Primer for the Upcoming Control Lab

In a lab assignment you will work on a robot car with two wheels, each driven with a separate electric motor. Let $d_l(t)$ and $d_r(t)$ be the distance traveled by the left and right wheels, and let $u_l(t)$ and $u_r(t)$ denote the control input (duty cycle of pulse width modulated current) we apply to the respective motor.

An appropriate model relating these variables is

$$\begin{aligned} d_l(t+1) - d_l(t) &= \theta_l u_l(t) - \beta_l \\ d_r(t+1) - d_r(t) &= \theta_r u_r(t) - \beta_r \end{aligned} \tag{5}$$

where the right hand sides approximate the speed for each wheel. Experimental data show that the speed instantly settles to a steady state proportional to the input, possibly with a bias term, hence the expressions $\theta_l u_l(t) - \beta_l$ and $\theta_r u_r(t) - \beta_r$.

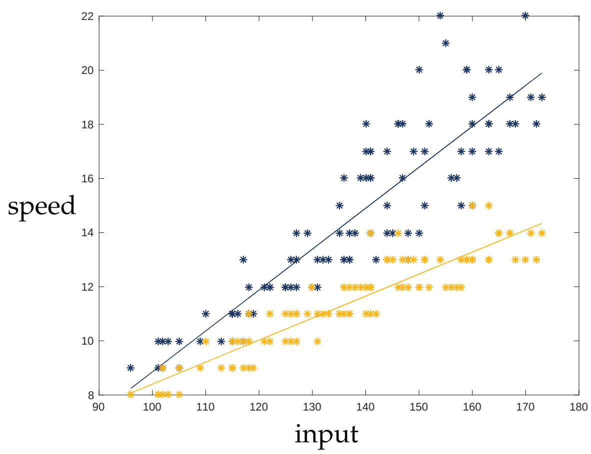

Figure 2: Speed vs. input data points from experiments. The left and right wheels are represented with different colors, and the lines show least squares estimates for each.

The parameters θ_l, θ_r, β_l, β_r, are estimated experimentally using least squares. We apply a time-varying input sequence $u_l(0)$, $u_l(1)$, $\ldots$, $u_l(k)$ to the left motor, collect the resulting data $d_l(1)$, $d_l(2)$, $\ldots$, $d_l(k+1)$, and set up the overdetermined equation

$$\underbrace{\begin{bmatrix} u_l(0) & -1 \\ u_l(1) & -1 \\ \vdots & \vdots \\ u_l(k) & -1 \end{bmatrix}}_{\triangleq S} \begin{bmatrix} \theta_l \\ \beta_l \end{bmatrix} = \begin{bmatrix} d_l(1) - d_l(0) \\ d_l(2) - d_l(1) \\ \vdots \\ d_l(k+1) - d_l(k) \end{bmatrix}.$$

Then the least squares estimate is

$$\begin{bmatrix} \hat{\theta}_l \\ \hat{\beta}_l \end{bmatrix} = (S^T S)^{-1} S^T \begin{bmatrix} d_l(1) - d_l(0) \\ d_l(2) - d_l(1) \\ \vdots \\ d_l(k+1) - d_l(k) \end{bmatrix}.$$

Parameter estimates $\hat{\theta}_r$, $\hat{\beta}_r$ for the right wheel are obtained similarly. However, the parameters for the two wheels may be significantly different. Thus, applying an identical input to both wheels would lead to nonidentical speeds, and the car would go in circles. We straighten the trajectory of the car in two steps:

Step 1: Apply the constant inputs

$$u_l = \frac{v^* + \hat{\beta}_l}{\hat{\theta}_l} \qquad u_r = \frac{v^* + \hat{\beta}_r}{\hat{\theta}_r} \tag{6}$$

which aim to set each speed to the same value v^*. Indeed, if our parameter estimates were exact, we would obtain v^* on the right hand side of each equation in (5).

While this step should improve the trajectory it may not completely straighten it, as the parameter estimates are unlikely to be perfectly accurate. In the next step we augment (6) with a feedback term.

Step 2: Modify (6) as

$$\begin{aligned} u_l(t) &= \frac{v^* + \hat{\beta}_l}{\hat{\theta}_l} + \frac{k_l}{\hat{\theta}_l}(d_r(t) - d_l(t)) \\ u_r(t) &= \frac{v^* + \hat{\beta}_r}{\hat{\theta}_r} + \frac{k_r}{\hat{\theta}_r}(d_l(t) - d_r(t)) \end{aligned} \tag{7}$$

where the feedback gains k_l and k_r are to be designed.

Assume for now that $\hat{\theta}_l = \theta_l$, $\hat{\beta}_l = \beta_l$, $\hat{\theta}_r = \theta_r$, $\hat{\beta}_r = \beta_r$, and substitute (7) in (5) to get

$$\begin{aligned} d_l(t+1) - d_l(t) &= v^* - k_l(d_l(t) - d_r(t)) \\ d_r(t+1) - d_r(t) &= v^* + k_r(d_l(t) - d_r(t)). \end{aligned} \tag{8}$$

Next, define $\delta(t) \triangleq d_l(t) - d_r(t)$ and note from (8) that it satisfies

$$\delta(t+1) - \delta(t) = (-k_l - k_r)\delta(t)$$

or, equivalently,

$$\delta(t+1) = (1 - k_l - k_r)\delta(t).$$

Thus, to ensure $\delta(t) \to 0$, we need to select k_l and k_r such that

$$|1 - k_l - k_r| < 1.$$

Without the feedback terms in (7), that is $k_l = k_r = 0$, we get

$$\delta(t+1) = \delta(t)$$

which means that the error accumulated in $\delta(t)$ persists and is in fact likely to grow when we incorporate a disturbance term to account for the mismatch between the parameters and their estimates $\hat{\theta}_l, \hat{\theta}_r, \hat{\beta}_l, \hat{\beta}_r$. The feedback in (7) is thus essential to dissipate the error $\delta(t)$ and to keep it bounded in the presence of parameter mismatch.