Feb. 22, 2006

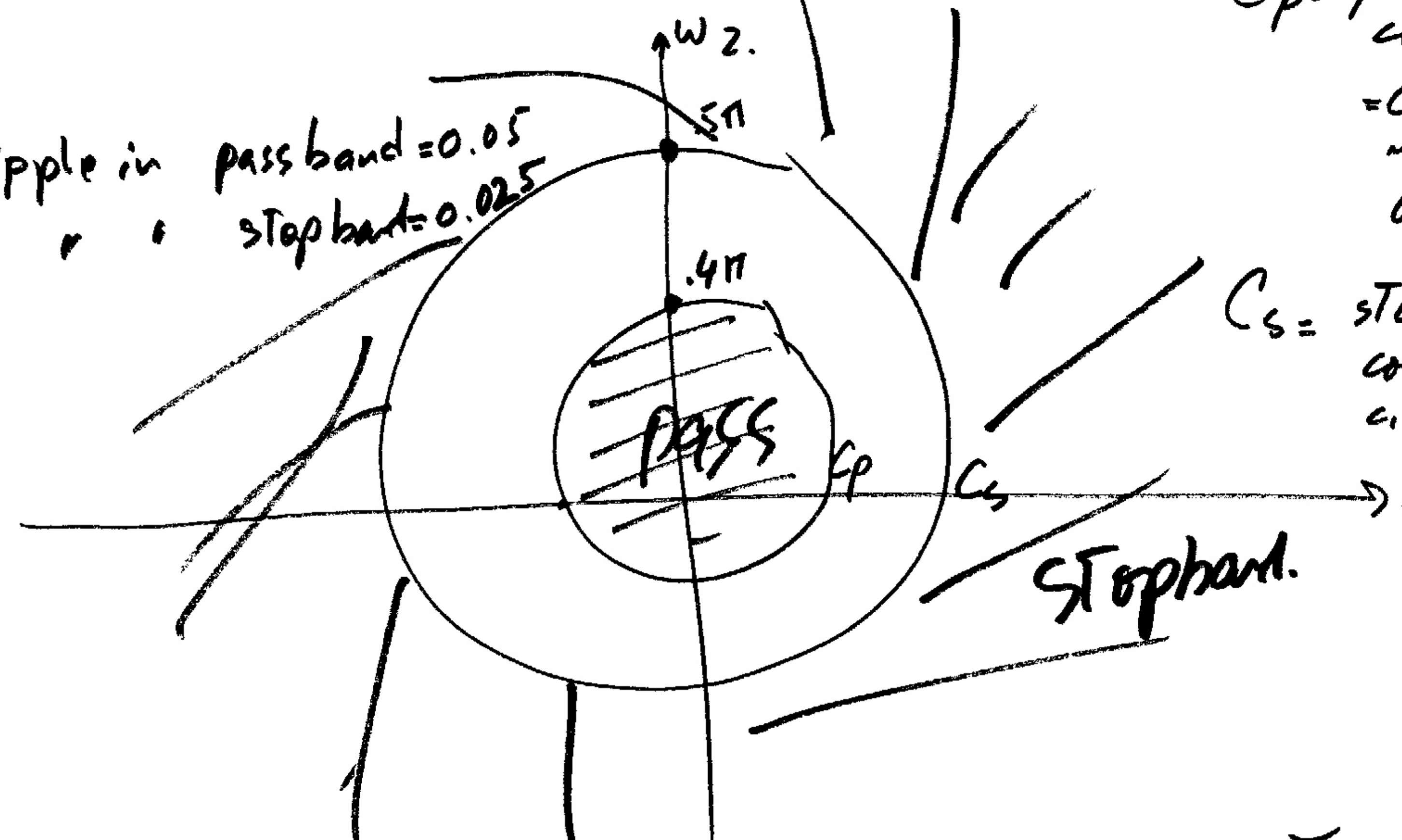
Example of 2D FIR Filter Design Using Transformation

- Design a LPF, circularly symmetric.

Specs:

δ_p = ripple in passband = 0.05

δ_s = " " stopband = 0.025



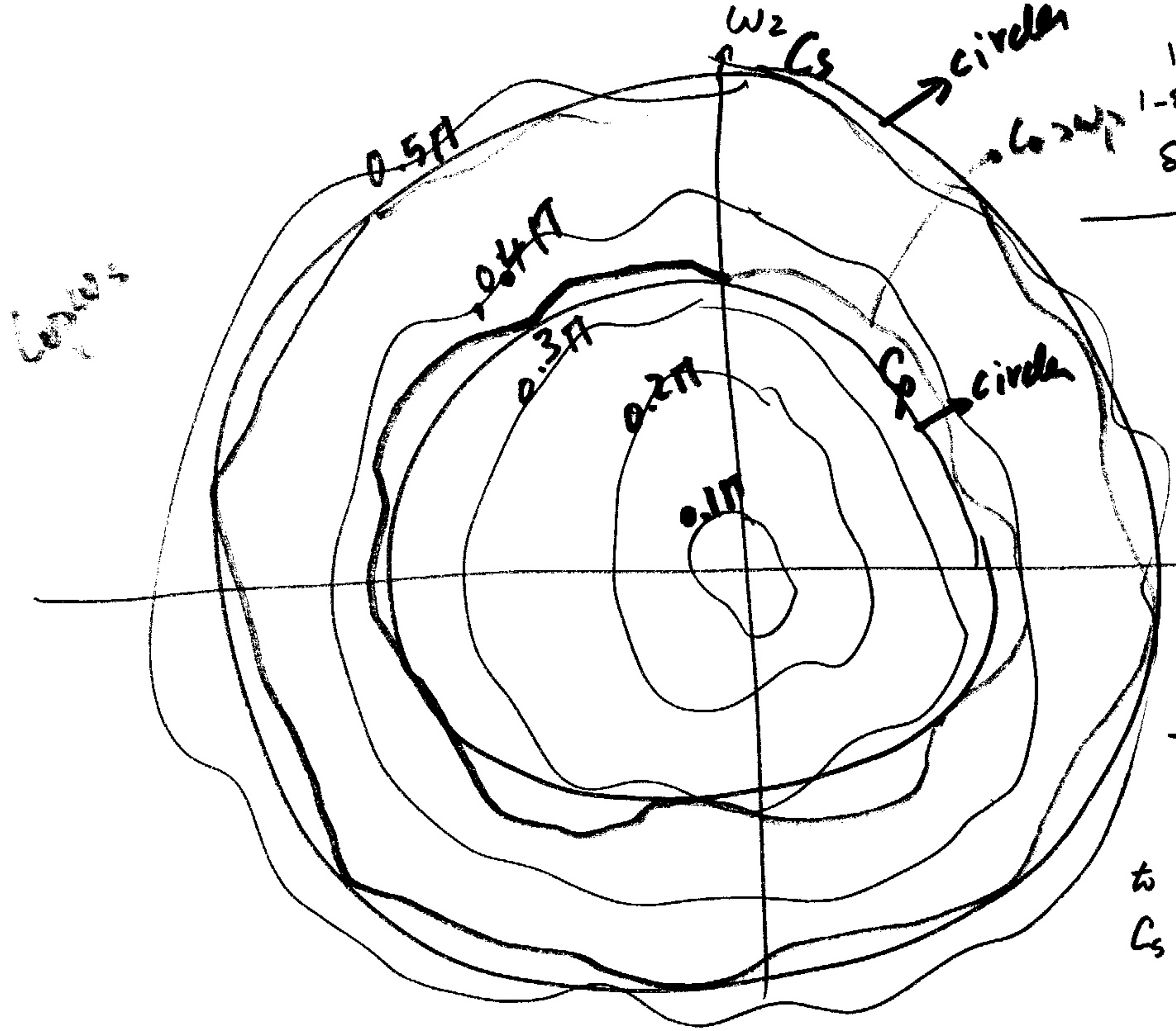
C_p = passband contour = circle of radius 0.4π

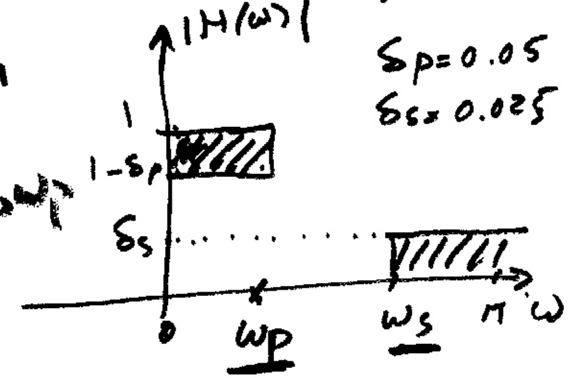
C_s = stopband contour: circle of radius 0.5π

Assume, i.e., $t(n_1, n_2)$ to be McClellan Transformation.

Transform specs from 2D to 1-D:

Recall: $H(\omega_1, \omega_2) = [H(\omega)]_{\cos\omega = T(\omega_1, \omega_2)}$





$\delta_p = 0.05$

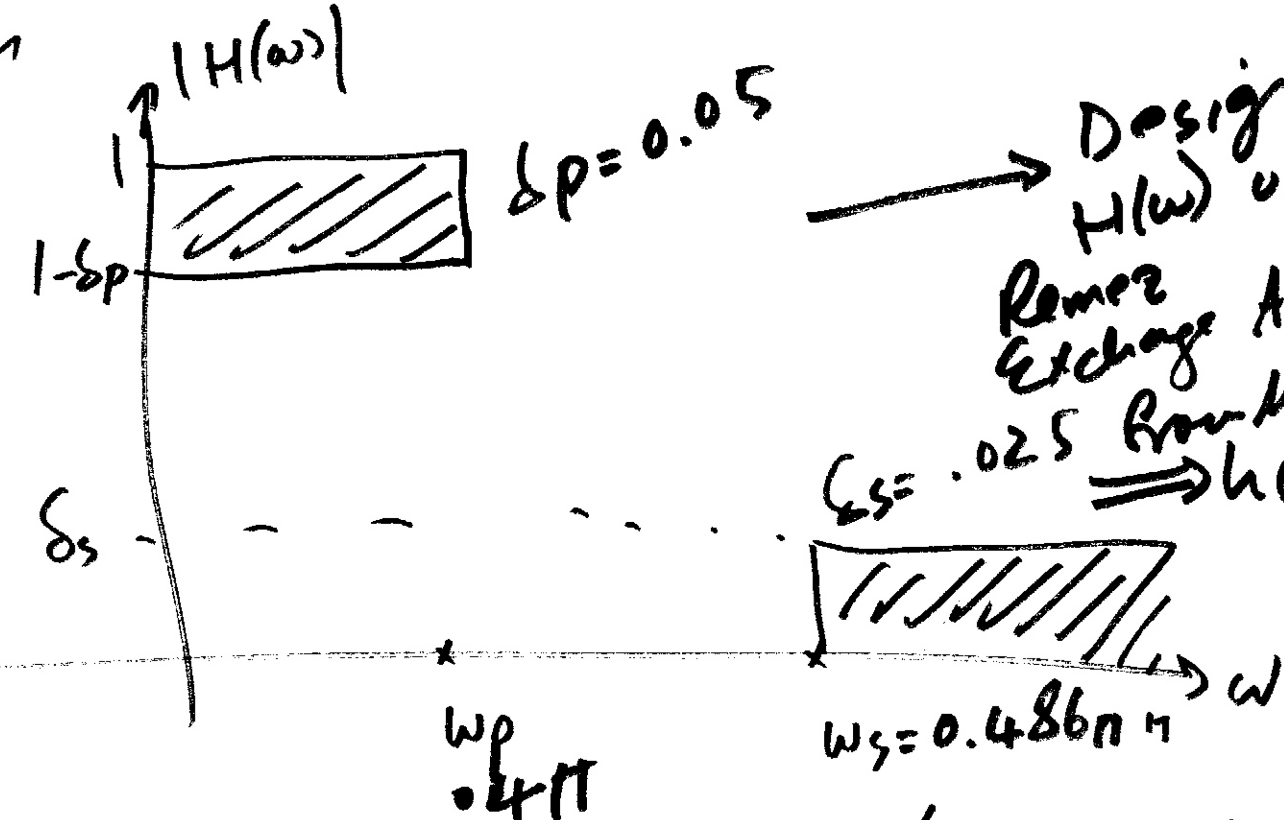
$\delta_s = 0.025$

- choose ω_p in such a way that iso.contour corresponding to ω_p is entirely outside C_p
- Choose largest ω_s so that its iso.contour corresponding to ω_s is barely inside C_s

1-D spec

$\delta_p = .05$ ← from 2D spec

$\delta_s = 0.025$ ← from 2D spec



→ Design H(ω) using Remez Exchange Alg. from Matlab ⟹ h(n).

- find numerically iso-contour corresponding to $\omega = 0.4\pi$ i.e. $\cos(0.4\pi)$ to completely encircle C_p = i.e. circle of radius 0.4π

- " " " " $\omega = 0.486\pi$ ie $\cos(0.486\pi)$ is entirely inside C_s = ie a circle of radius 0.5π

ω_p, ω_s → for 1D filter

3

$\Rightarrow$ One design $h(n)$.
know $t(n_1, n_2)$ $\Bigg\}$ $\Longrightarrow$ $h(n_1, n_2)$.

$$H(\omega_1, \omega_2) = \left[H(\omega) \right]_{\cos\omega = T(\omega_1, \omega_2)}$$

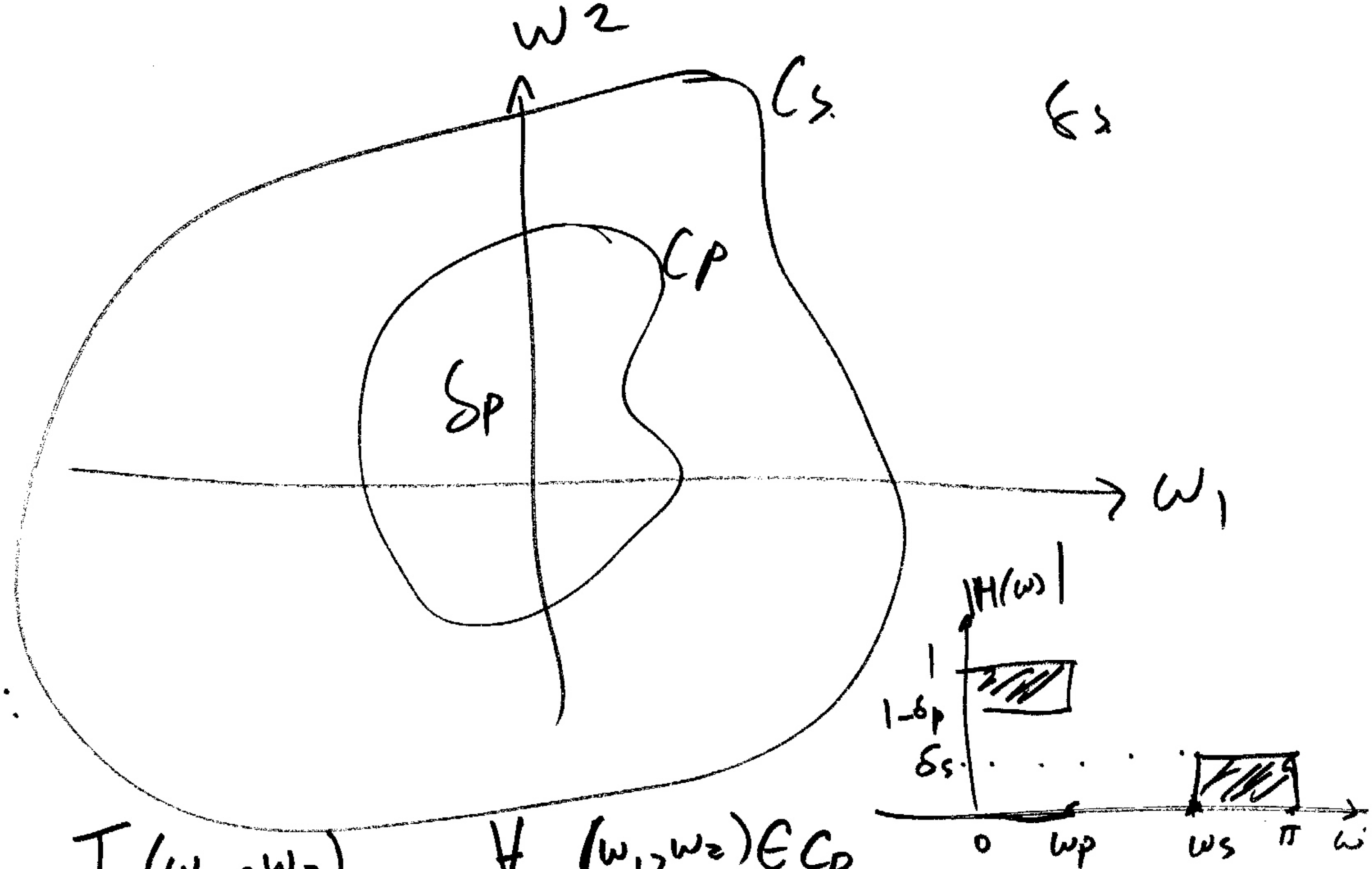
Example 2 :

- Observation : shape of iso-contours for $t(n_1, n_2)$ i.e. $T(\omega_1, \omega_2)$ are similar to shape of iso-contours of final $H(\omega_1, \omega_2)$.

- How to from scratch design $t(n_1, n_2)$?

Soln:- Use shape of C_p, C_s (ie. spec. contour) To design $t(n_1, n_2)$.

Ideally.



Ideally:

- $\cos\omega_p = T(\omega_1, \omega_2) \quad \forall (\omega_1, \omega_2) \in C_p$
- $\cos\omega_s = T(\omega_1, \omega_2) \quad \forall (\omega_1, \omega_2) \in C_s$

Would be nice if.

(*) $$\left[\cos\omega\right]_{0 \le \omega \le \omega_p} = \left[T(\omega_1, \omega_2)\right]_{(\omega_1, \omega_2) \text{ inside } C_p.}$$

(*) $$\left[\cos\omega\right]_{\omega_s \le \omega \le \pi} = \left[T(\omega_1, \omega_2)\right]_{(\omega_1, \omega_2) \text{ outside } C_s.}$$

How design $t(n_1, n_2)$?

1. Steps for Designing $t(n_1, n_2)$

1.1 Assume shape, size, region of support for $t(n_1, n_2)$

1.2. Impose Constraints.

1.3. Choose ω_p, ω_s Tentatively in 1-D filter.

Design $t(n_1, n_2)$ such that

$$\cos\omega_p \lesssim T(\omega_1, \omega_2) \quad \forall (\omega_1, \omega_2) \in C_p$$

$$\cos\omega_s \gtrsim T(\omega_1, \omega_2) \quad \forall (\omega_1, \omega_2) \in C_s$$

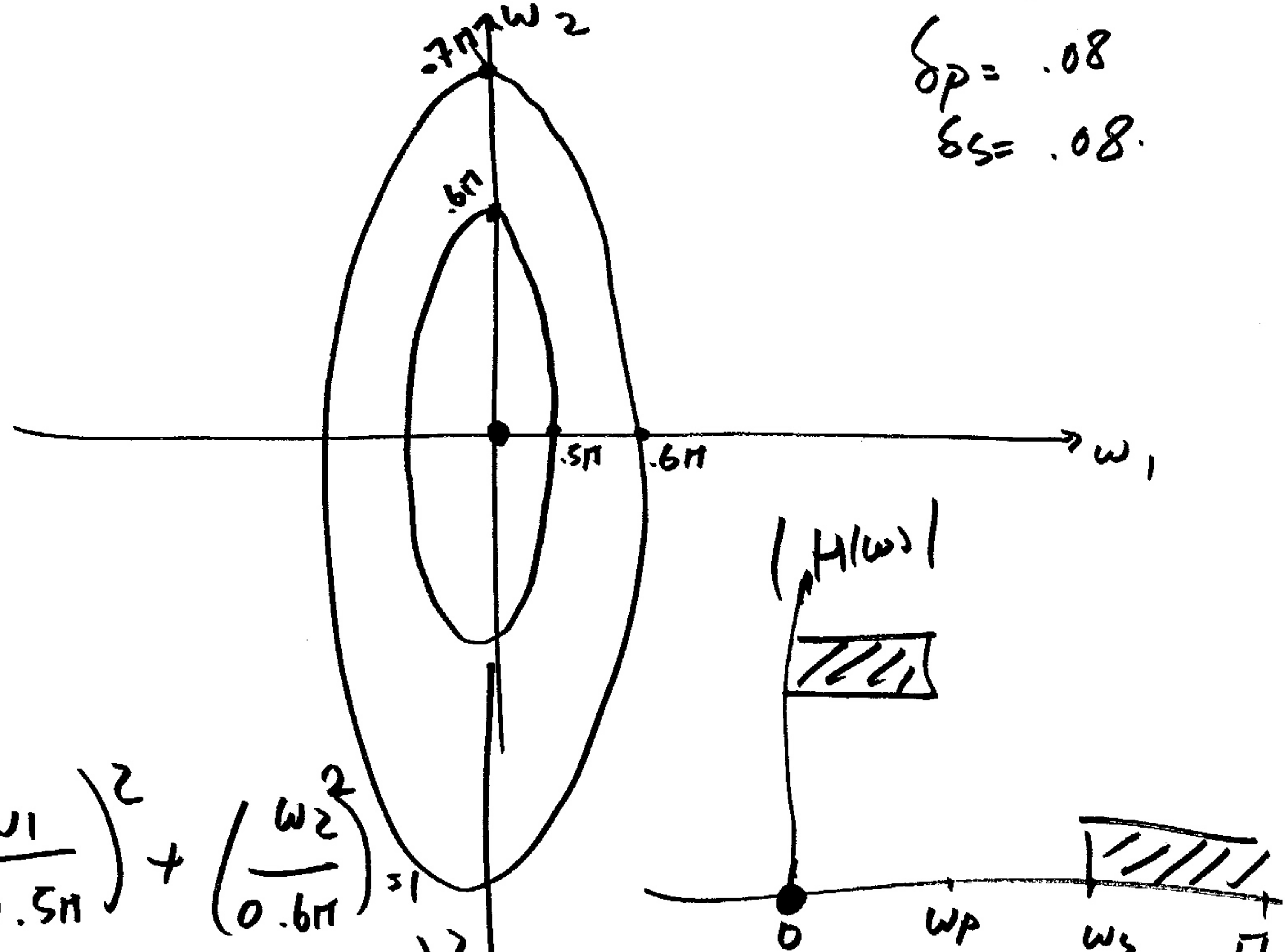
1.4. Start $t(n_1, n_2)$ in step 1.3.

design $t'(n_1, n_2)$ to ensure

(*) are satisfied.

<u>Ex</u> Design a 2D filter with eliptic passband and stop band.

$\delta_p = .08$
$\delta_s = .08$

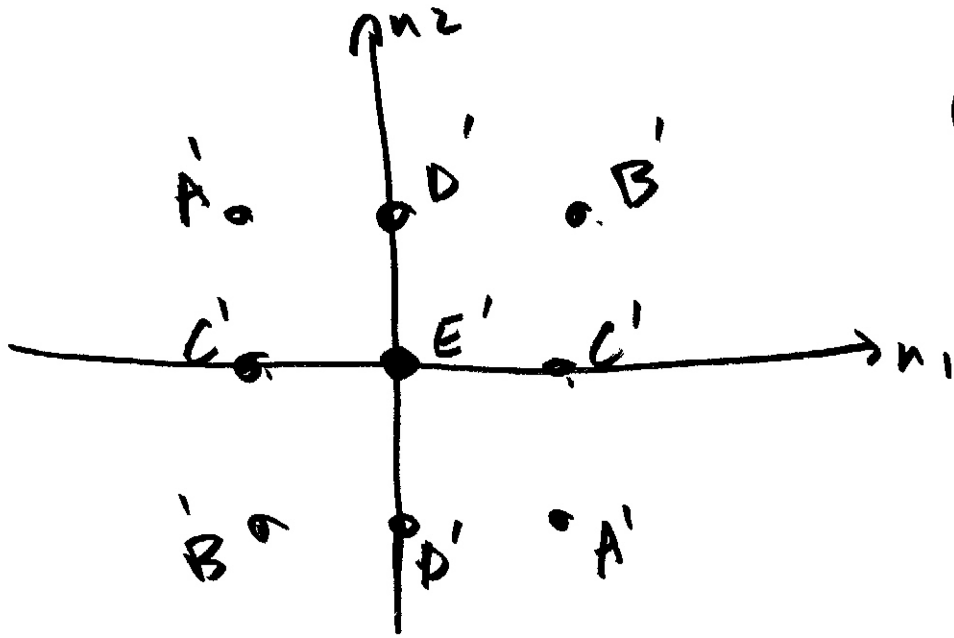


$$C_p: \left(\frac{\omega_1}{0.5\pi}\right)^2 + \left(\frac{\omega_2}{0.6\pi}\right)^2 = 1$$

$$C_s: \left(\frac{\omega_1}{0.6\pi}\right)^2 + \left(\frac{\omega_2}{0.7\pi}\right)^2 = 1$$

(1.1) Assume shape size, Region of support $t(n_1, n_2)$

ROS: $3 \times 3 \Longrightarrow 9$ unknowns $A, B, C, D, E, F, G, H, I$



(1.2) Impose Constraint:

(a) zero phase, real impulse response.

$t(n_1, n_2) = t(-n_1, -n_2)$

$\Longrightarrow$ 5 unknowns.

A', B', C', D', E

(b) 4 fold symmetry:-

$A' = B'$

$\Longrightarrow$ 4 unknowns: A', C', D', E'

$$T(\omega_1, \omega_2) = E' + 2C' \cos\omega_1 + 2D' \cos\omega_2 + 2A' \cos\omega_1 \cos\omega_2$$

(c) map $\omega = 0$ to $(\omega_1, \omega_2) = (0, 0)$ in 2D space.

$$\left[\cos\omega\right]_{\omega=0} = 1 = \left[T(\omega_1, \omega_2)\right]_{(\omega_1,\omega_2)=(0,0)}$$

$$= E' + 2C' + 2D' + 2A' = 1$$

$\Longrightarrow$ one less unknown.

$\Longrightarrow$ 3 unknowns

$$E' = 1 - 2C' - 2D' - 2A'$$

$$T(\omega_1, \omega_2) = (1 - 2C' - 2D' - 2A') + 2C' \cos\omega_1 + 2D' \cos\omega_2 + 2A' \cos\omega_1 \cos\omega_2$$

$\Longrightarrow$ 3 unknowns: A', C', D'

1.3 choose ω_p, ω_s in LD arbitrarily.

say, $\omega_p = 0.5\pi$ $\quad \omega_s = 0.6\pi$

Design $t(n_1, n_2)$ such that.

$\cos\omega_p \lesssim T(\omega_1, \omega_2) \quad (\omega_1, \omega_2) \in C_p$

$\cos\omega_s \gtrsim T(\omega_1, \omega_2) \quad (\omega_1, \omega_2) \in C_s$

$$Error_p = E_p = \iint_{(\omega_1,\omega_2)\in C_p} \left(T(\omega_1,\omega_2) - \cos\omega_p\right)^2 d\omega_1 d\omega_2$$

$$Error_s = E_s = \iint_{(\omega_1,\omega_2)\in C_s} \left(T(\omega_1,\omega_2) - \cos\omega_s\right)^2 d\omega_1 d\omega_2$$

Minimize E_p and E_S with respect to A', C', D'.

$$\Rightarrow \begin{cases} \dfrac{\partial E_p}{\partial A'} = 0 , & \dfrac{\partial E_p}{\partial C'} = 0 & \dfrac{\partial E_p}{\partial D'} = 0 \\ \dfrac{\partial E_s}{\partial A'} = 0 , & \dfrac{\partial E_s}{\partial C'} = 0 & \dfrac{\partial E_p}{\partial D'} = 0 \end{cases}$$

$T(\omega_1, \omega_2)$ is linear in A', C', D'

$(T(\omega_1, \omega_2) - \cos\omega_p)^2$ is quadratic in A', C', D'

$\Rightarrow$

$$\frac{\partial E_p}{\partial A'} = \iint_{(\omega_1,\omega_2)\in C_p} \frac{\partial}{\partial A'}\left(T(\omega_1,\omega_2) - \cos\omega_p\right)^2 d\omega_1 d\omega_2$$

$$\frac{\partial E_p}{\partial A'} = \iint_{(\omega_1,\omega_2)\in C_p} 2\left(T(\omega_1,\omega_2) - \cos\omega_p\right)\frac{\partial}{\partial A'}\left(T(\omega_1,\omega_2) - \cos\omega_p\right) d\omega_1 d\omega_2$$

T is linear w.r.t. A', $\frac{\partial}{\partial A'}(\;)$ is constant $= -2 + 2\cos\omega_1\cos\omega_2$

$$\frac{\partial E_p}{\partial A'} = \iint_{(\omega_1,\omega_2)\in C_p} \left(2\cos\omega_1\cos\omega_2 - 2\right) 2\left(T(\omega_1,\omega_2) - \cos\omega_p\right) d\omega_1 d\omega_2$$

T is linear in A', C', D'

$= 0$

$\Rightarrow$ 6 linear system of Eqns

3 unknowns

$\Rightarrow$ Apply least square

$\downarrow$

A', C', D'

$\downarrow$

E'

$\downarrow$

B'

$\Downarrow$

$t(n_1, n_2)$