March 15, 2006

Image Enhancement

- unsharp masking + high boost filtering

- unsharp Masking.

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

processed   original   LPF = blurred.

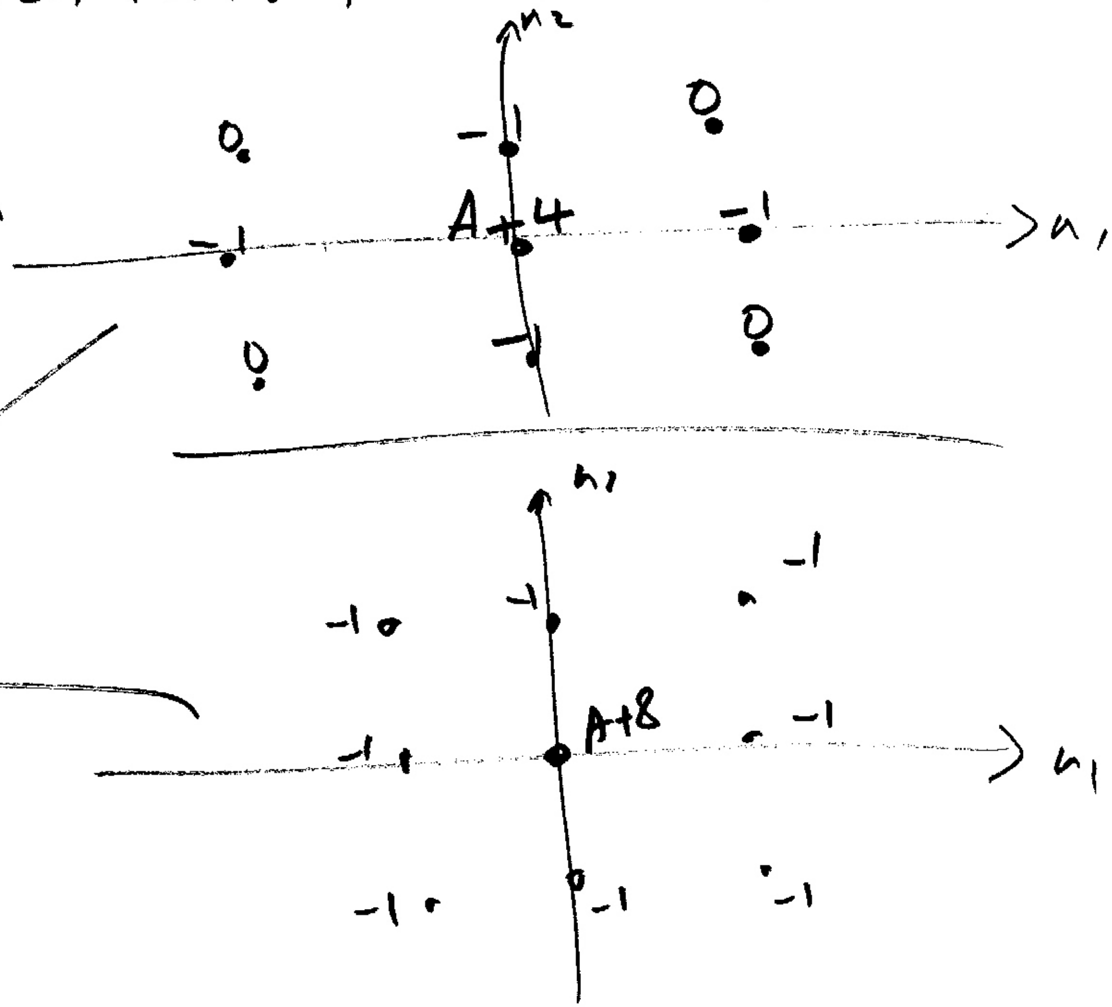
- General version unsharp masking. : highboost filtering

$$f_{hb}(x,y) = A\,f(x,y) - \bar{f}(x,y)$$

scaling factor.
$A > 1$

$$f_{hb}(x,y) = (A-1)\,f(x,y) + f_s(x,y).$$

hb = linear operator



possible impulse response

HB filtering.

Using Gradients for Enhancement

First derivative:

$$\vec{\nabla} f = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} G_x \\ G_y \end{bmatrix}$$

$$|\vec{\nabla} f| = \sqrt{G_x^2 + G_y^2} \longrightarrow \text{Edge Detection.}$$

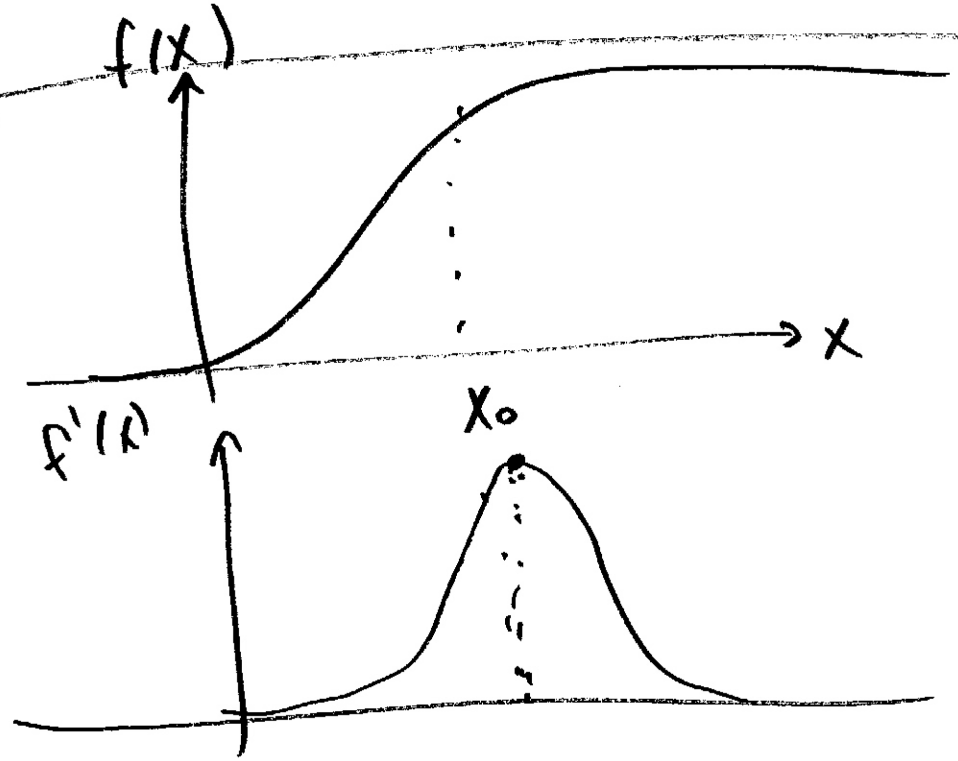
$\downarrow$ To simplify computationally

$$|\vec{\nabla} f| \simeq |G_x| + |G_y|$$

Edge Detection

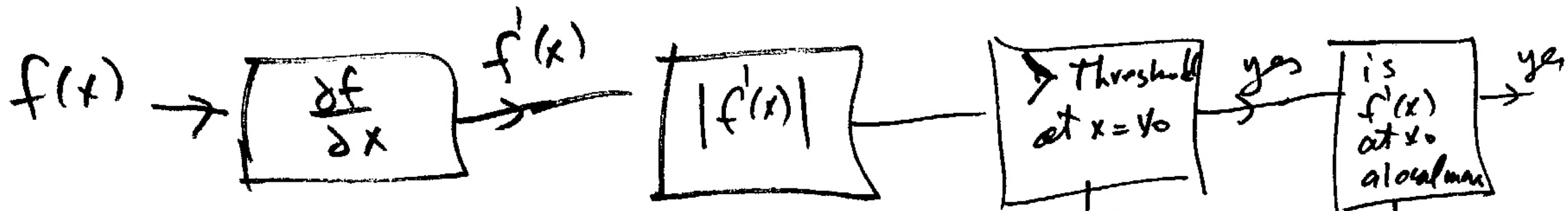
- Gradient Method:
- Laplacian $\nabla^2 f$
- L of G = Laplacian of Gaussian: Hildreth/Marr.

1-D



<u>Derivative</u>

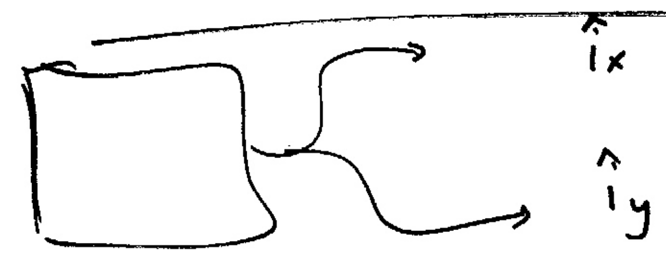
if $\frac{\partial f}{\partial x}$ large $\longrightarrow$ likely an edge.



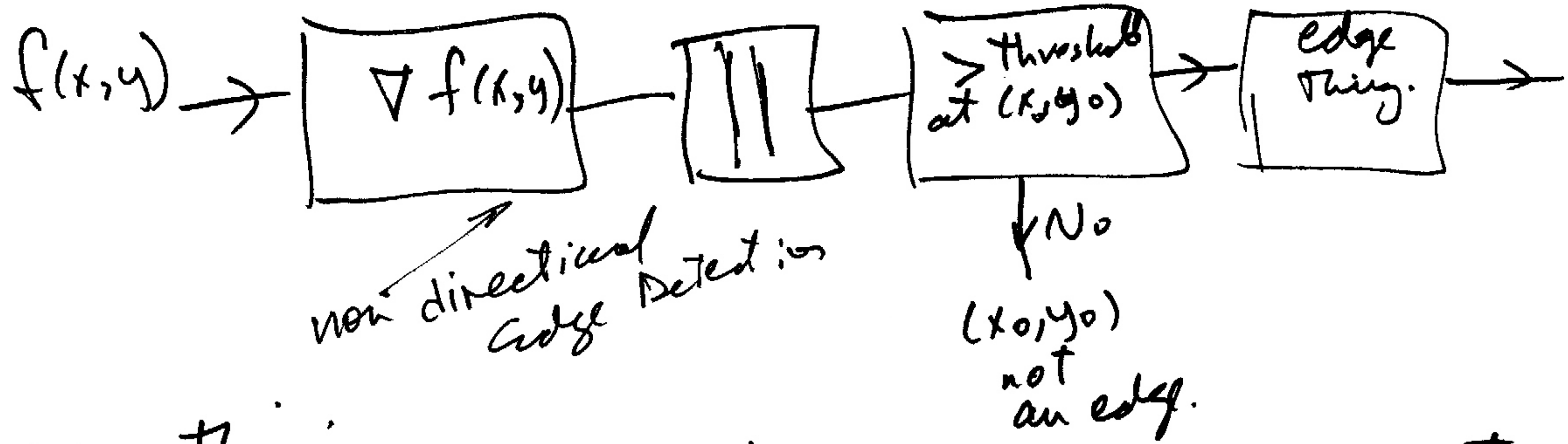
1D Edge Detect.

show 10.7 of G/W

Extension To 2D



$$\nabla f(x,y) = \frac{\partial f(x,y)}{\partial x}\,\hat{i}_x + \frac{\partial f(x,y)}{\partial y}\,\hat{i}_y$$



Edge Thing:

(a) If $|\nabla f(x,y)|$ has a local max at (x_0,y_0) in horizontal direction, but not vertical, we declare (x_0,y_0) an edge when

$$\left|\frac{\partial f(x,y)}{\partial x}\right|_{(x_0,y_0)} > K\left|\frac{\partial f(x,y)}{\partial y}\right|_{(x_0,y_0)}$$

$K \simeq 2$

(b) If $|\nabla f(x,y)|$ has a local max at (x_0, y_0) in vertical direction but not horizontal, declare (x_0, y_0) an edge. when

$$\left|\frac{\partial f}{\partial y}(x,y)\right|_{(x_0,y_0)} > K \left|\frac{\partial f}{\partial x}(x,y)\right|_{(x_0,y_0)}$$

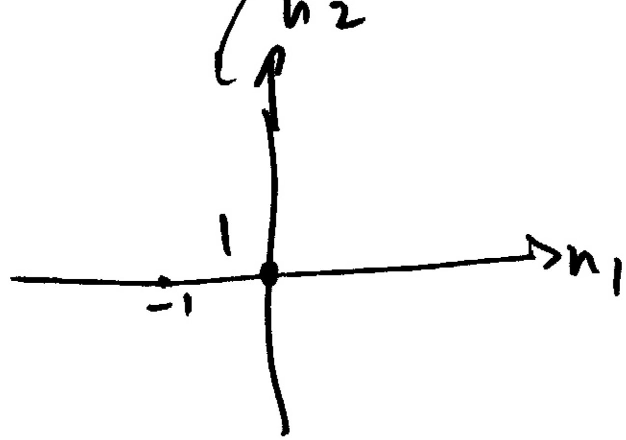
Directional Gradient Edge Detectors

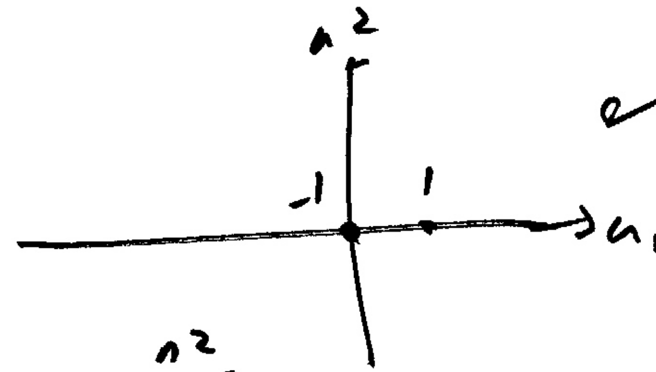
- Bias Towards a particular direction.

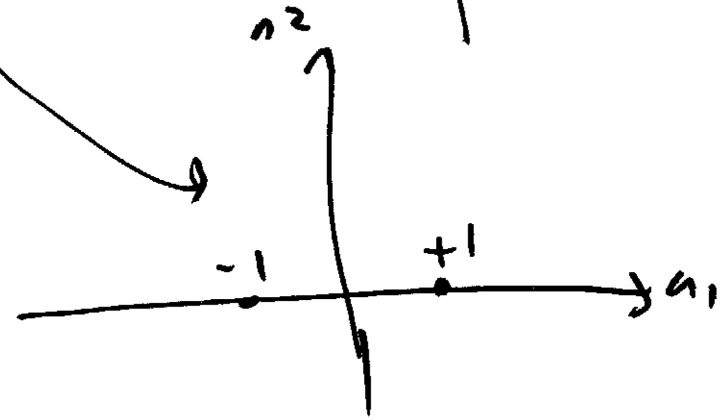
compute vertical edge: $\dfrac{\partial f(x,y)}{\partial x}$

horizontal edge $\dfrac{\partial f(x,y)}{\partial y}$

$$\frac{\partial f(x,y)}{\partial x} = \begin{cases} \dfrac{f(x,y) - f(x-1,y)}{T} \\[2ex] \dfrac{f(x+1,y) - f(x,y)}{T} \\[2ex] \dfrac{f(x+1,y) - f(x-1,y)}{2T} \end{cases}$$

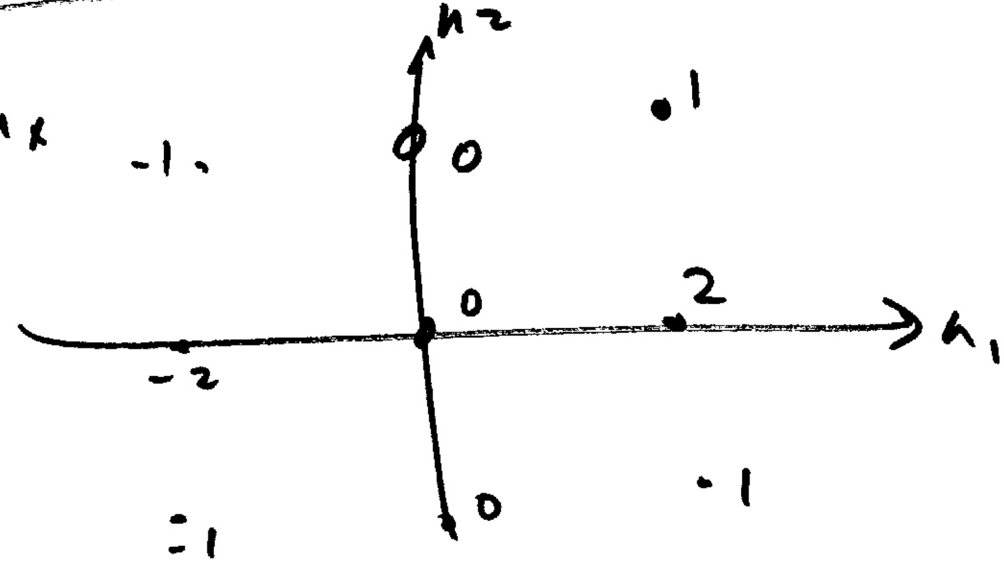






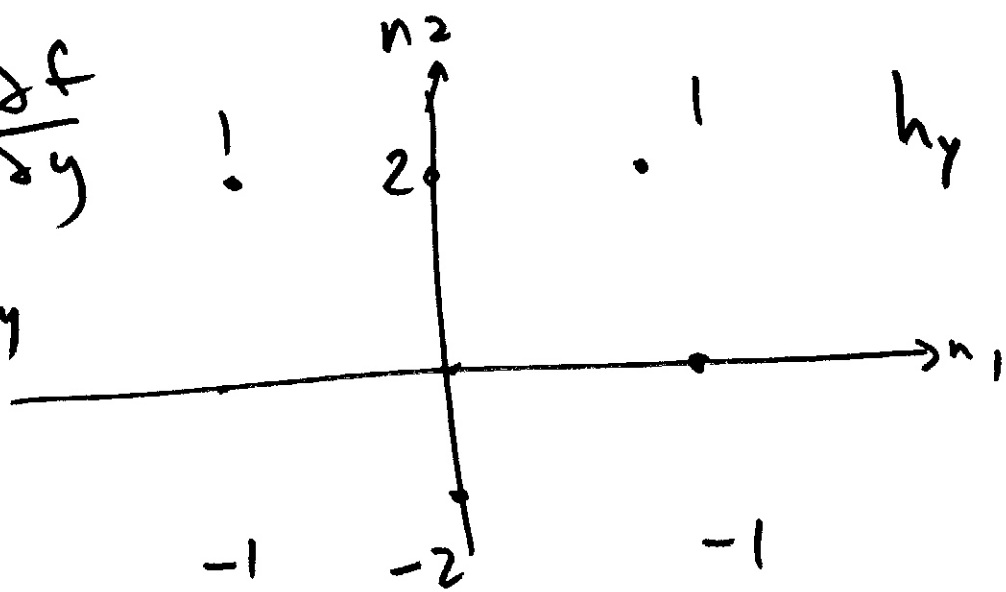
Famous Directional Edge Detection — SoBel

$\frac{\partial f}{\partial x}$ $f * h_x$



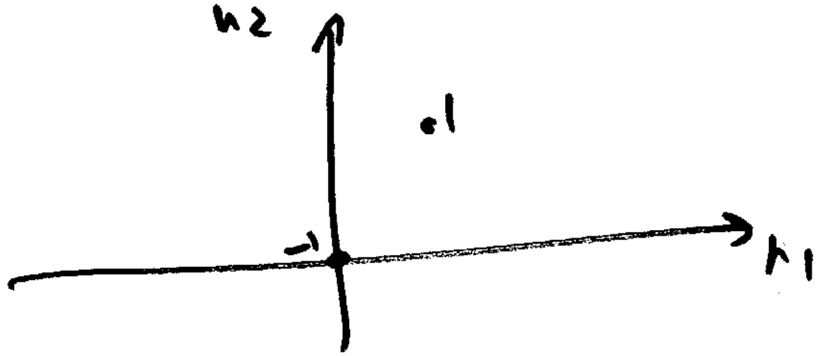
$= \frac{\partial f}{\partial y}$

$= f * h_y$

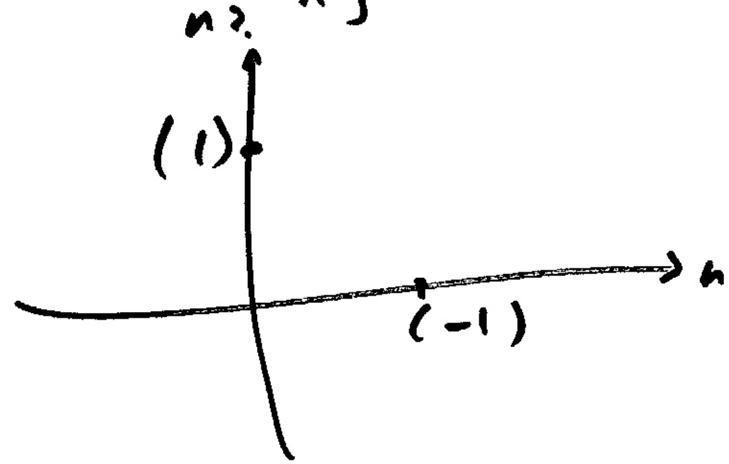


Roberts's edge detector

h_x



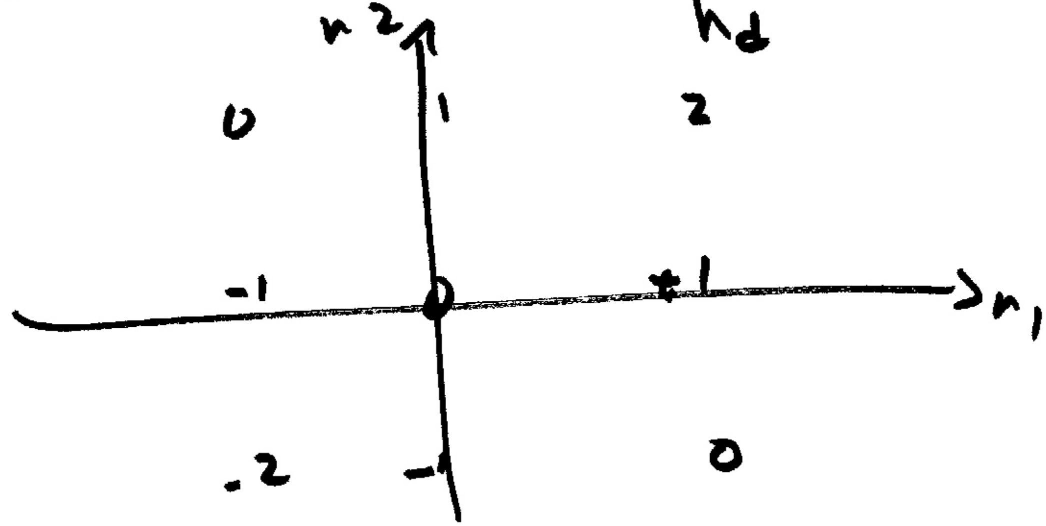
h_y

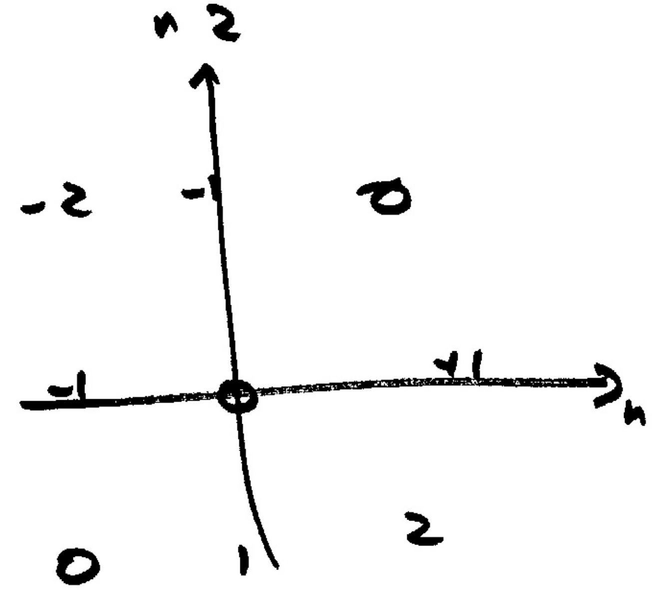


Diagonal Directional Gradient filters

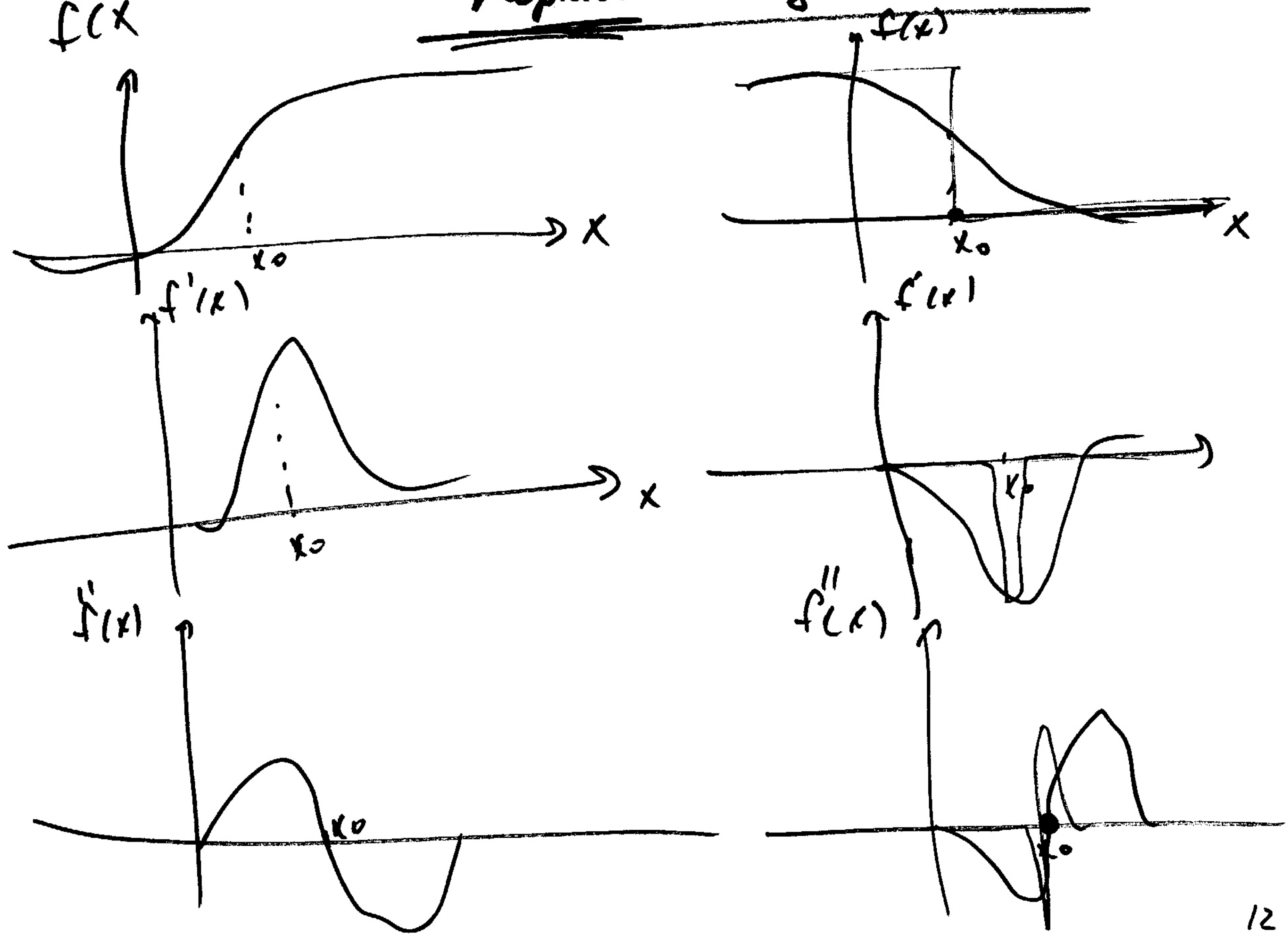
Sobel $\rightarrow$ Diagonal.

h_d





Laplacian Edge Detection



- zero crossing of $f''(x) \longrightarrow$ edge.

Extension To 2D

$$\nabla^2 f(x,y) = \nabla\left(\nabla f(x,y)\right)$$

$$= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

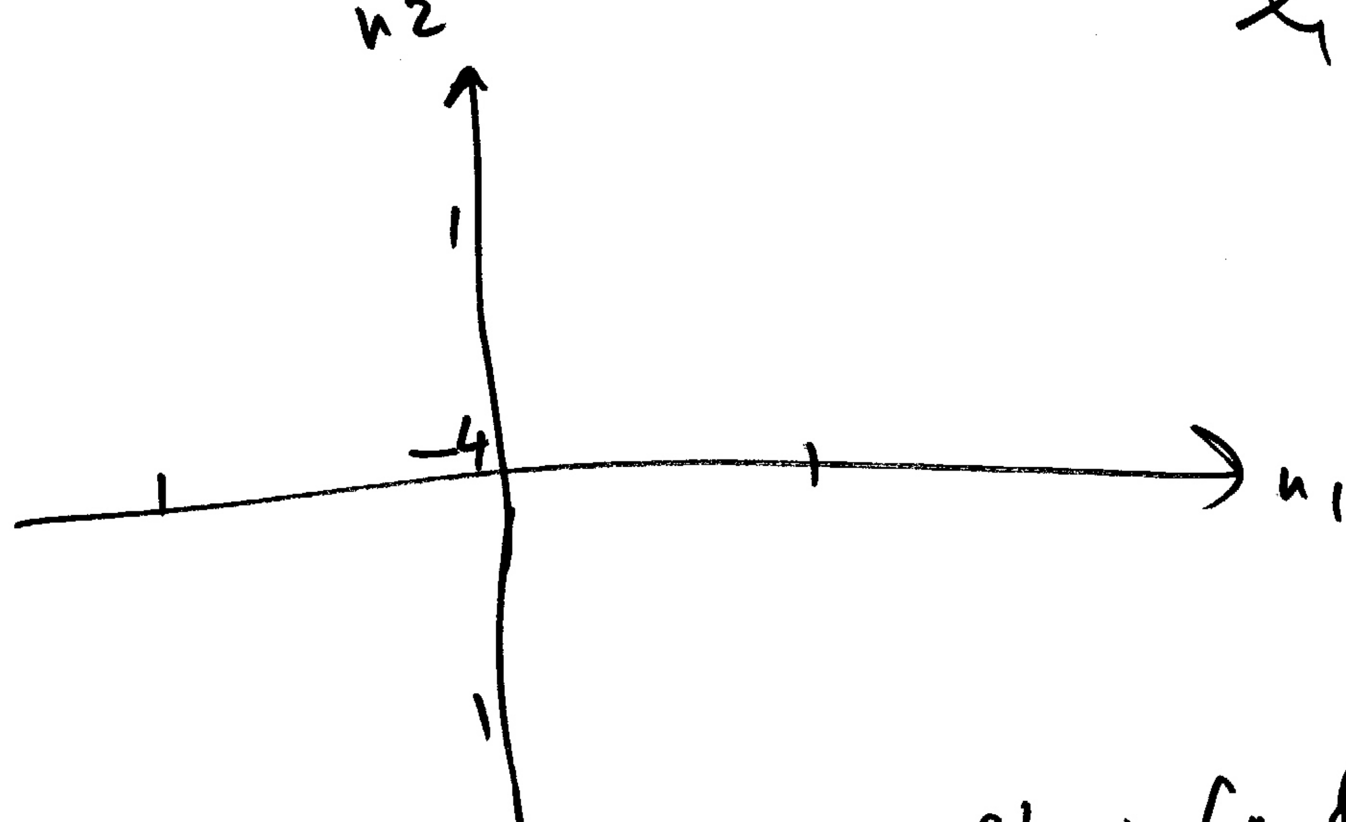
$$\left.\begin{aligned} \frac{\partial f}{\partial x} &\longrightarrow f(n_1+1, n_2) - f(n_1, n_2) \\ \frac{\partial^2 f}{\partial x^2} &\longrightarrow f_x(n_1, n_2) - f_x(n_1-1, n_2) \end{aligned}\right\} \Rightarrow$$

$$\frac{\partial^2 f}{\partial x^2} \Longrightarrow f(n_1+1, n_2) - 2f(n_1, n_2) + f(n_1-1, n_2)$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \longrightarrow f(n_1+1, n_2) + f(n_1-1, n_2) + f(n_1, n_2+1) + f(n_1, n_2-1) - 4f(n_1, n_2)$$

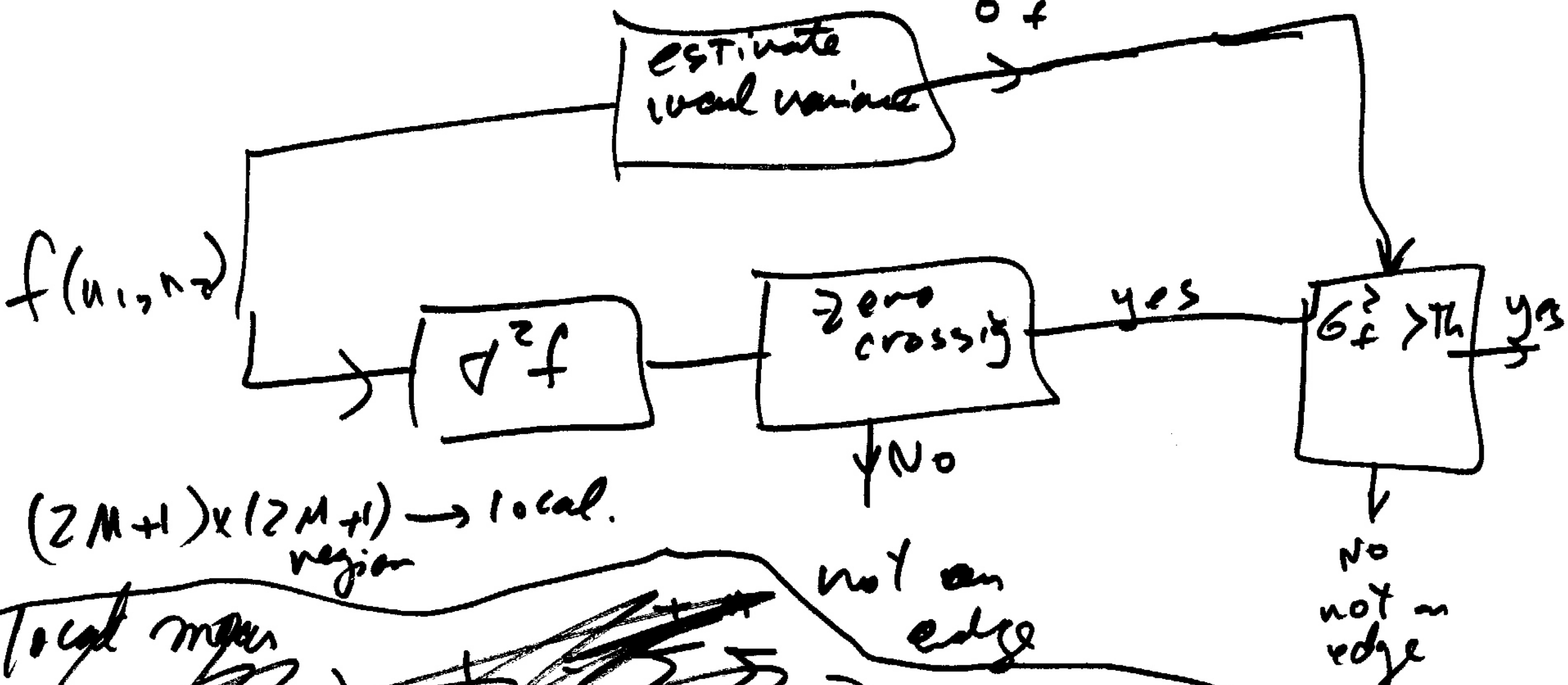
$$= f * h.$$

h



Show fig 8.33
J Lim

How To use $\nabla^2 f$ operator to detect edges?



$(2M+1) \times (2M+1) \rightarrow$ local region

~~Local mean~~

~~$m_f(n_1, n_2) = \frac{1}{(2M+1)^2} \sum_{k_1} \sum f(k_1, k_2)$~~

~~local variance~~

~~$\sigma_f^2(n_1, n_2) = \frac{1}{(2M+1)^2} \sum \sum$~~

15

local mean

$$m_f(n_1,n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} f(k_1,k_2)$$

local variance

$$\sigma_f^2(n_1,n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} \left(f(k_1,k_2) - m_f(k_1,k_2)\right)^2$$

Window $(2M+1) \times (2M+1)$

centered at (n_1,n_2)

Edge Detection Laplacian of Gaussian

Gaussian $h(x,y) = e^{-\frac{(x^2+y^2)}{2\pi\sigma^2}}$

$$H(\Omega_x, \Omega_y) = 2\pi^2 \sigma^2 e^{-2\pi\sigma^2 \frac{(\Omega_x^2+\Omega_y^2)}{2}}$$

"Laplacian of Gaussian" = LoG.

$$\nabla^2 \left[f(x,y) * \underset{\text{Gaussian}}{h(x,y)} \right] =$$

$$= f(x,y) * \left[\nabla^2 h(x,y) \right]$$

$$= f(x,y) * \left[\frac{\partial^2 h(x,y)}{\partial x^2} + \frac{\partial^2 h(x,y)}{\partial y^2} \right]$$

$$\nabla^2 h(x,y) = \frac{e^{-(x^2+y^2)/2\pi\sigma^2}}{(2\pi\sigma^2)^2}\left(x^2+y^2-2\pi\sigma^2\right)$$

$$F\{\quad\} = -2\pi^2\sigma^2 e^{-\pi\sigma^2(\Omega_x^2+\Omega_y^2)/2}(\Omega_x^2+\Omega_y^2)$$

Fig. 8.36 J. Lim