March 8, 2006

Mathematical Aspects/Derivation of Histogram Equalization

- Consider continuous values of intensity rather Than discretized.

- r = grey level of image To be equalized

  r normalized to $[0, 1]$

  $0 \rightarrow$ dark = Black

  $1 \rightarrow$ Bright = White.

- **Goal**: Design a Transformation.

  histogram matching or equalization $\longleftarrow T(r) = S$ $\quad 0 \leq r \leq 1$, $\quad 0 \leq S \leq 1$

- Assumptions about $T(r)$.

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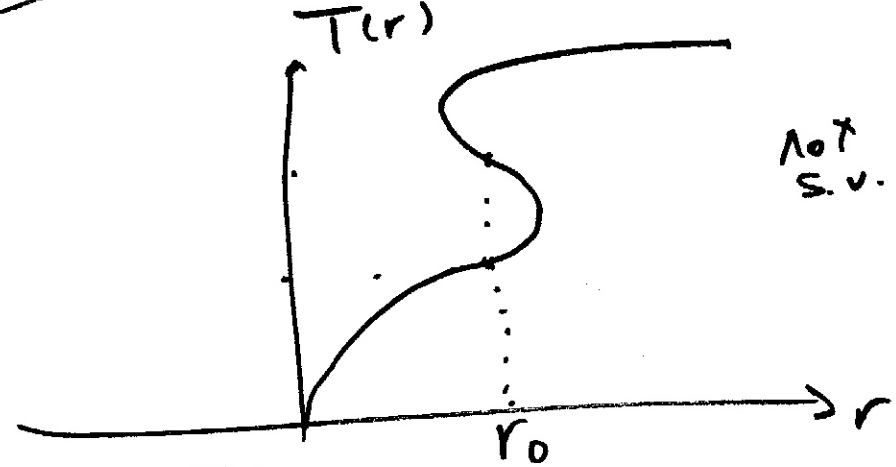
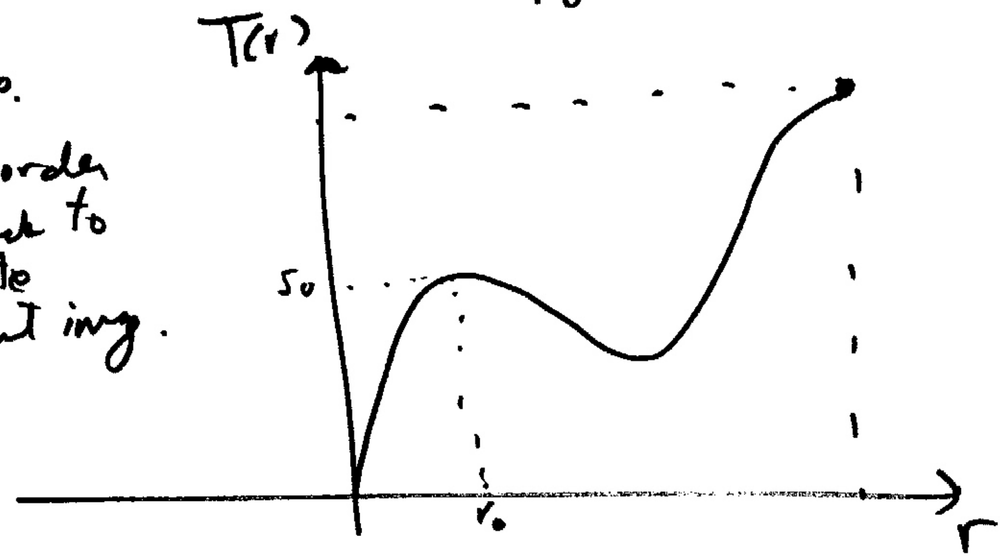
(a) $T(r)$ single valued and monotically increasing in the interval $0 \leq r \leq 1$

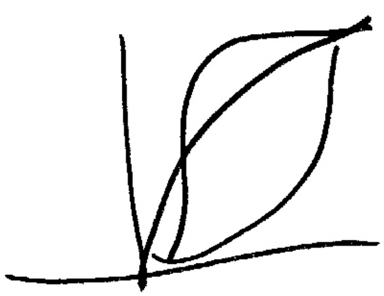
→ for inverse Transformation to exist



not s.v.

→ preserve increasing order from black to the white in output img.





(b) $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$

output grey levels in the same range as input levels.

$T(r) = s$

$T^{-1}(s) = r$

r = input intensity
s = output intensity.

<u>Result from prob Theory</u>:

If $T^{-1}(s)$ satisfies condition (a) Then.

$$P_s(s) = P_r(r) \left| \frac{dr}{ds} \right|$$

Consider CDF: as a Transformation.

$$s = T(r) = \int_0^r P_r(w)\,dw$$

$$\frac{ds}{dr} = \frac{d}{dr}\left\{\int_0^r P_r(w)\,dw\right\} = P_r(r)$$

$$P_s(s) = P_r(r)\left|\frac{1}{P_r(r)}\right| = 1 \qquad 0 \leq s \leq 1$$

<u>Words</u>: If $T(r)$ is just a CDF or just the integral of input pdf ($P_r(r)$) Then ~~and~~ applying $T(r)$ results in a image whose pdf ($P_s(s)$) is Uniform

Discrete Case

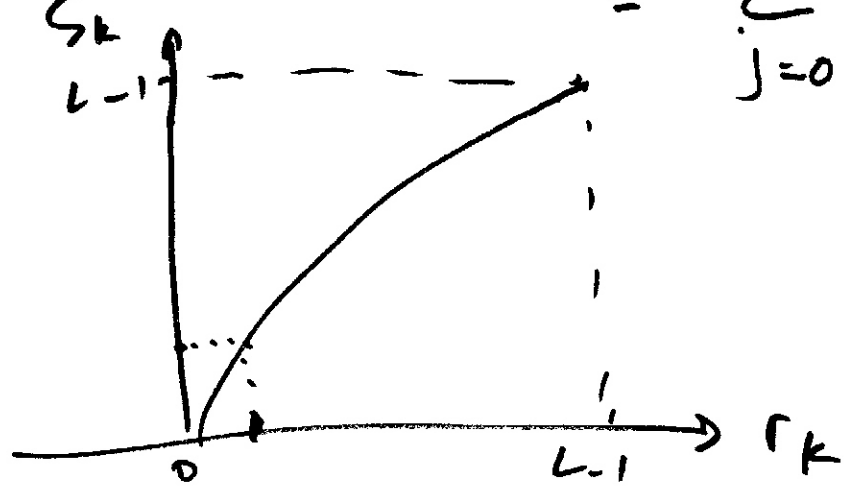
r_k = discrete intensity value. $k = 0, \ldots, L-1$

$p_r(r_k) = \frac{n_k}{n}$ $k = 0, \ldots, L-1$

n_k = # of pixels that have intensity r_k.

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) \leftarrow$$

$$= \sum_{j=0}^{k} \frac{n_j}{n}. \qquad k = 0, \ldots, L-1$$



Histogram Matching

$T(r) = s$

Rather than $p_s(s)$ uniform, we want $p_s(s)$ to match a "desired" pdf
given

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r = pixel values before matching

z = pixel " after "

Can compute $p_r(r)$ from given image.

Know, given $p_z(z)$

<u>Goal</u>: what is the transformation $r \longrightarrow z$?

Approach:

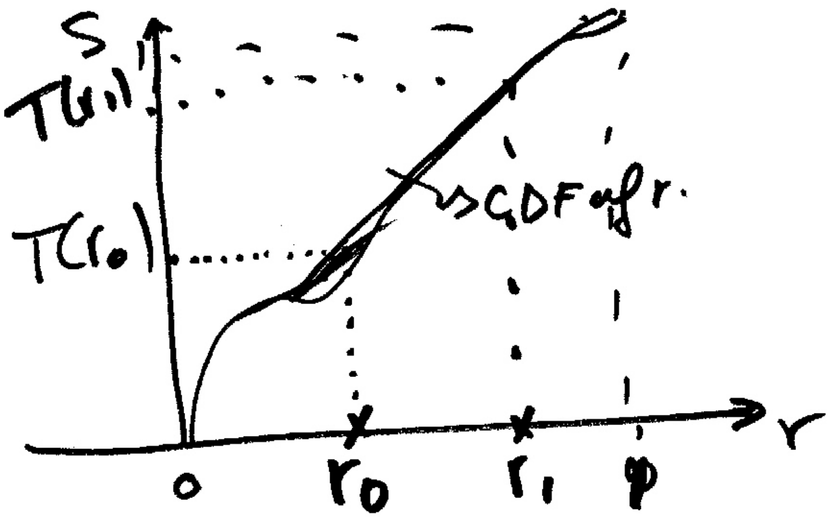
$$s = T(r) = \int_0^r P_r(w)\,dw \longrightarrow \text{CDF of } r.$$

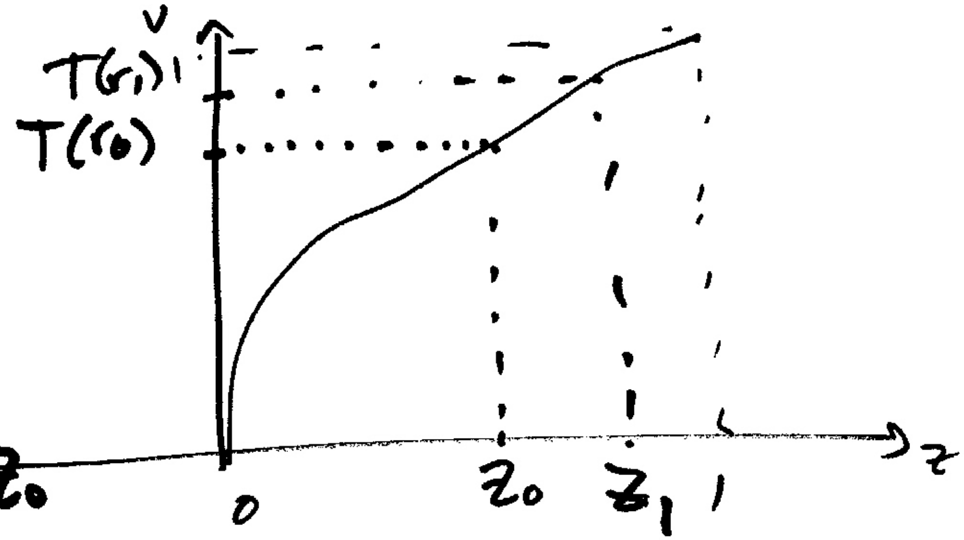
$$v = G(z) = \int_0^z P_z(t)\,dt \longleftarrow \text{CDF of } z.$$

Histogram matching

$$G(z) = T(r)$$

$$z = G^{-1}\{T(r)\}$$





$r_0 \longrightarrow$ how to find z_0

Lookuptable

r_0	z_0
r_1	z_1

→ Matched histogram of $p_z(z)$ desired