Feb 25, 06

2D. Filter Design using Transformation

If $|T(\omega_1, \omega_2)| > 1$ for some $(\omega_1, \omega_2) \in [-\pi, +\pi] \times [-\pi, +\pi]$

Then resulting $H(\omega_1, \omega_2)$ is "ill behaved" for those (ω_1, ω_2).

$\Rightarrow$ Problematic.

(1.4) Make sure regions inside C_p (passband) in 2-D filter correspond to ω: $0 \longrightarrow \omega_p$. in 1-D filter

Ditto for C_s.

$$H(\omega_1, \omega_2) = \left[H(\omega) \right]_{\cos\omega = T(\omega_1, \omega_2)}$$

$|\cos\omega| < 1$ make sure $|T(\omega_1, \omega_2)| < 1$

<u>Proposal</u>: Renormalize $T(\omega_1, \omega_2)$ such that it is in the range

$$|T| < 1$$

Minimally perturb T to get T'

$$T'(\omega_1, \omega_2) = K_1 T(\omega_1, \omega_2) + K_2$$

find K_1, K_2 s.t. $|T'(\omega_1, \omega_2)| < 1$

$\forall (\omega_1, \omega_2)$ in $[-\pi, +\pi] \times [-\pi, +\pi]$.

$$\max_{(\omega_1, \omega_2)} T(\omega_1, \omega_2) = T_{max}.$$

$$\min_{(\omega_1, \omega_2)} T(\omega_1, \omega_2) = T_{min}$$

Find K_1 and K_2 s.th. $\begin{cases} \max T' = 1 \\ \min T' = -1 \end{cases}$

$$T'(\omega_1, \omega_2) = K_1 T(\omega_1, \omega_2) + K_2.$$

$$\left\{ \begin{array}{l} +1 = K_1 T_{max} + K_2 \\ -1 = K_1 T_{min} + K_2 \end{array} \right\} \longrightarrow$$

$\Rightarrow$ find K_1 & K_2

$\Rightarrow$ new Transformation T'

$$K_1 = \frac{2}{T_{max} - T_{min}} \qquad K_2 = \frac{T_{max} + T_{min}}{T_{max} - T_{min}}$$

Need to modify ω_p, ω_s (for T)

$\omega_p = 0.5\pi$ $\quad \omega_s = 0.6\pi$

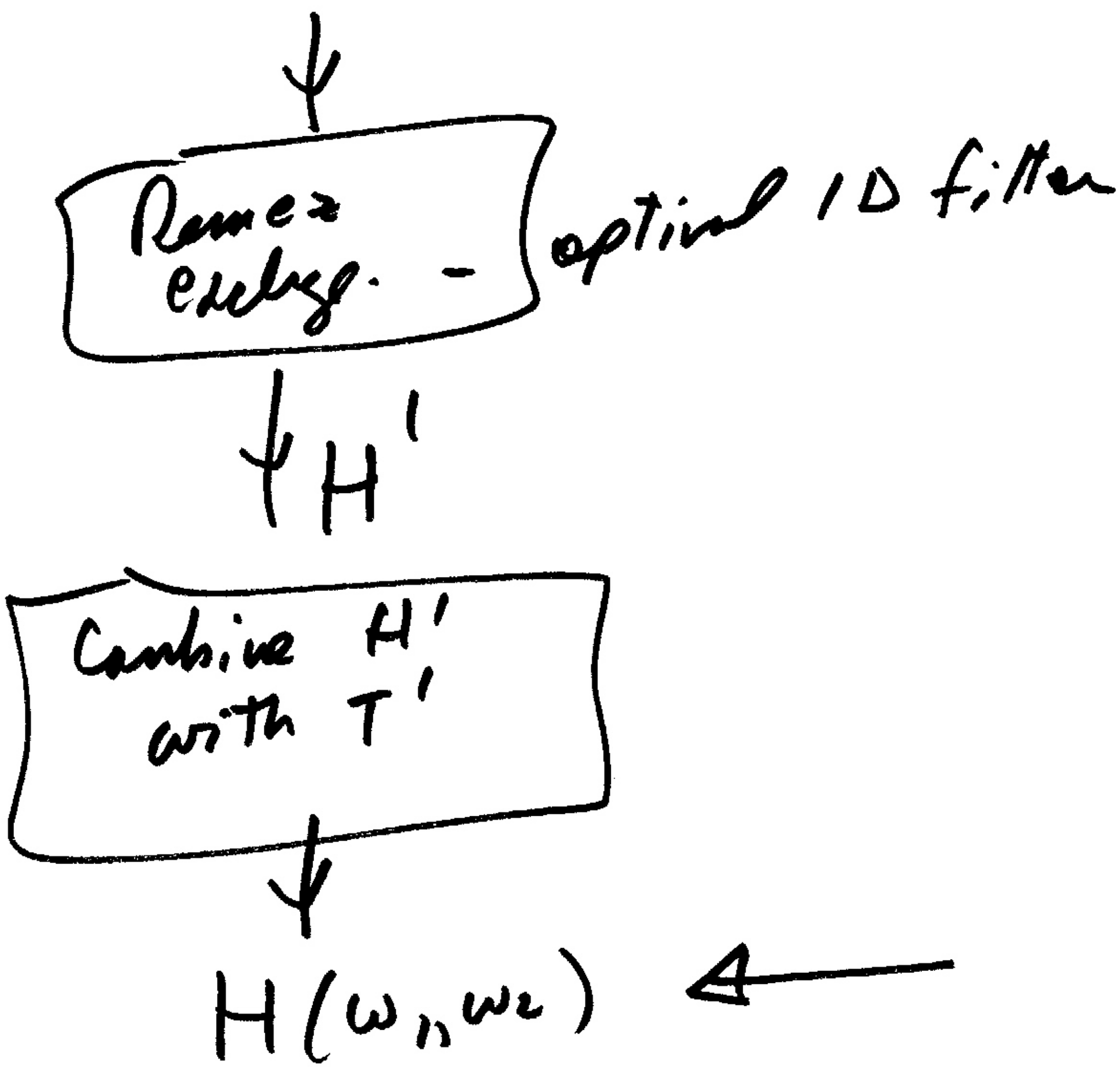
$$T \longrightarrow T' \quad \begin{cases} T(\omega_1, \omega_2) \approx \cos \omega_p. & (\omega_1, \omega_2) \in C_p \\ T(\omega_1, \omega_2) \approx \cos \omega_s & (\omega_1, \omega_2) \in C_s \end{cases}$$

$$T'(\omega_1, \omega_2) = K_1 T(\omega_1, \omega_2) + K_2$$

$$\cos \omega_p' \approx T'(\omega_1, \omega_2) \qquad (\omega_1, \omega_2) \in C_p.$$

$$\begin{cases} \cos \omega_p' = K_1 \cos \omega_p + K_2 \\ \cos \omega_s' = K_1 \cos \omega_s + K_2 \end{cases}$$

Plug in ω_p', ω_s', δ_p, δ_s



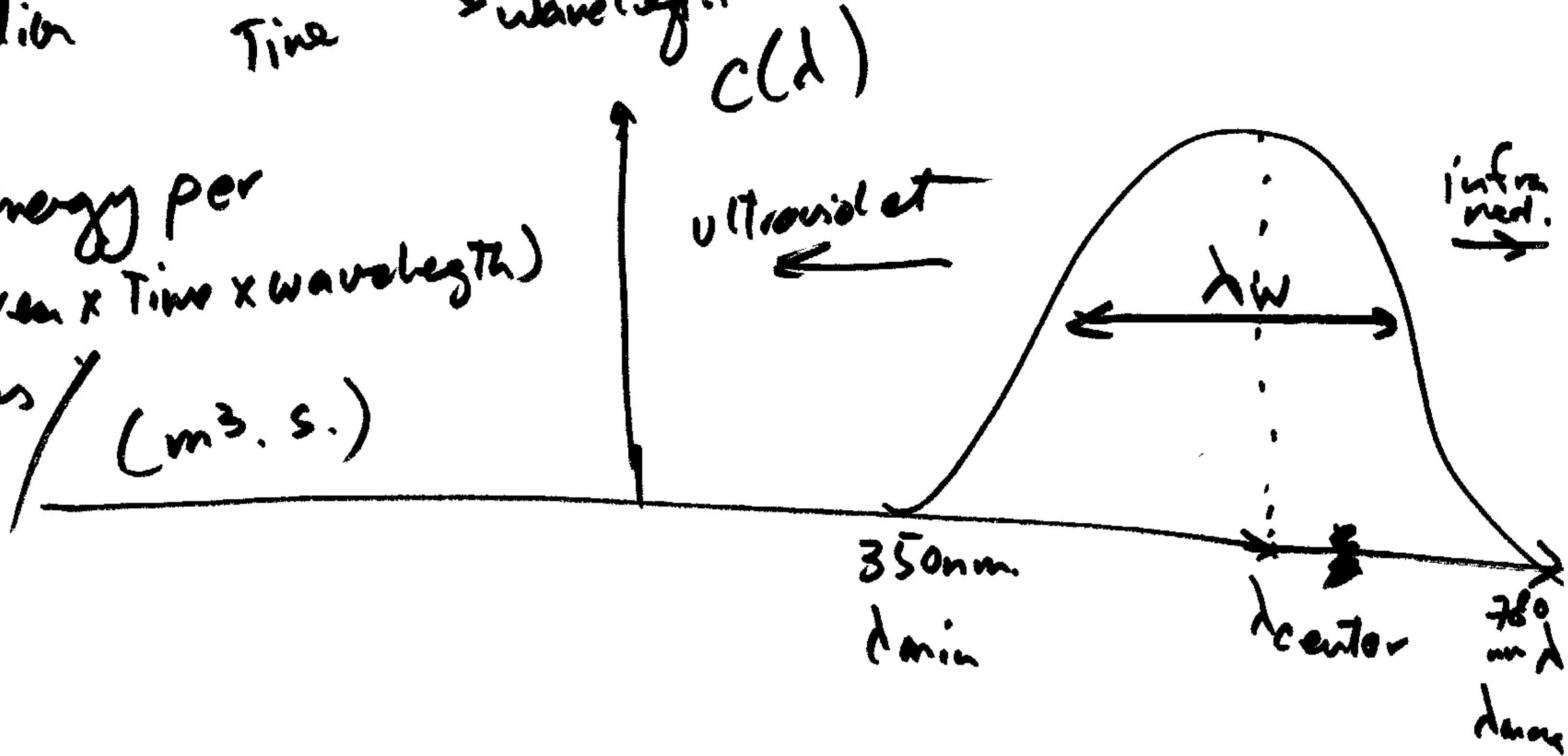
Light as Electromagnetic Wave.

$C(x, y, t, \lambda)$ = energy density of electromagnetic wave.

- x, y → location
- t → Time
- λ → wavelength.

→ energy per (area × Time × wavelength)

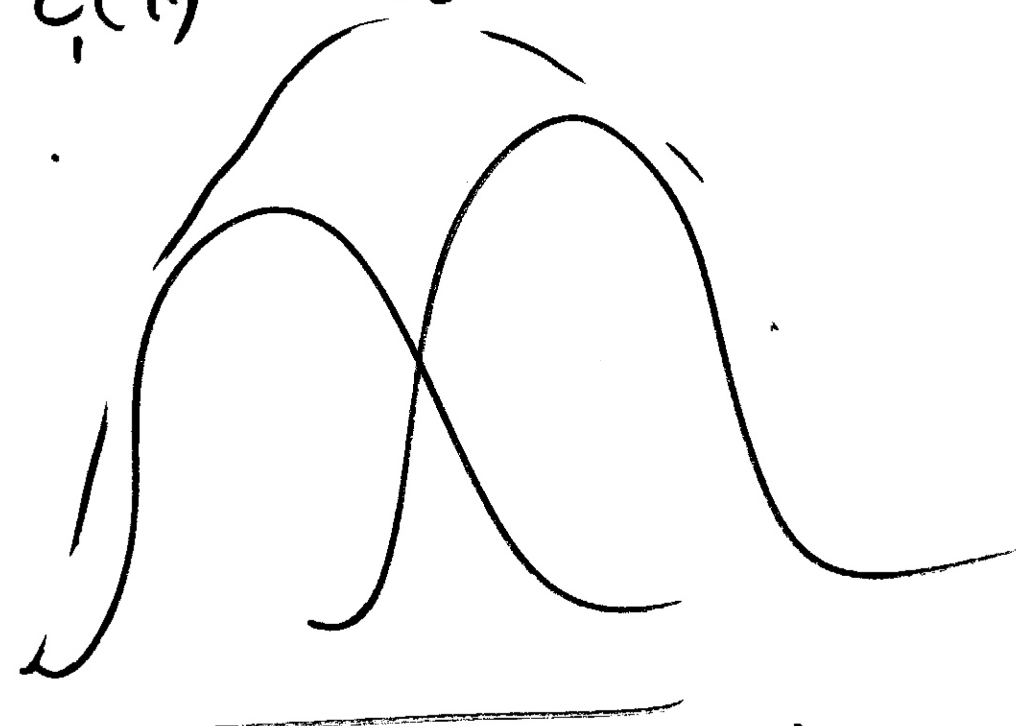
joules / $(m^3 . s.)$



Physical Domain.	Very approximate	Perceptual Domain
$\int_{\lambda_{min}}^{\lambda_{max}} C(\lambda)\, d\lambda$	how bright?	Brightness.
λ_{center}:	color.	Hue
$\simeq 700nm$		Red.
$\simeq 550nm$		green.
$\simeq 450nm$		Blue.
λ_w	vivid or dull.	Saturation

Additive / subtractive color system

Additive: Two $c_1(\lambda)$ $c_2(\lambda)$ combine.



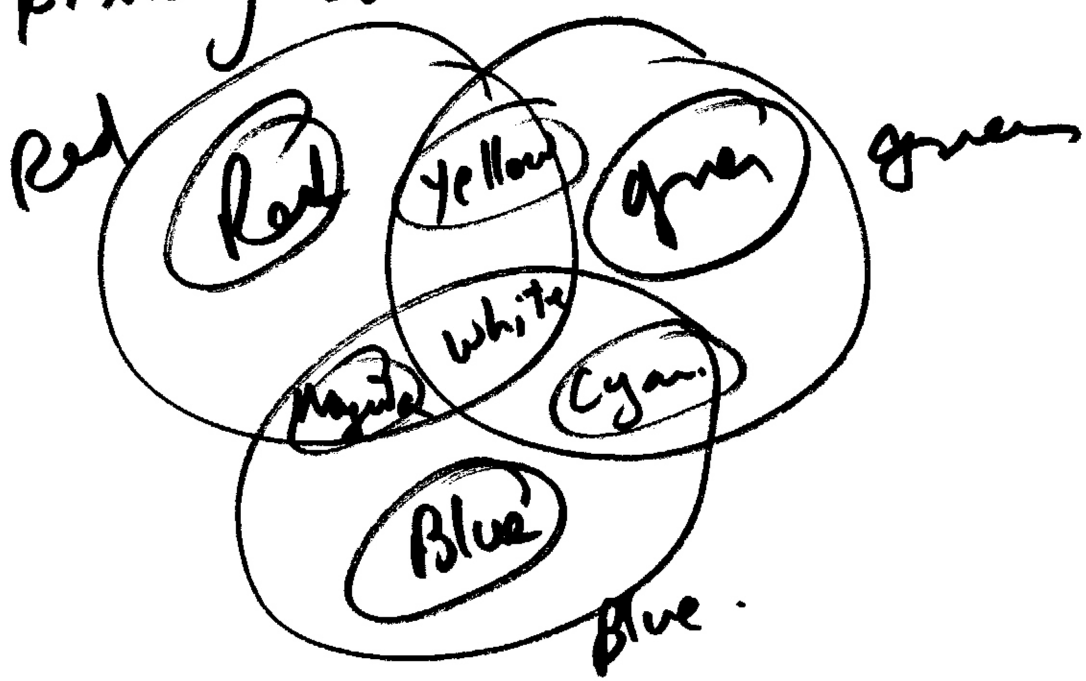
$C(\lambda) = c_1(\lambda) + c_2(\lambda)$.

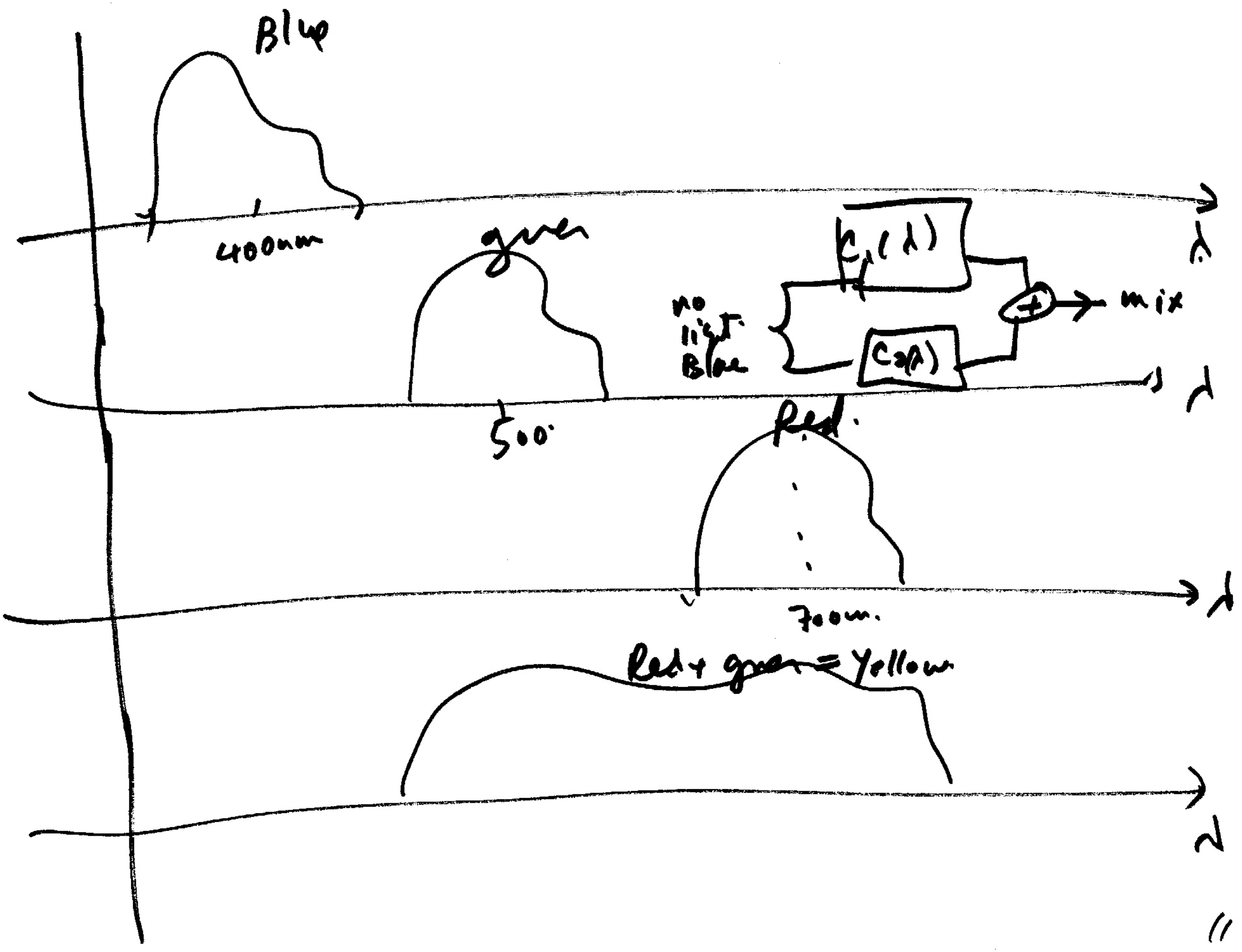
additive system: → Add light at diff. wavelength a new color

Ex additive syst → TV.

Red Gun
Blue "
Green gun

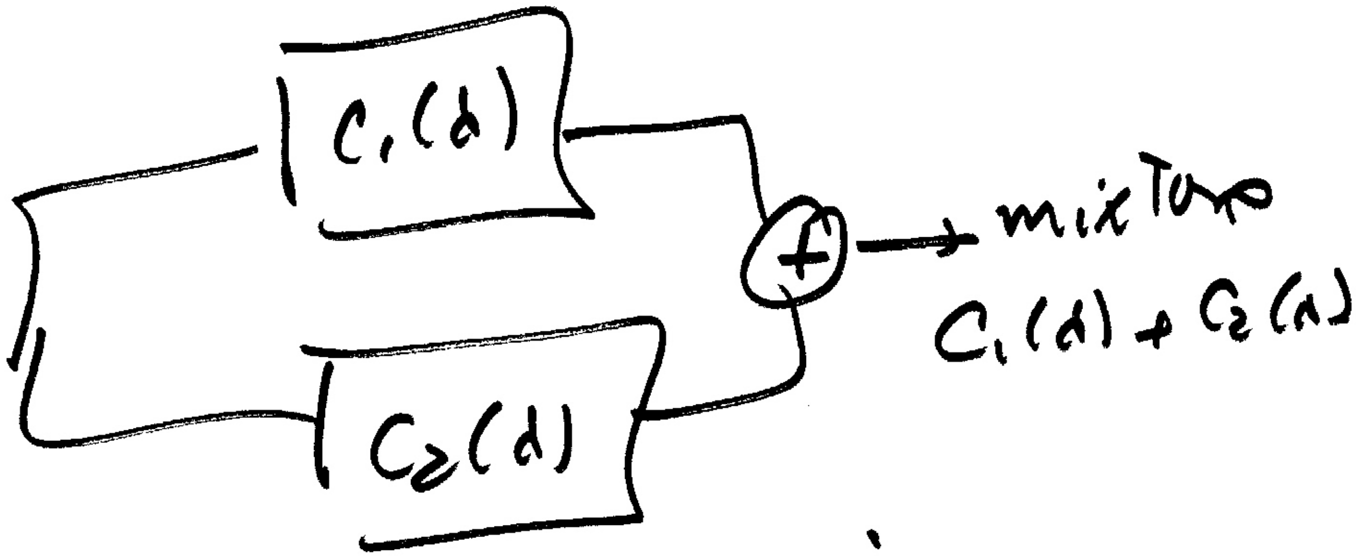
3 primary colors → Red, green, Blue





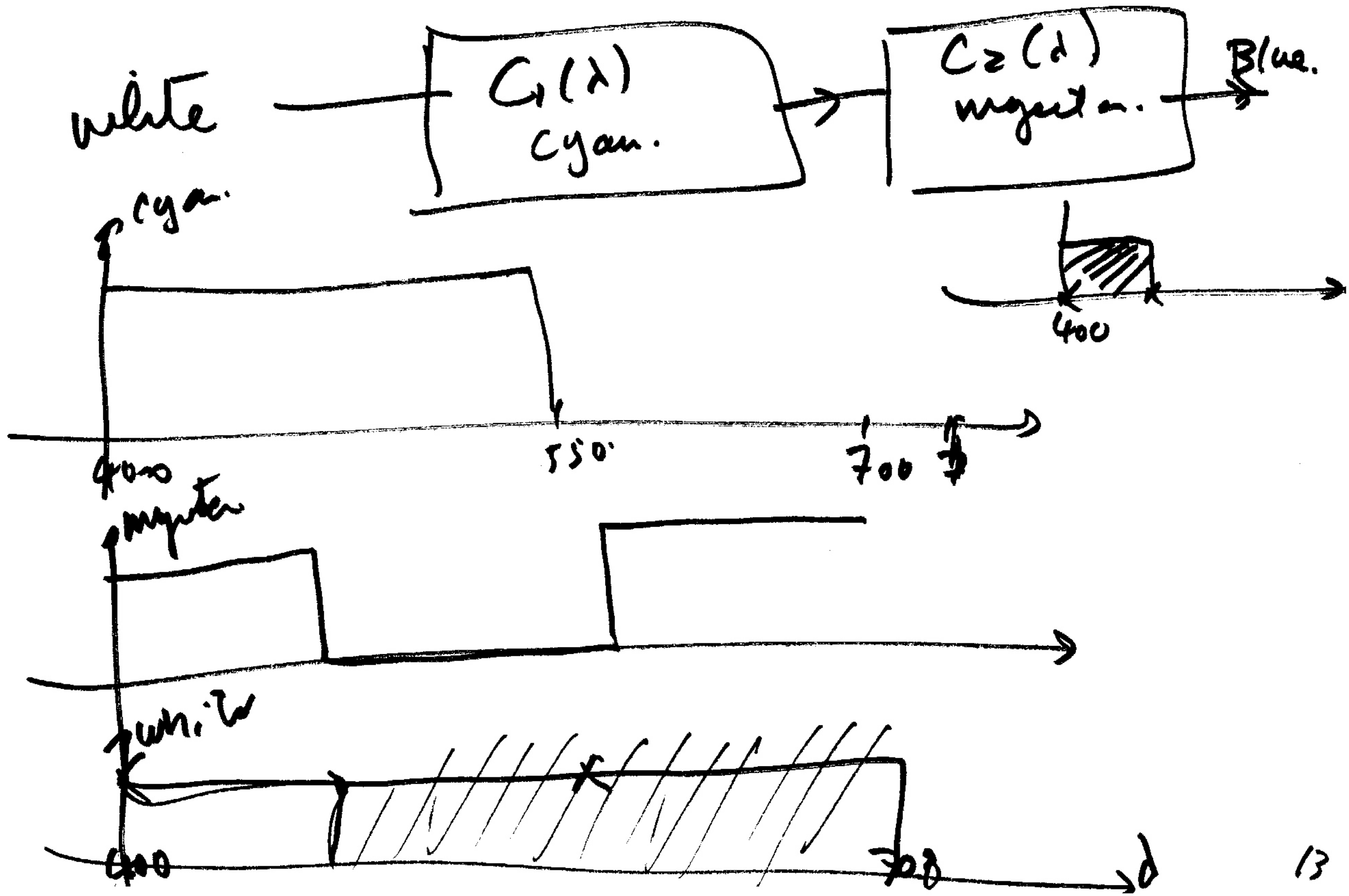
Blue
400nm
green
500
$C_1(\lambda)$
$C_2(\lambda)$
no light Blue
+
mix
Red
700nm
Red + green = Yellow
λ
λ
λ
λ
11

Blace



Ex: Camera sensor.

Subtractive Color system.



3 primary colors of subtractive sys