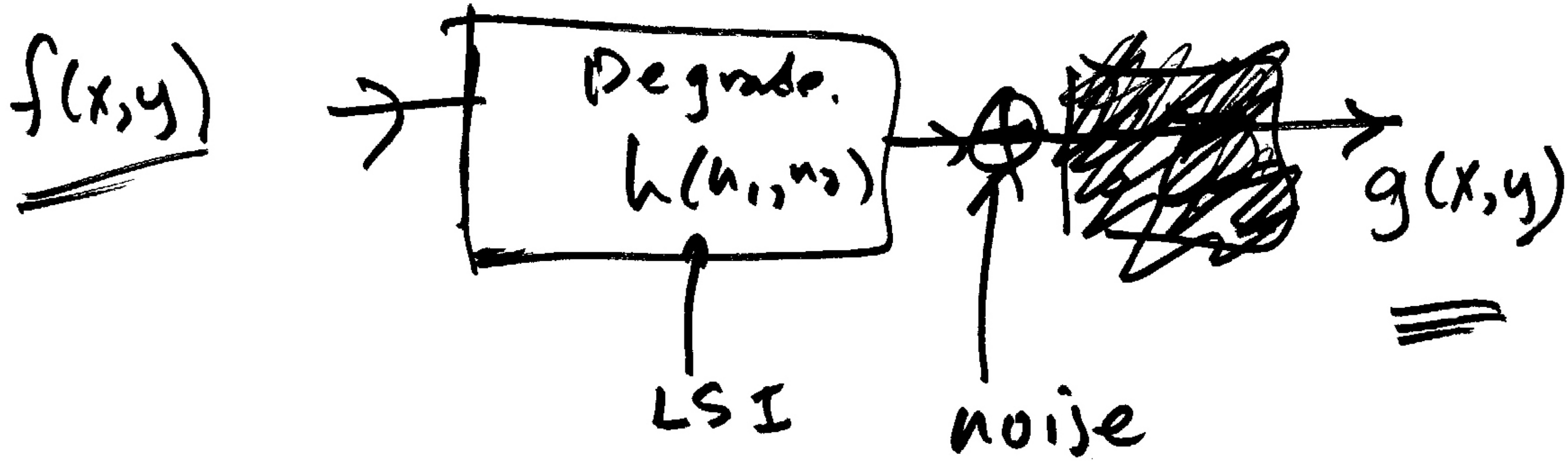
March 17, 2006

Image Restoration

1. Enhancement $\longrightarrow$ image "looks" better
   $\downarrow$
   subjective improvement.

2. Restoration.
   Image has been degraded by something
   - noise
   - blur
   - Atmospheric Turbulence.

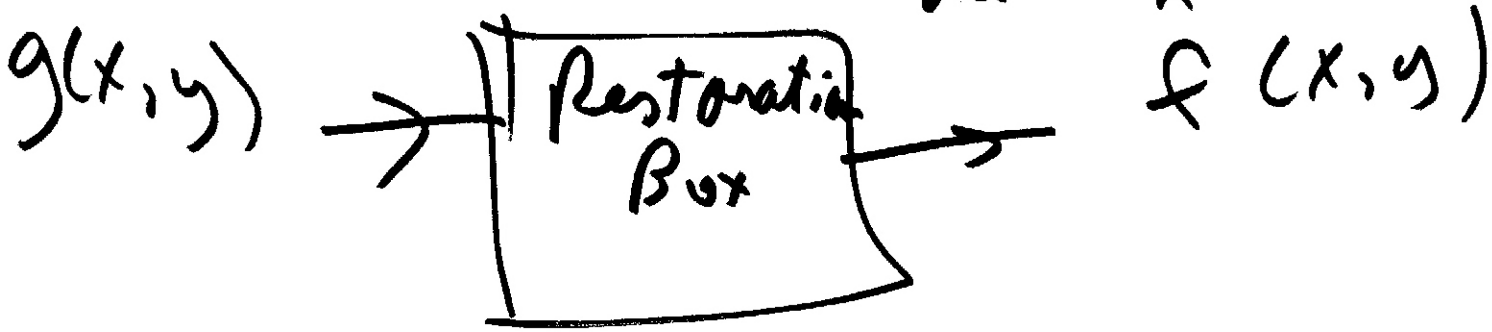
   objective models $\rightarrow$ Error $\rightarrow$ MSE



$$g(x,y) = h * f + \eta$$

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

noise



minimize $E\left[\left(f(x,y) - \hat{f}(x,y)\right)^2\right]$

Today.

H is identity.

only corruption $\longrightarrow$ noise.

$$g(x,y) = f(x,y) + \eta(x,y)$$

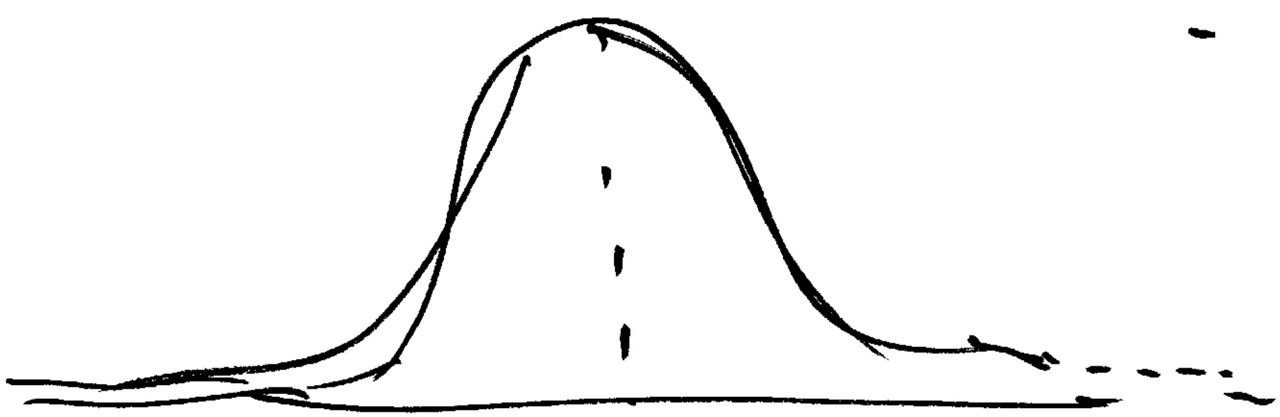
observed. original image noise

Assume noise is independent of spatial coordinates uncorrelated w.r.t. image itself.

① Gaussian

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(z-\mu)^2}{2\pi\sigma^2}}$$

$var = \sigma^2$ mean $= \mu$



μ

- electronic noise

- sensor noise due to poor illumination or high temperature

-

70% of values are within one sigma $[\mu - \sigma, \mu + \sigma]$

95% within 2 sigma.

② Rayleigh.

$$p(z) = \begin{cases} \frac{z}{b}(z-a)\, e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

$$\mu = a + \sqrt{\frac{b\pi}{4}} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$

noise in range imaging apps.

3 Erlang (Gamma) noise.

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1} e^{-az}}{(b-1)!} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

laser imaging.

$\mu = b/a \qquad \sigma^2 = \dfrac{b}{a^2}$

4 Exponential special case of Erlang.

$$p(z) = \begin{cases} a e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

Networking

(5) Uniform.

$$p(z) = \begin{cases} \frac{1}{b-a} & a \le z \le b \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu = \frac{a+b}{2} \qquad \text{var} = \frac{(b-a)^2}{12}$$

random # generation.

(6) Impulse, salt/pepper noise.

quick transit, faulty switching.

$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & \text{otherwise.} \end{cases}$$

if $b > a$, b show up as light dots

a " " " dark dot.

① Mean Filters

1. Arithmetic Mean.

$S_{xy} \rightarrow$ window.

$m \times n$.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

2. Geometric Mean.

similar to arithmetic but loose less detail

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$$

3. Harmonic mean filter.

works for salt noise. fails for pepper

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

4. Contra Harmonic.

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$Q > 0 \rightarrow$ remove pepper noise

$Q < 0 \rightarrow$ remove salt noise

order statistics filter

1. Median

look at S_{xy}. find median
replace (x,y) with the
median value.

2. Max $\hat{f}(x,y) = \max_{S_{xy}} g(s,t)$

also removes some dark pixels.

3. Min $\hat{f}(x,y) = \min_{S_{xy}} g(s,t)$

also removes some white pixels

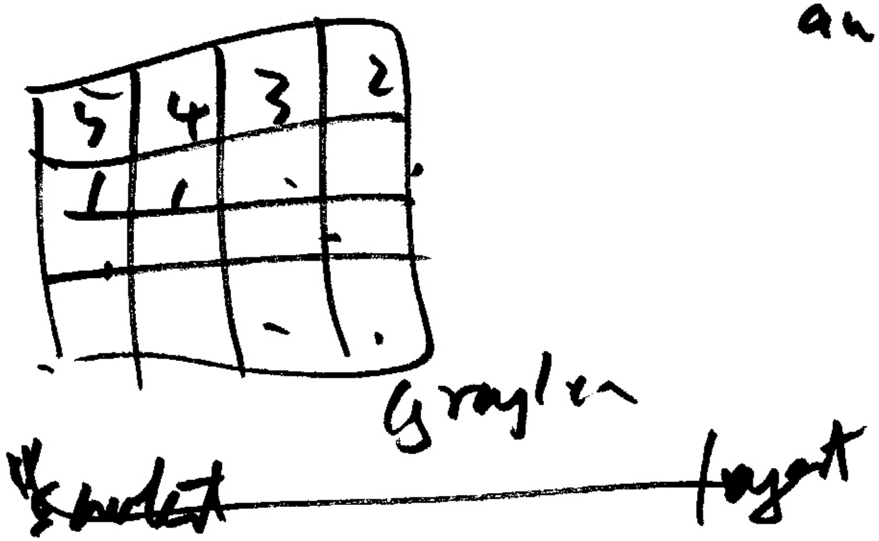
4. Midpoint $\frac{1}{2}\{\max + \min\}$.

Alpha Trimed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(x,y)$$

S_{xy} $m \times n$.

$g_r(s,t) \longrightarrow g(s,t)$ excloding.
$d/2$ brightest graylevel
and $d/2$ darkest " "



Adaptive local noise Reduction

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_N^2}{\sigma_L^2}\left[g(x,y) - m_L \right]$$

m_L = local mean = $\frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$

σ_L^2 = local variance.

σ_N^2 = noise variance.

if $\sigma_N^2 << \sigma_L^2 \longrightarrow \hat{f} \simeq g$ good

if $\sigma_N^2 >> \sigma_L^2$ then approximate $\frac{\sigma_N^2}{\sigma_L^2} \simeq 1$

$\longrightarrow \hat{f} \simeq m_L$