega/\omega_c \ll 1$ and when $\omega/\omega_c \gg 1$.

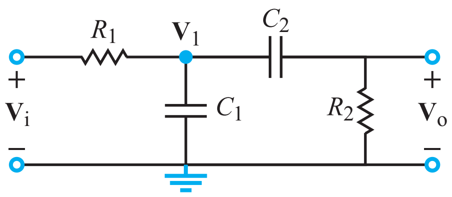


Figure P9.31: Circuit for Problem 9.31.

Solution: (a) At node $\mathbf{V}_1$, KCL gives:

$$\frac{\mathbf{V}_1 - \mathbf{V}_i}{R_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_{c_1}} + \frac{\mathbf{V}_1}{\mathbf{Z}_{c_2} + R_2} = 0.$$

Voltage division gives

$$\mathbf{V}_o = \frac{R_2 \mathbf{V}_1}{\mathbf{Z}_{c_2} + R_2}.$$

With $\mathbf{Z}_{c_1} = 1/j\omega C_1$ and $\mathbf{Z}_{c_2} = 1/j\omega C_2$, solution is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega R_2 C_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_1 C_2}.$$

To express denominator in standard form, we expand it to get

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{j\omega R_2 C_2}{1 + j\omega(R_1 C_1 + R_2 C_2 + R_1 C_2) + (j\omega\sqrt{R_1 R_2 C_1 C_2})^2} \\ &= \frac{j\omega K}{1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2},\end{aligned}$$

where

$$K = R_2 C_2, \qquad \omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}},$$

$$\xi = \frac{1}{2}\,\omega_c[R_1 C_1 + R_2 C_2 + R_1 C_2].$$

(b) For $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 1\ \mu$F, and $C_2 = 2\ \mu$F.

$$K = 4 \times 10^{-6}, \qquad \omega_c = 0.5 \times 10^6 \text{ rad/s},$$

$$\xi = \frac{7}{4}.$$

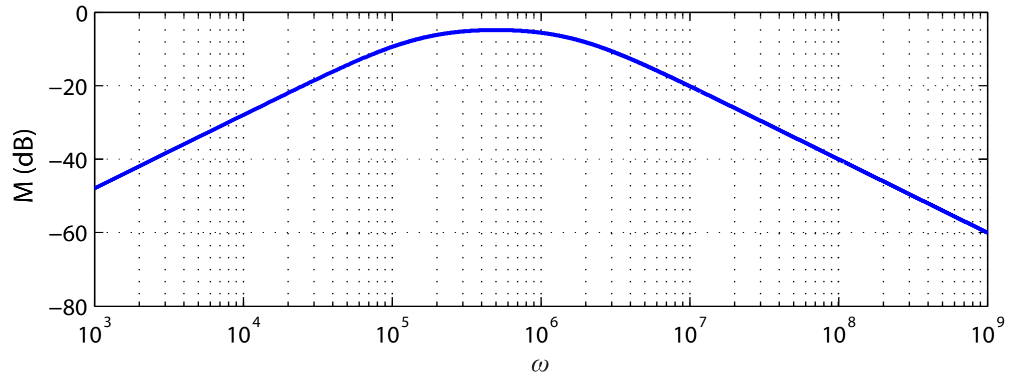
Hence,

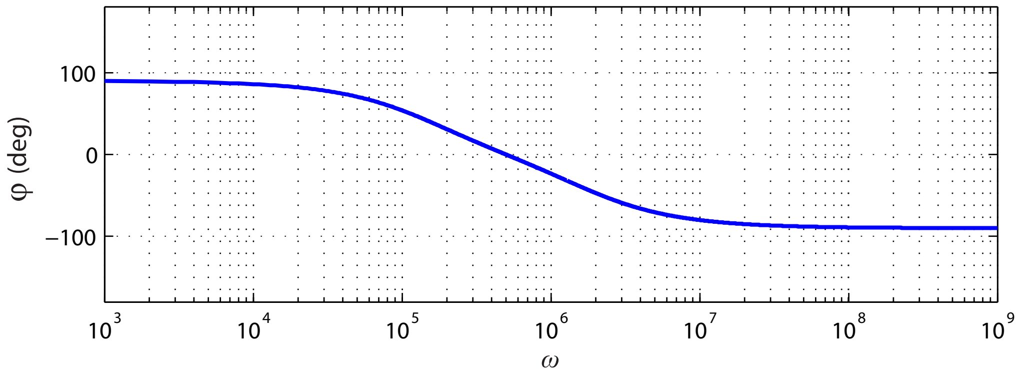
$$\mathbf{H}(\omega) = \frac{j4 \times 10^{-6}\omega}{1 + j\frac{7}{2}\omega/\omega_c + (j\omega/\omega_c)^2},$$

with $\omega_c = 0.5 \times 10^6$ rad/s.

$$M\ [\text{dB}] = 20\log|\mathbf{H}(\omega)|$$

Spectral plots of M [dB] and $\phi(\omega)$ are shown in Figs. P9.31(a) and (b).





Figures P9.31(a) and (b)

(c) From part (b), the cutoff frequency of the quadratic pole is

$$\omega_c = 0.5 \times 10^6 \text{ rad/s}.$$

Low-frequency asymptote $(\omega/\omega_c \ll 1)$:

$$\mathbf{H}(\omega) \simeq j4\times 10^{-6}\omega \quad \Longrightarrow \quad \text{slope of } M\ [\text{dB}] \text{ is } +20 \text{ dB/decade.}$$

High-frequency asymptote $(\omega/\omega_c \gg 1)$:

$$\mathbf{H}(\omega) \simeq \frac{j4\times 10^{-6}\omega}{j^2\omega^2/\omega_c^2} = -j\frac{4\times 10^{-6}\omega_c^2}{\omega} = \frac{-j10^6}{\omega} \quad \Longrightarrow \quad \text{slope} = -20 \text{ dB/decade.}$$