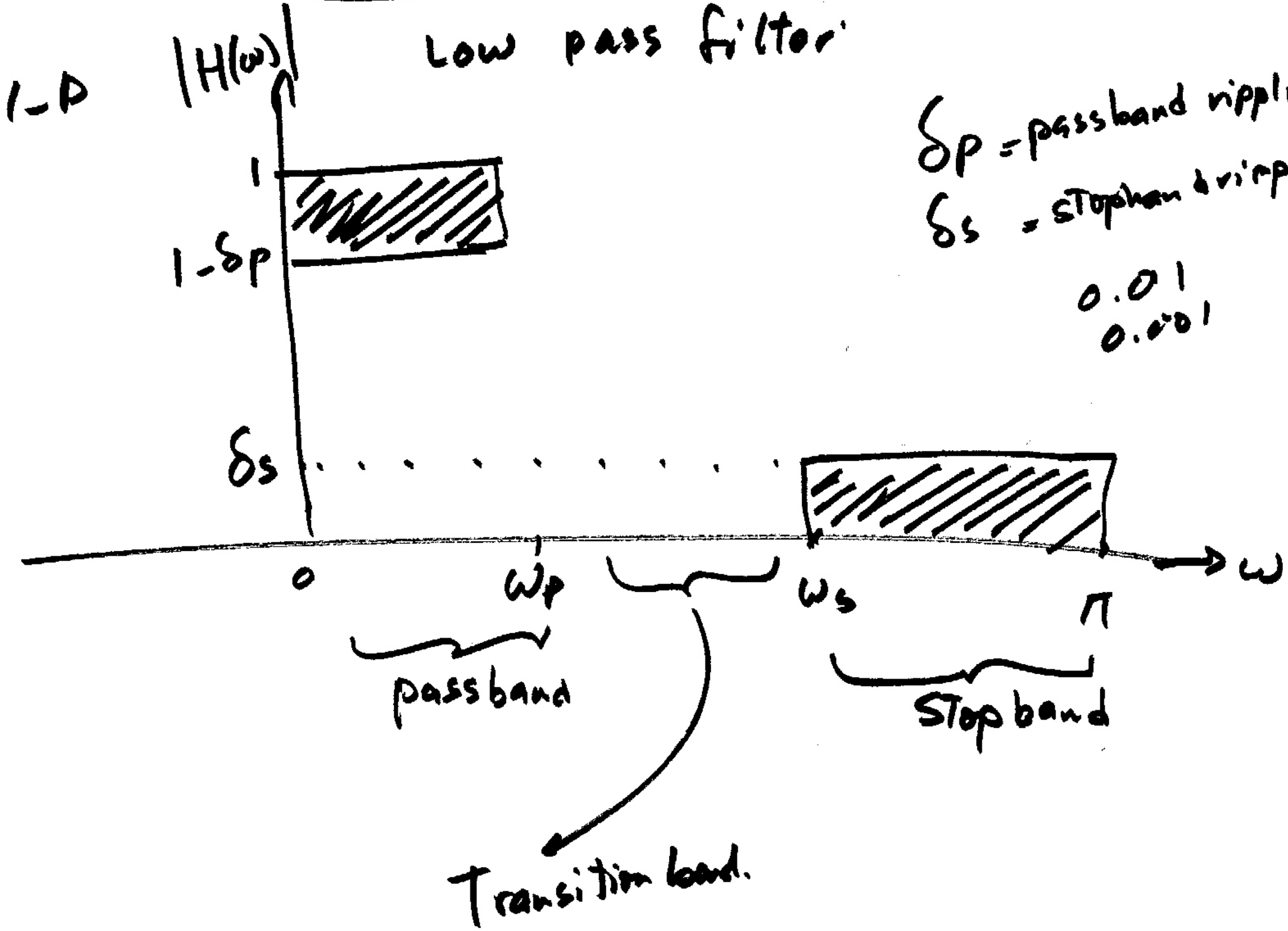
Feb 17, 06

Filter Specs

Low pass filter

Ex 1-D



δp = passband ripple

δs = stopband ripple.

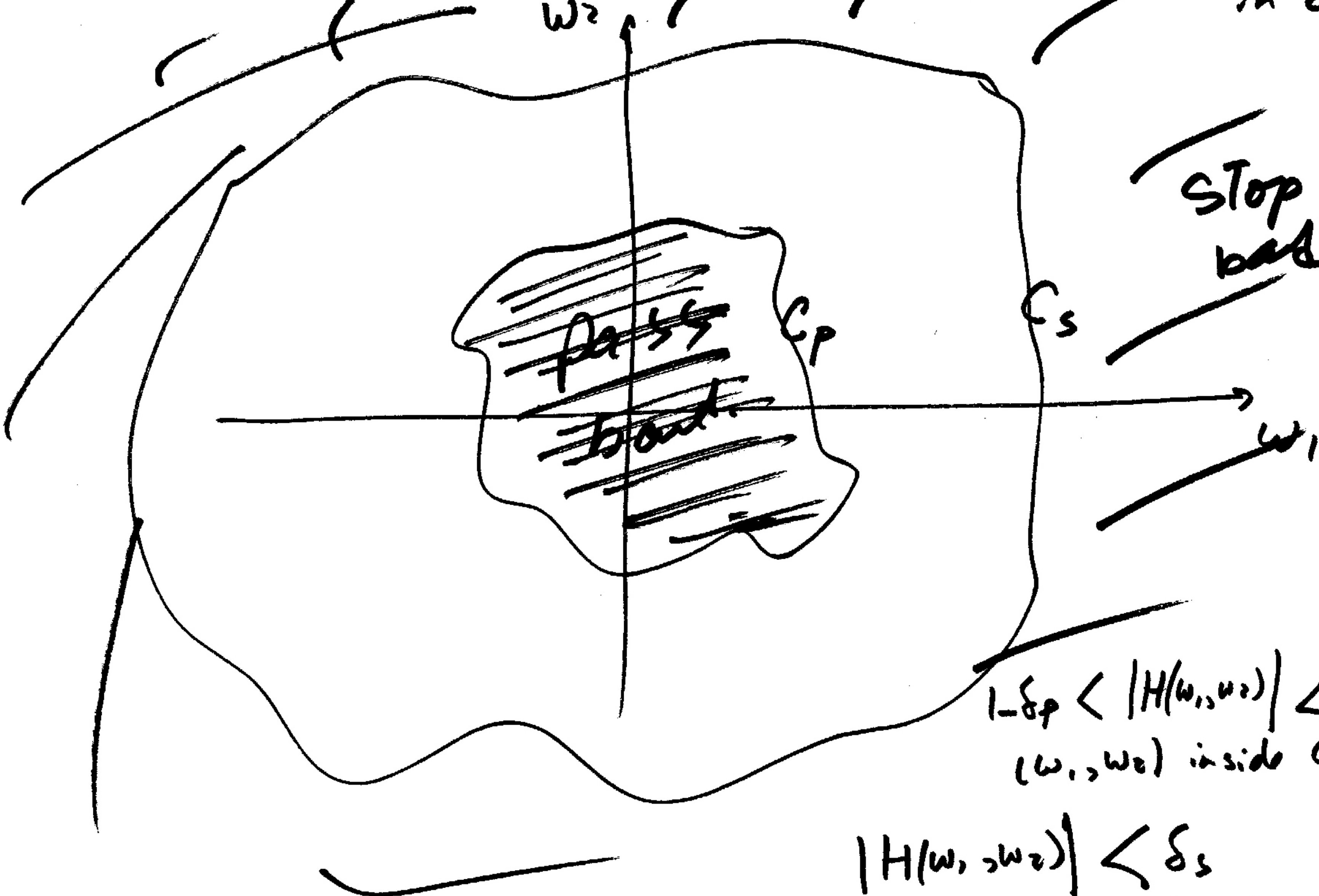
0.01

0.001

1

2D specs

low Pass filter in 2D.



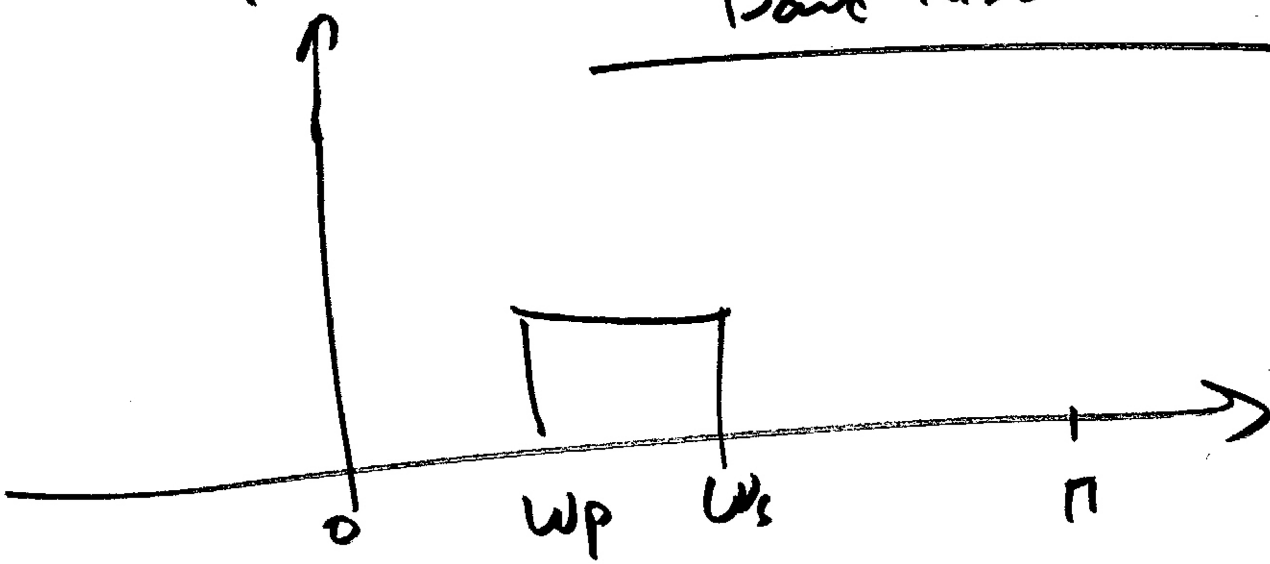
$1-\delta_p < |H(w_1, w_2)| < 1$

(w_1, w_2) inside C_p

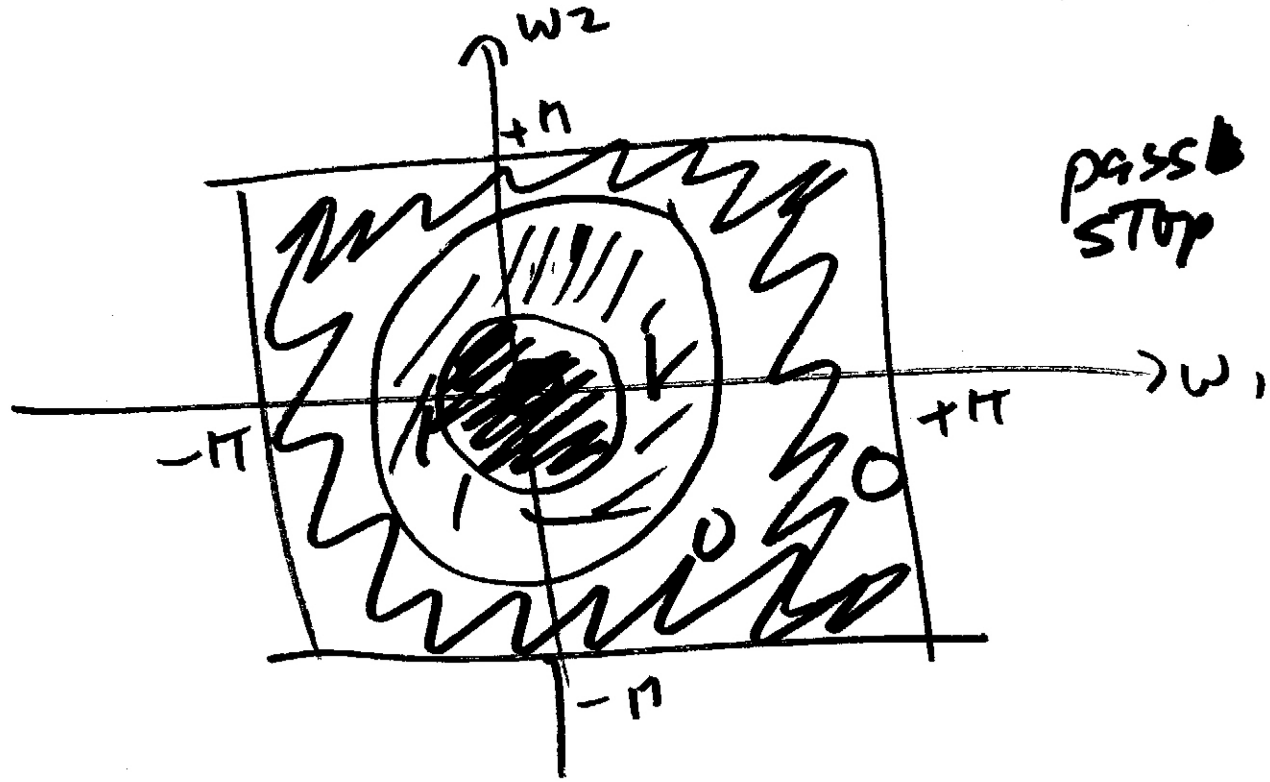
$|H(w_1, w_2)| < \delta_s$

(w_1, w_2) outside C_s

Band Pass filters



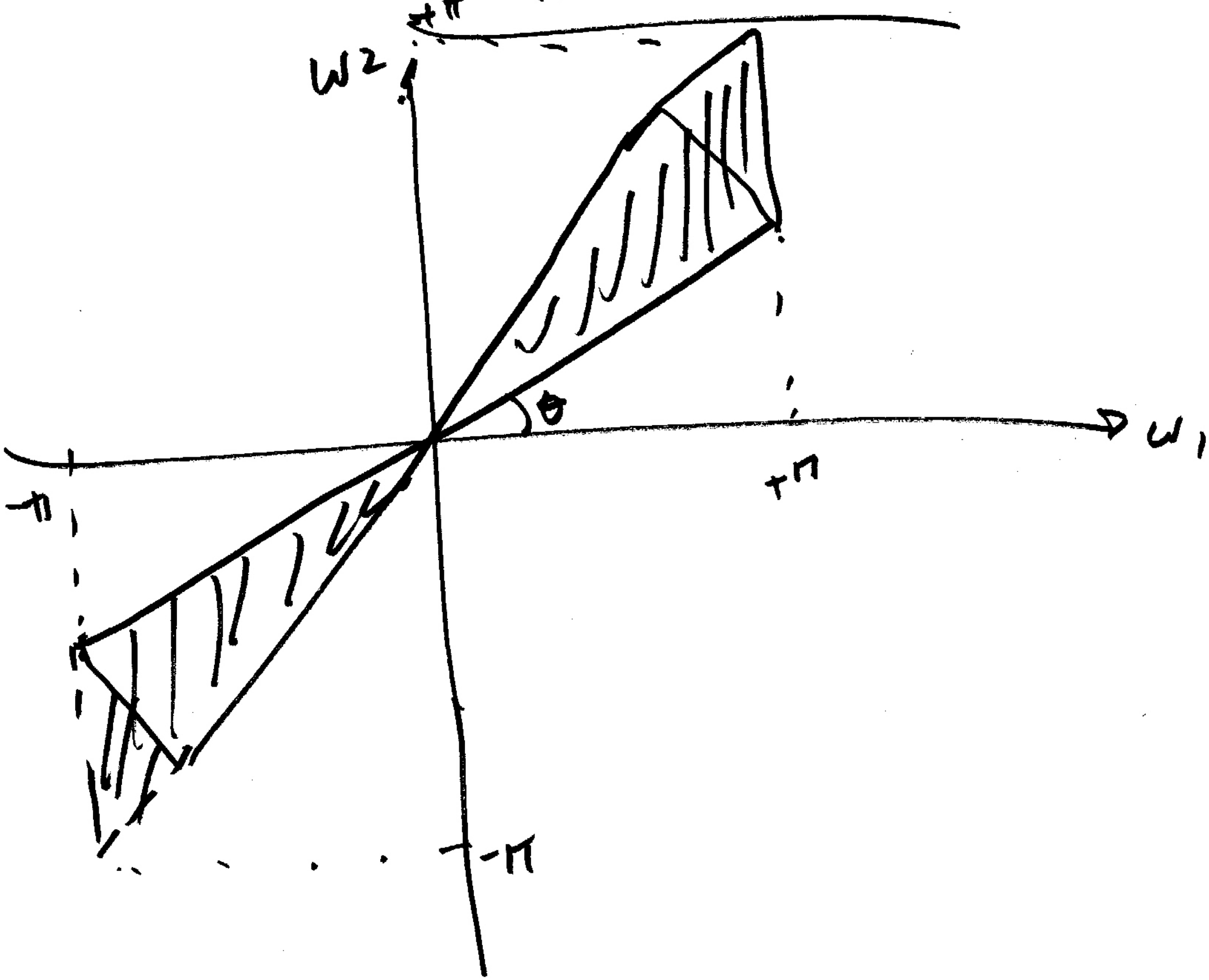
1D



2D

pass green
stop Red

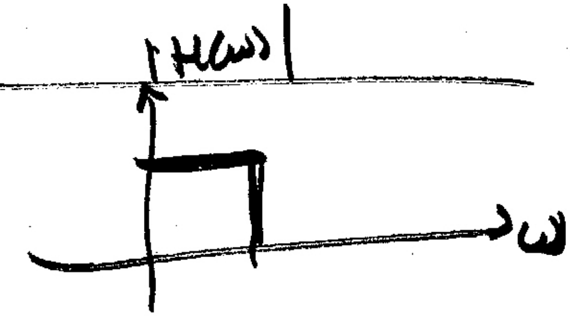
Directional Filters



3 ways of designing FIR Filters

1. Window $\quad 1D \longrightarrow 2D$.
2. Frequency sampling $\quad 1D \longrightarrow 2D$.
3. Design by Transformation : McClellan.

Window

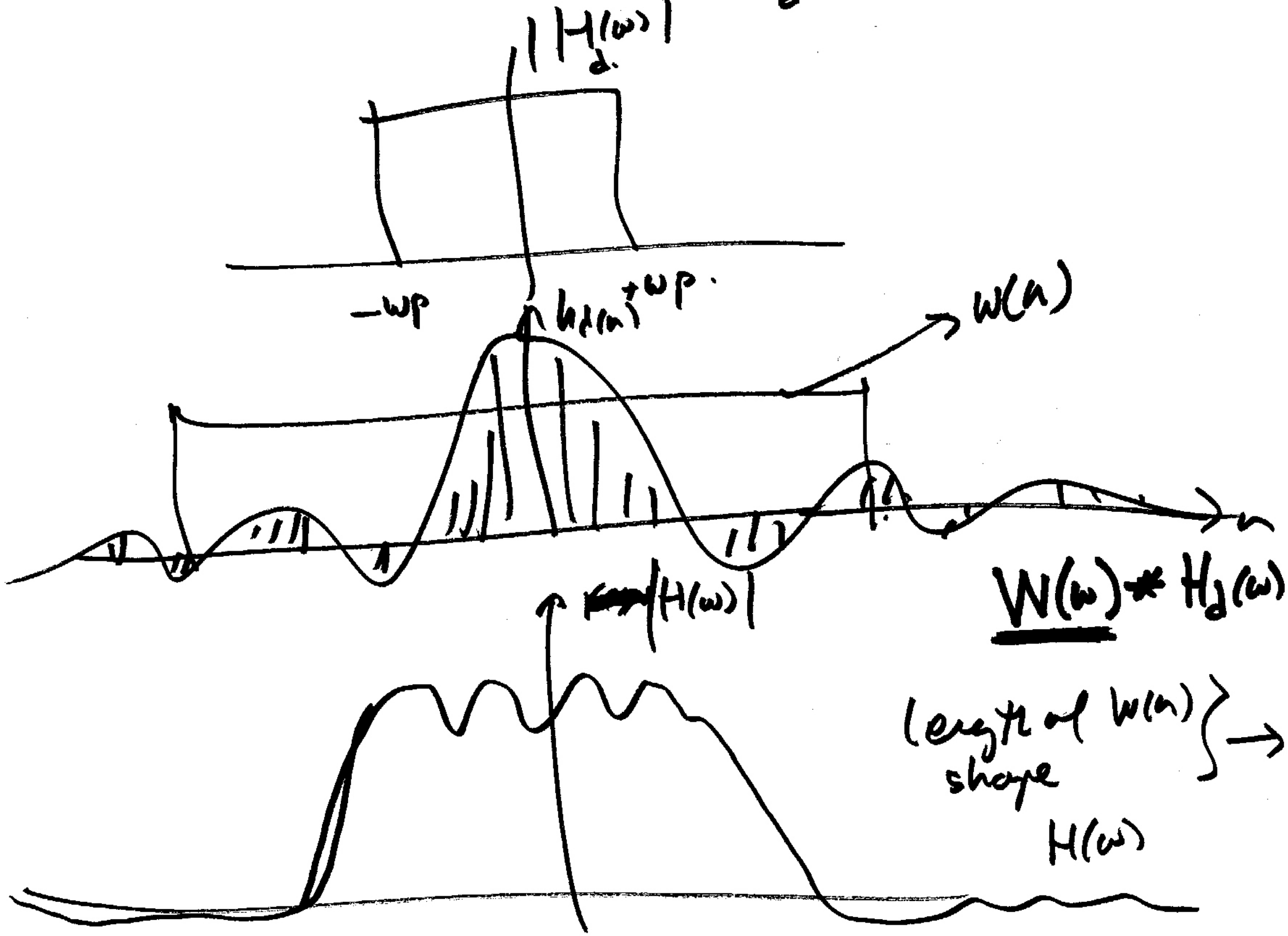


1. Start with ideal freq. response $H_d(\omega)$

2. take inverse DTFT. $h_d(n)$ $\quad h_d(n) \longrightarrow$ sinc. ∞ ly long $\longrightarrow$ IIR.

3. $h(n) = w(n)\, h_d(n)$ $\quad w(n) \longrightarrow$ window fn.

$$h(n_1,n_2) = W(n_1,n_2)\, h_d(n_1,n_2)$$

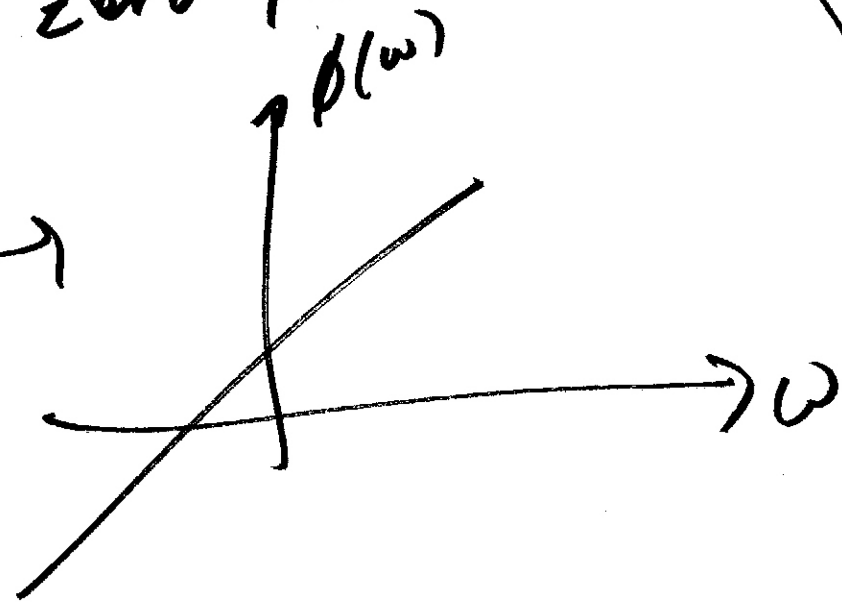


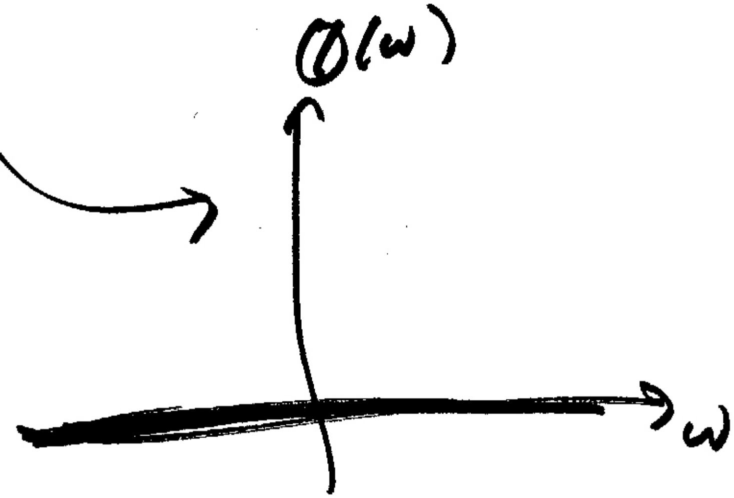
2D windows

$H(\omega) = |H(\omega)| e^{j\phi(\omega)}$

linear phase $\phi(\omega)$ is linear in ω →

Zero phase $\phi(\omega)$ is zero —





$$h(n_1,n_2) = \underbrace{h_d(n_1,n_2)}_{\text{desired impulse resp.}} \underbrace{W(n_1,n_2)}_{\text{zero phase window}}$$

$$\left.\begin{array}{l}\text{If } h_d \text{ is zero phase} \\ \text{If } W \text{ " " "}\end{array}\right\} \longrightarrow h(n_1,n_2) \text{ is zero phase}$$

Kaiser window :

$$w_a(t) = \begin{cases} \dfrac{I_0\left(\alpha\sqrt{1-\left(\frac{t}{\tau}\right)^2}\right)}{I_0(\alpha)} & |t| \tau \\ 0 & \text{otherwise.} \end{cases}$$

duration $\approx 2\tau$

α = Kmb Tradeoff main lobe width with sidelobe height

$I_0(x)$ modified zeroth order Bessel fn.

$$I_0(x) = \sum_{i=0}^{\infty} \frac{x^{2i}}{2^{2i}(i!)^2}$$

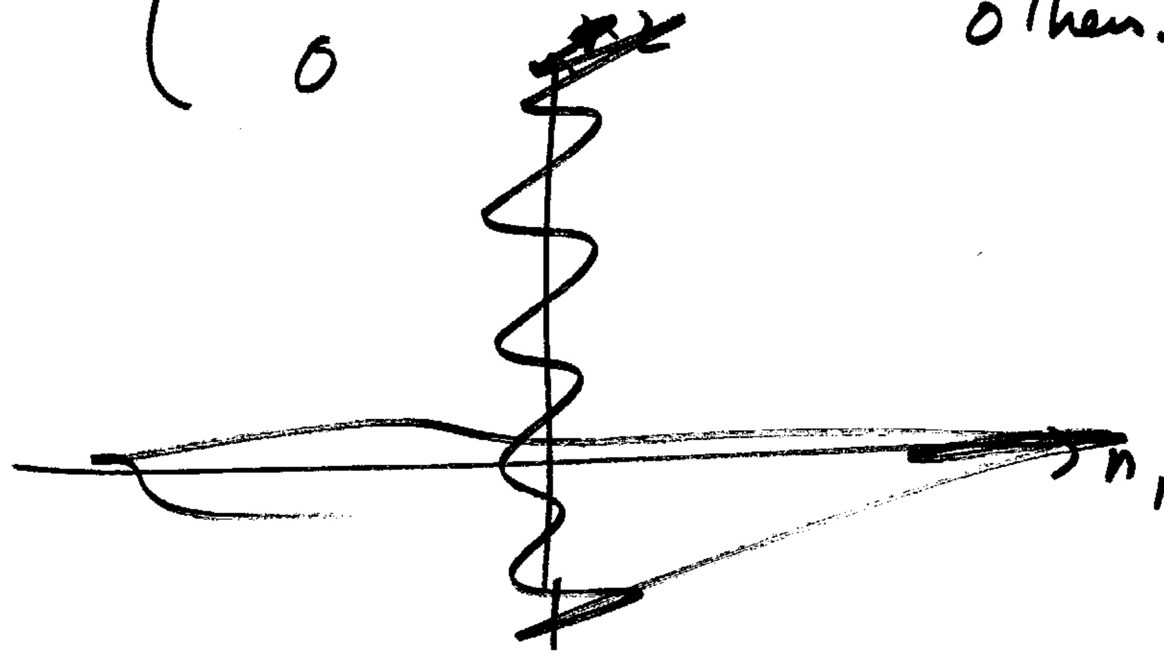
2D Window

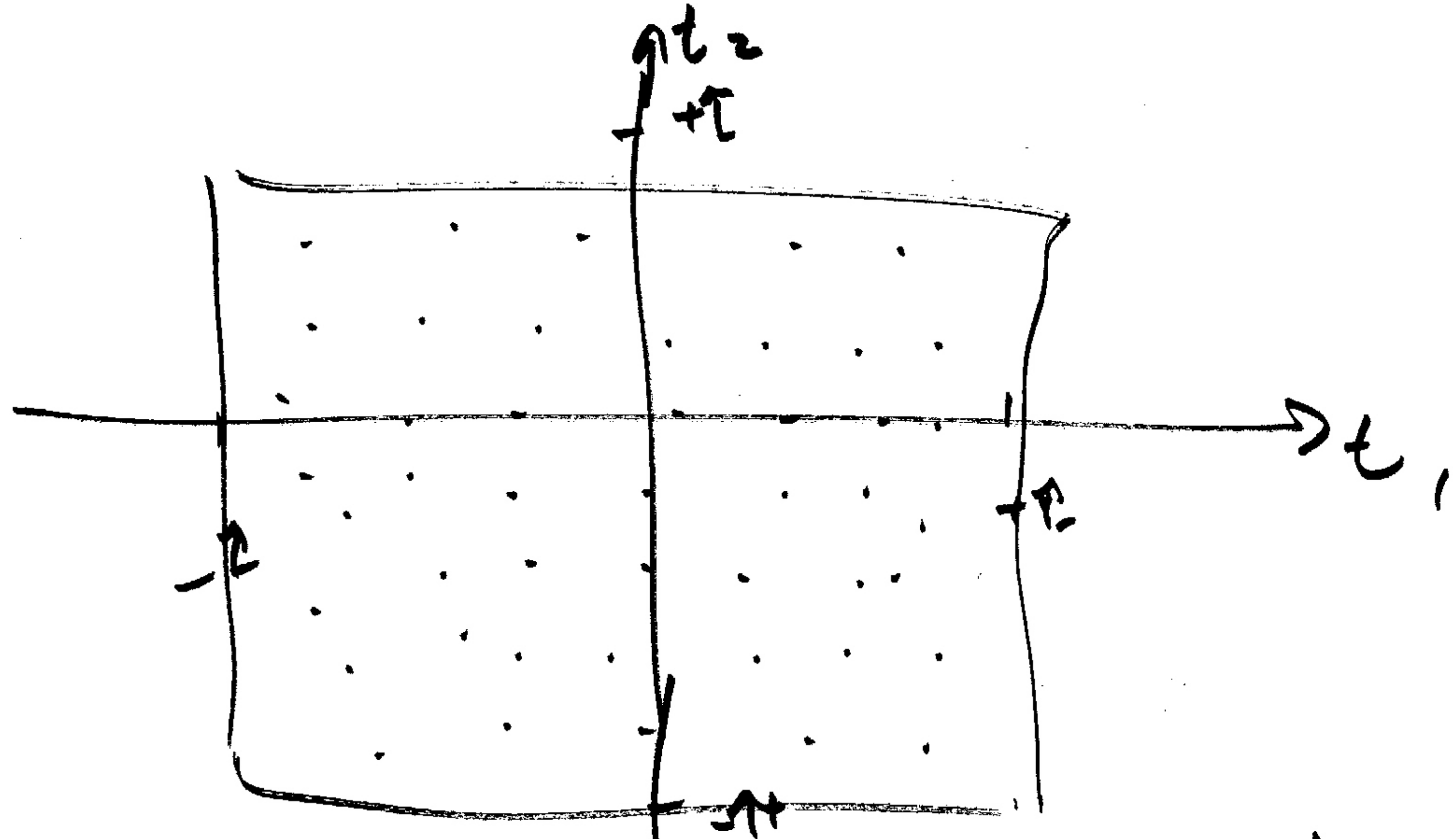
(1) Continuous Time 2D window.

$$W_a(t_1, t_2) = W_a(t_1)\, W_a(t_2)$$

(2) - sample This to make it discrete.

$$w(n_1, n_2) = \begin{cases} W_a(t_1, t_2)\Big|_{t_1 = n_1,\ t_2 = n_2} & (n_1, n_2) \in R_w \\ 0 & \text{otherwise} \end{cases}$$





$$W(n_1, n_2) = W_a(n_1)\, W_a(n_2)$$

sampled version of $w_a(t)$

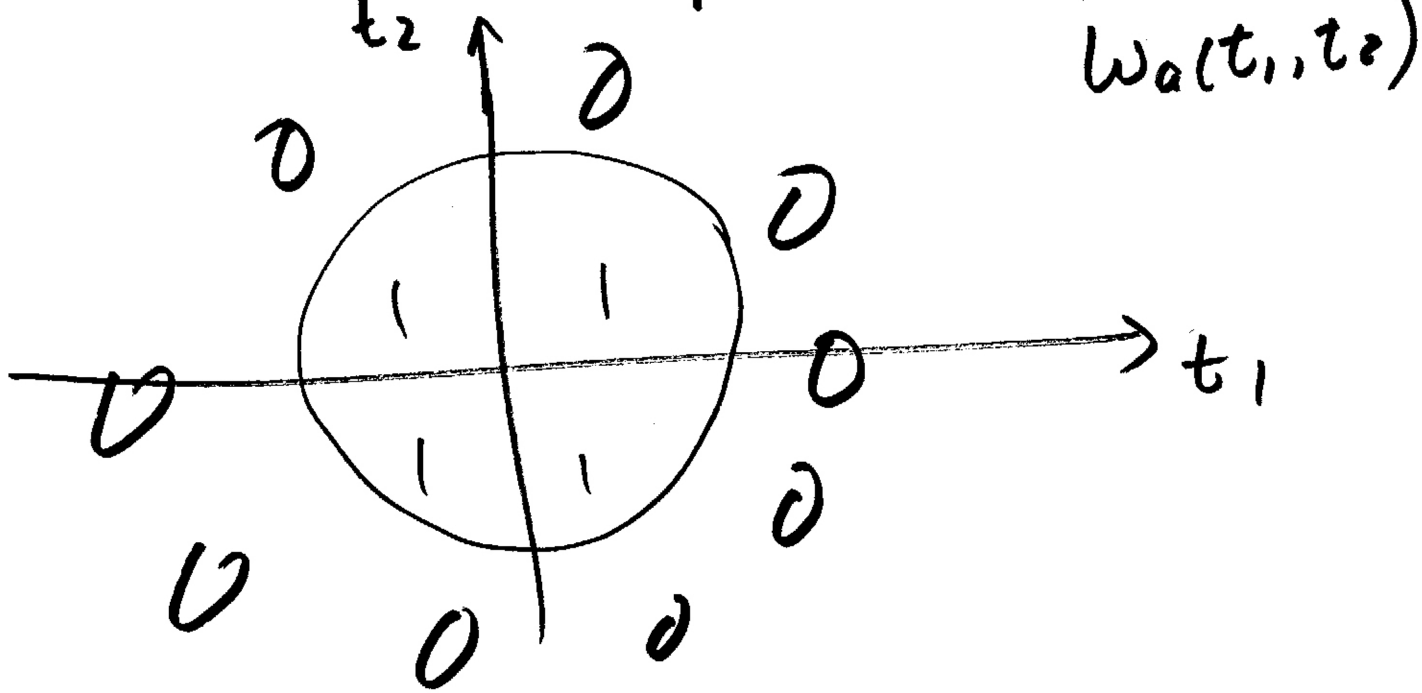
$$W(\omega_1, \omega_2) = W_a(\omega_1)\, W_b(\omega_2)$$

Another way 2D windows

Rotate $w_a(t)$ to get $w_a(t_1, t_2)$

Sample $w_a(t_1, t_2)$ to get $w(n_1, n_2)$

$$w_a(t_1, t_2) = w_a(t)\Big|_{t=\sqrt{t_1^2 + t_2^2}}$$



12

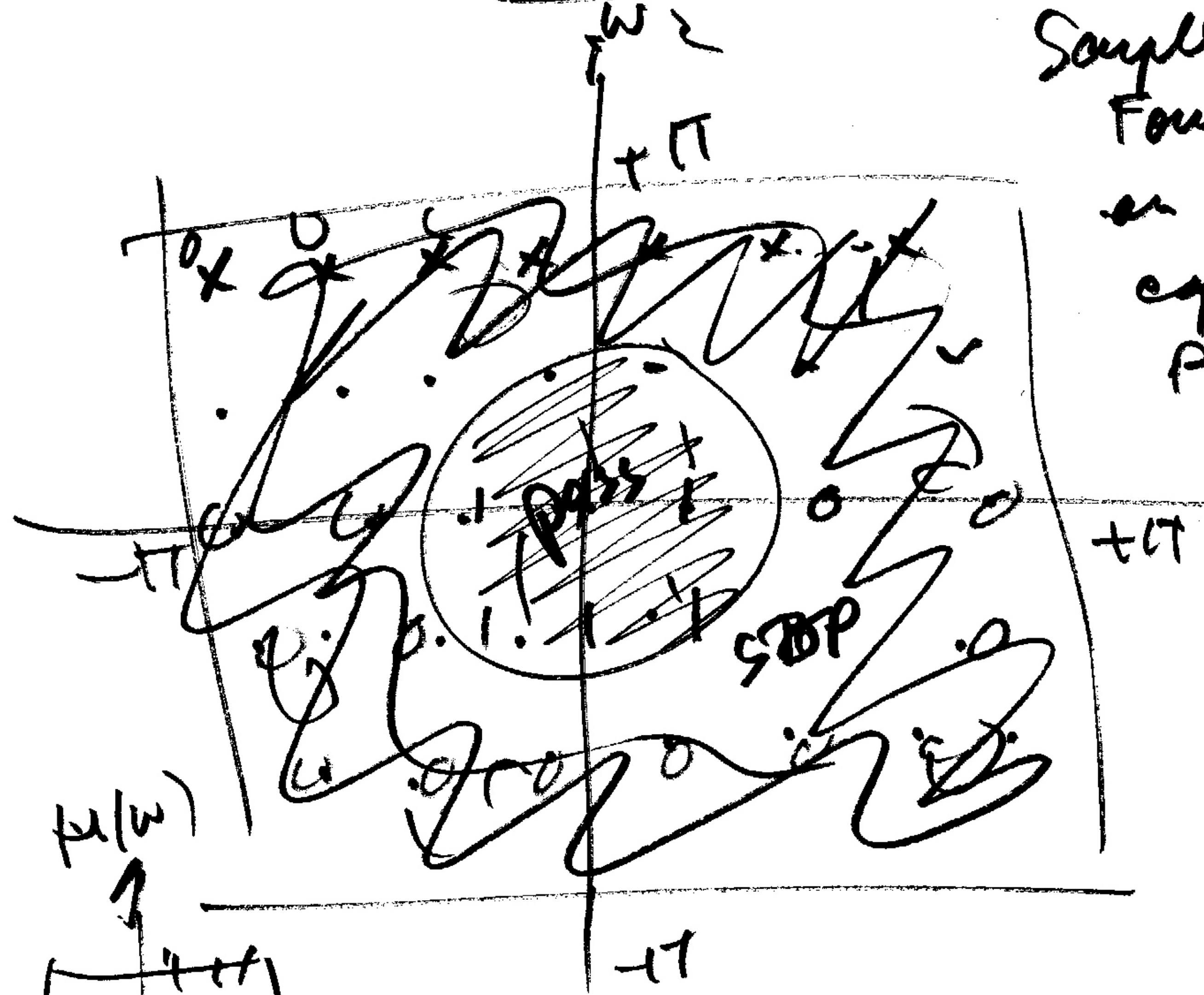
$$w(n_1, n_2) = \begin{cases} w_a(t_1, t_2)\Big|_{\substack{t_1 = n_1 \\ t_2 = n_2}} & (n_1, n_2) \in R_w \\ 0 & \text{otherwise} \end{cases}$$

circularly symmetric window

Show pictures from J. Lim

4.5 → 4.9

Freq. Sampling Technique



Sample 2D Fourier space on a $M \times M$ ~~grid~~ equally spaced points.

$\downarrow$

IDFT

$\downarrow$

$M \times M$ FIR filter

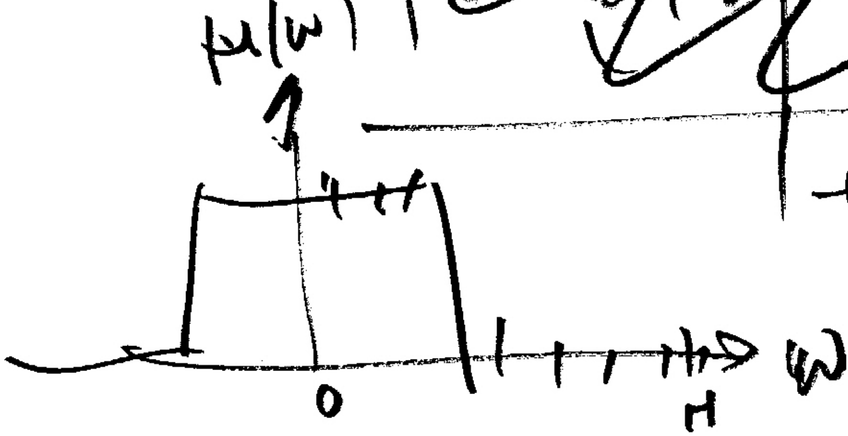


Fig. 4.10 → 4.11

2D Filter Design using Transformation

Idea: $H(\omega_1, \omega_2) = \left[H(\omega) \right]_{\omega = G(\omega_1, \omega_2)}$

$$H(\omega_1, \omega_2) = \left[H(\omega) \right]_{\cos\omega = T(\omega_1, \omega_2)}$$

Q: (1) How to design $T(\omega_1, \omega_2)$

(2) How to design $H(\omega)$ ← optimal.

How to make $H(\omega)$ in 1-D be zero phase?

If Filter is zero phase

$H(\omega) = |H(\omega)| e^{j\phi(\omega)}$ $\quad \phi(\omega) = 0 \quad \forall \omega$

$\Rightarrow H(\omega) = H^*(\omega) \Longrightarrow h(n) = h^*(-n)$

assume $h(n)$ has real coeff

$\Rightarrow h(n) = h(-n)$.

Zero phase in 1D $\Longrightarrow h(n) = h(-n)$

Assume $H(\omega)$ is zero phase

$h(n) = h(-n)$

$h(n)$ has $2N+1$ sample.

$$H(\omega) = \sum_{n=-N}^{+N} h(n) e^{-j\omega n}$$

zero phase

$$= h(0) + \sum_{n=1}^{N} 2h(n)\cos(\omega n)$$

$$H(\omega) = \sum_{n=0}^{N} a(n) \cos(\omega n)$$

$$H(\omega) = \sum_{n=0}^{N} b(n) (\cos\omega)^n$$

chebychev polynomial

$$\cos(2\omega) = \cos(\omega+\omega) = 2\cos^2\omega - 1$$

$$\cos(3\omega) = \cos(\omega + 2\omega) = 2\cos^3\omega - \cos\omega$$

$$- 2(1 - \cos^2\omega)\cos\omega$$

$$H(\omega_1,\omega_2) = \left[H(\omega) \right]_{\cos\omega = T(\omega_1,\omega_2)}$$

$$H(\omega_1,\omega_2) = \sum_{n=0}^{N} b(n) \left[T(\omega_1,\omega_2) \right]^n$$

Can show: if $T(\omega_1, \omega_2)$ is zero phase and $H(\omega)$ is zero-phase

Then $\longrightarrow$ $H(\omega_1, \omega_2)$ will be zero phase.

Suppose $T(\omega_1, \omega_2)$ is freq. response

$t(n_1, n_2) \longleftarrow (2M+1) \times (2M+1)$

$H(\omega) \longleftarrow 2N+1$ point.

$$T(\omega_1, \omega_2) = \sum_{n_1=-M}^{+M} \sum_{n_2=-M}^{+M} t(n_1, n_2)\, e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

$$H(\omega_1,\omega_2) = \sum_{n=0}^{N} b(n) \left(\sum_{n_1=-M}^{+M} \sum_{n_2=-M}^{+M} t(n_1,n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2} \right)^n$$

$$H(\omega_1,\omega_2) = \sum_{n_1=-NM}^{+NM} \sum_{n_2=-NM}^{+NM} h(n_1,n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

$$\to (2NM+1) \times (2NM+1)$$

$$\left.\begin{array}{l} M=1 \to t(n_1,n_2) : 3\times 3 \\ N=10 \to h(n) : 21 \text{ pt} \end{array}\right\} \to \begin{array}{c} h(n_1,n_2) \\ 21\times 21 \end{array}$$

Steps for 2D Filter Using Transformation

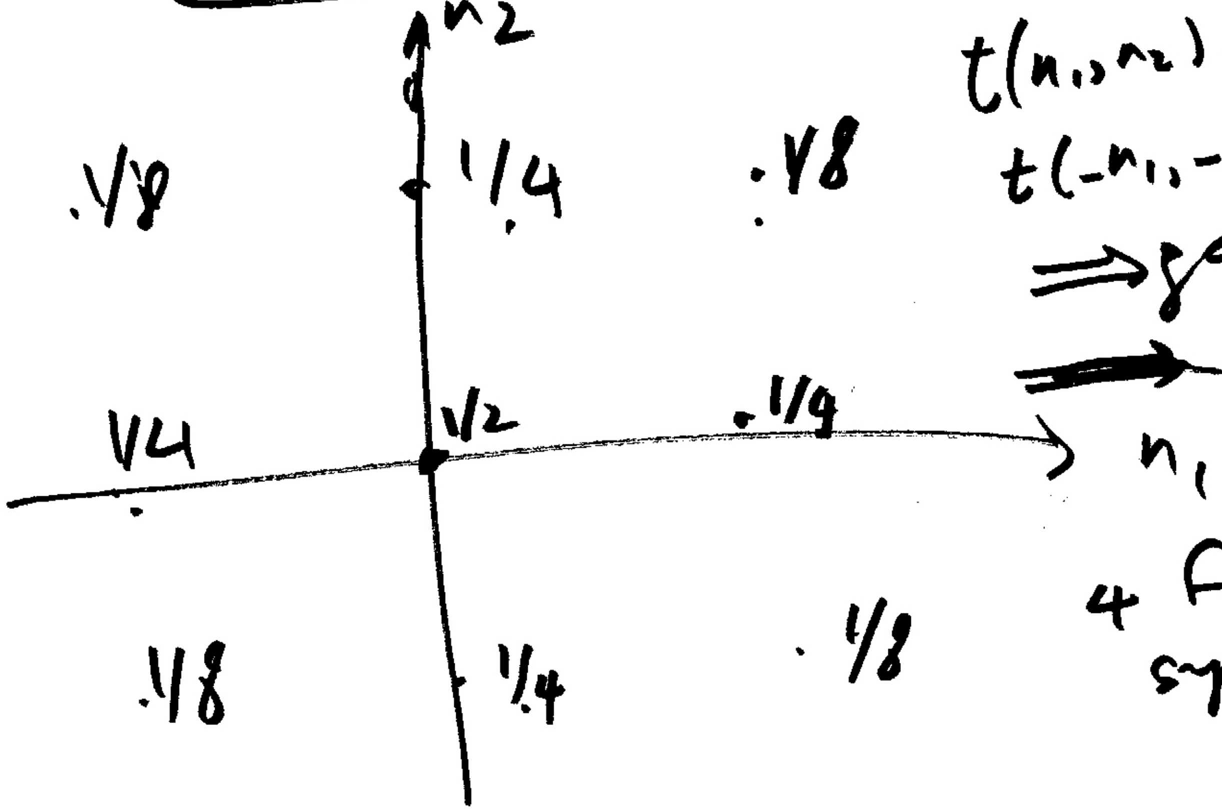
1. Start with specs in 2D
2. Either choose or design $t(n_1, n_2)$
3. Derive specs for $H(\omega)$ from specs in 2D for $H(\omega_1, \omega_2)$ and given $t(n_1, n_2)$

   $\omega_s, \omega_p, \delta_s, \delta_p$ for $H(\omega)$

4. Design $H(\omega)$, $h(n)$ (optimal filter design)

5. Combine $h(n)$ and $t(n_1, n_2)$ to get $h(n_1, n_2)$ $\quad H(\omega_1, \omega_2) = \left[H(\omega)\right]_{\cos\omega = T(\omega_1, \omega_2)}$

McMullen Transformation



$t(n_1, n_2) = t(-n_1, -n_2)$

$\Longrightarrow$ geo phase

4 fold symmetry