March 22, 06

Image ~~[illegible]~~ Restoration

- Spatial domain
  - "additive Gaussian noise"
  - salt/pepper
    - median
    - Adaptive Median filters
- Frequency domain

Adaptive Median Filter

- looking at neighborhood $S_{x,y}$ around pixel (x,y)
- z_{xy} = pixel value at center of $S_{x,y}$
- z_{max} = max value of intensity over S_{xy}

- z_{min} = min value in S_{xy}
- z_{med} = median value in S_{xy}

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outline

- Keep increasing window size until z_{med} is not an impulse.
  $z_{min} < z_{med} < z_{max}$
- When this happen, check z_{xy}.
  - If z_{xy} is not an impulse $\rightarrow$ output z_{xy}
  - If z_{xy} is an impulse $\rightarrow$ output z_{med}
    since z_{med} is not an impulse.

Psuedo code

Part A   if $z_{min} < z_{med} < z_{max}$

Then go to part B. $\rightarrow$ z_{med} is not an impulse.

else. if windowsize $< S_{max}$

window $\leftarrow$ window + 1, go to part A

else. output z_{xy}.

part B.   If $z_{min} < z_{xy} < z_{max}$ $\rightarrow$ z_{xy} is not an impulse

output z_{xy}

else. output z_{med}

Image Restoration

Frequency domain technique

Periodic noise.

what if interference Pattern is not "clean"?

Sources of periodic interference patterns:
Coupling + amplification of low level signals in electro optical scanners's electronic circuitry.

Approach (1) First isolate principal contributions (spikes). of the interference pattern.

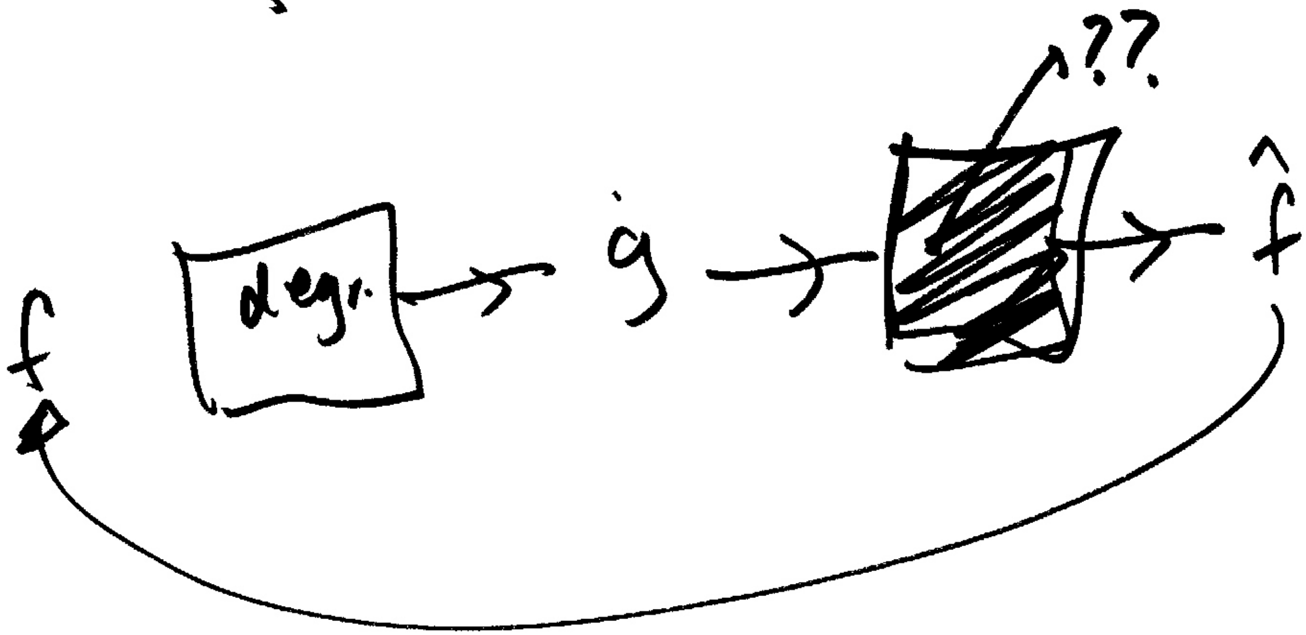
(2) subtract a variable weighted portion of the pattern from the corrupt image.

objective: i.e. e.g. minimize local variance of a processed image.

$g(x,y) \longleftrightarrow G(\omega_1, \omega_2)$ observed degraded signal.

$f(x,y) \longleftrightarrow F(\omega_1, \omega_2) \rightarrow$ clean signal. original undegraded.

$\hat{f}(x,y) \longleftrightarrow \hat{F}(\omega_1, \omega_2) \rightarrow$ processed version of g approximate f.



Step ① put a notch filter $H(\omega_1, \omega_2)$ at location of each spike.

Find filter

$$N(\omega_1, \omega_2) = H(\omega_1, \omega_2)\, G(\omega_1, \omega_2)$$

noise.

$$\eta(x, y) = \mathcal{F}^{-1}\left\{ H(\omega_1, \omega_2)\, G(\omega_1, \omega_2) \right\}$$

noise in space domain.

Step 2

$$\hat{f}(x,y) = g(x,y) - w(x,y)\,\eta(x,y)$$

weight fn.

Optimization: choose local weights $w(x,y)$ to minimize local variance of $\hat{f}$ at (x,y)

neighborhood $(2a+1) \times (2b+1)$

local variance over this neighborhood:

$$\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x,y) \right]^2$$

$\bar{\hat{f}}$ = local mean = $\bar{\hat{f}}(x,y)$

$$= \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \hat{f}(x+s, y+t)$$

plug in $\hat{f} = g - W\eta$ into

assume $W(x,y)$ is constant over $[2a+1] \times [2b+1]$ region.

$$W(x+s, y+t) \simeq W(x,y)$$

$$s,t \in [-a, +a] \times [-b, +b]$$

$$\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{-a}^{+a} \sum_{-b}^{+b}$$

$$\Big\{ \big[g(x+s, y+t) \big] - \big[w(x,y)\, \eta(x+s, y+t) \big] - \big[\bar{g}(x,y) - w(x,y)\, \bar{\eta}(x,y) \big] \Big\}^2$$

optimal weight

$$\frac{\partial \sigma^2_{xy}}{\partial w(x,y)} = 0 \Rightarrow \boxed{w(x,y) = \frac{\overline{g(x,y)\,\eta(x,y)} - \bar{g}(x,y)\,\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}}$$

Restoration

$$f \longrightarrow \boxed{\text{deg.}} \longrightarrow g \longrightarrow \boxed{\text{Rest oration}} \longrightarrow \hat{f}$$

observe

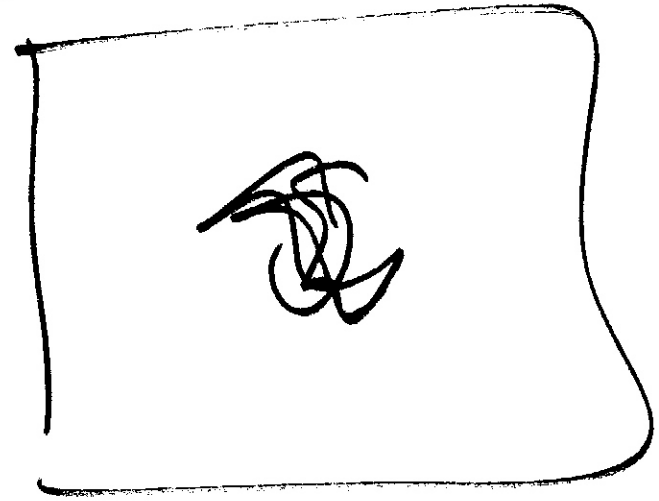
$$f(x,y) \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ \text{LSI} \\ h(n_1,n_2)\end{matrix}} \longrightarrow \oplus \longrightarrow g(x,y)$$

noise

Estimating the degradation fn

① Observation

go to parts of g
that you have
apriori knowledge.



see how it looks.
compare to what it should have looked like.
⟹ make some intelligent guess about
h and noise

② Experimentation : put a known signal
into your system to calibrate

(3) Modeling

Starley + Hufnagel.

Atmospheric turbulence

$$H(\omega_1, \omega_2) = e^{-K(\omega_1^2 + \omega_2^2)^{5/6}}$$

Motion Blur $\leftarrow$