March 24, 2006

Restoration

$$f \longrightarrow \boxed{\text{degrad}} \longrightarrow g \longrightarrow \boxed{\text{Restoration}} \longrightarrow \hat{f}$$

as close as possible.

Degradation can be modelled as.

$$f \longrightarrow \boxed{\begin{array}{c}\text{LSI} \\ h(n_1, n_2)\end{array}} \longrightarrow \oplus \longrightarrow g$$

noise

Use domain specific knowledge to model degradation.

Motion Blur modeling

image $f(x,y)$

$x_0(t)$ Time varying component of motion along x.

$y_0(t)$ " " " " " " y.

T = duration of exposure.

$g(x,y)$ = observed or captured signal.

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))\, dt$$

$$\text{F.T.}\{g(x,y)\} = G(\omega_x, \omega_y) = \int_{-\infty}^{+\infty}\int g(x,y)\, e^{-j2\pi(\omega_x x + \omega_y y)}\, dx\, dy.$$

$$G(\omega_x,\omega_y)=\int_0^T\left(\iint_{-\infty}^{+\infty} f(x-x_0(t),y-y_0(t))\,e^{-j2\pi(\omega_x x+\omega_y y)}\,dx\,dy\right)dt$$

$$=\int_0^T F(\omega_x,\omega_y)\,e^{-j2\pi(\omega_x x_0(t)+\omega_y y_0(t))}\,dt$$

$$\boxed{G(\omega_x,\omega_y)=F(\omega_x,\omega_y)\underbrace{\int_0^T e^{-j2\pi(\omega_x x_0(t)+\omega_y y_0(t))}\,dt}_{H(\omega_x,\omega_y)}}$$

- $x_0(t)=0 \quad y_0(t)=0 \Longrightarrow g(x,y)=T\,f(x,y)$
- $x_0(t)=\frac{at}{T}$ = constant speed along x direction $y_0(t)=0$

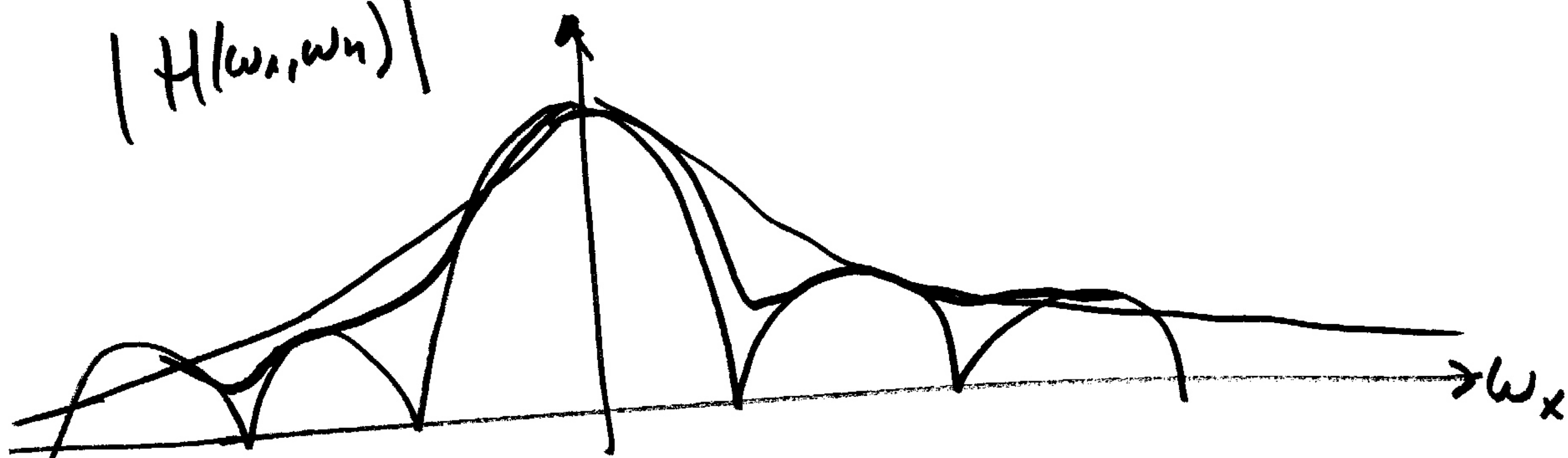
$$\boxed{G(\omega_x,\omega_y)=F(\omega_x,\omega_y)\,H(\omega_x,\omega_y)}$$

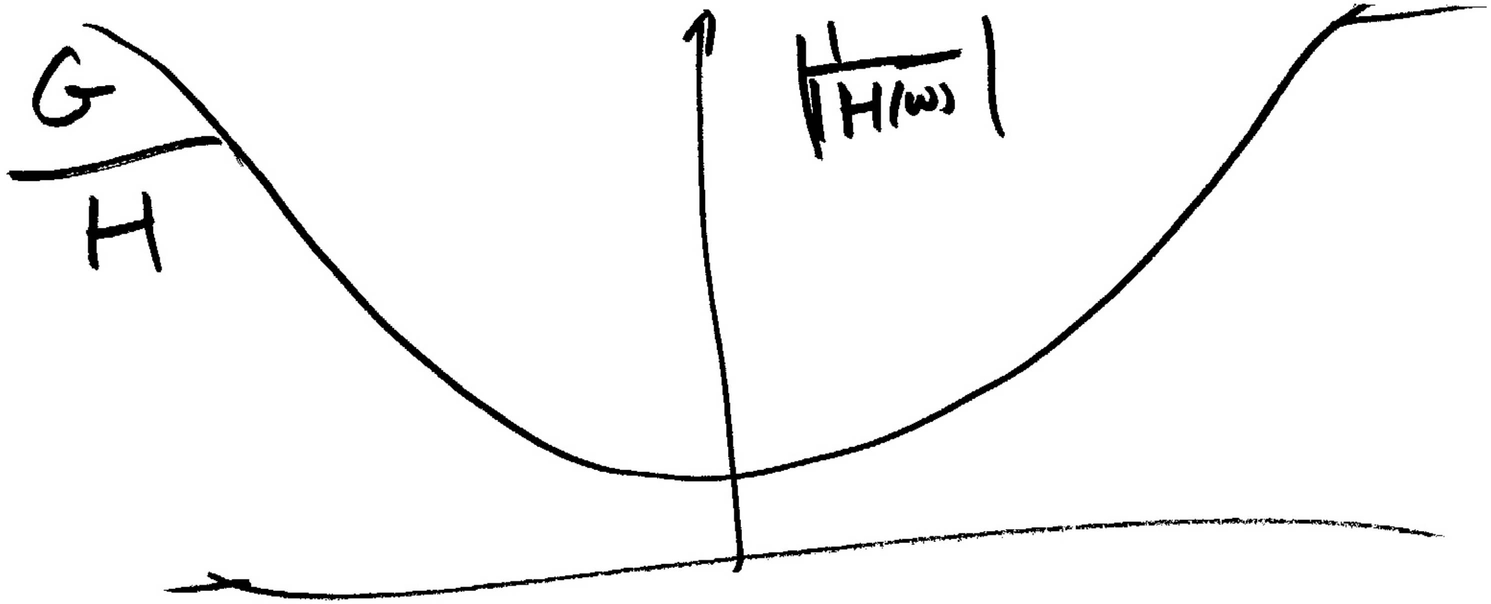
$$H(\omega_x, \omega_y) = \int_0^T e^{-j2\pi(\omega_x x_0(t))} \, dt$$

$$= \int_0^T e^{-j2\pi \frac{\omega_x a t}{T}} \, dt$$

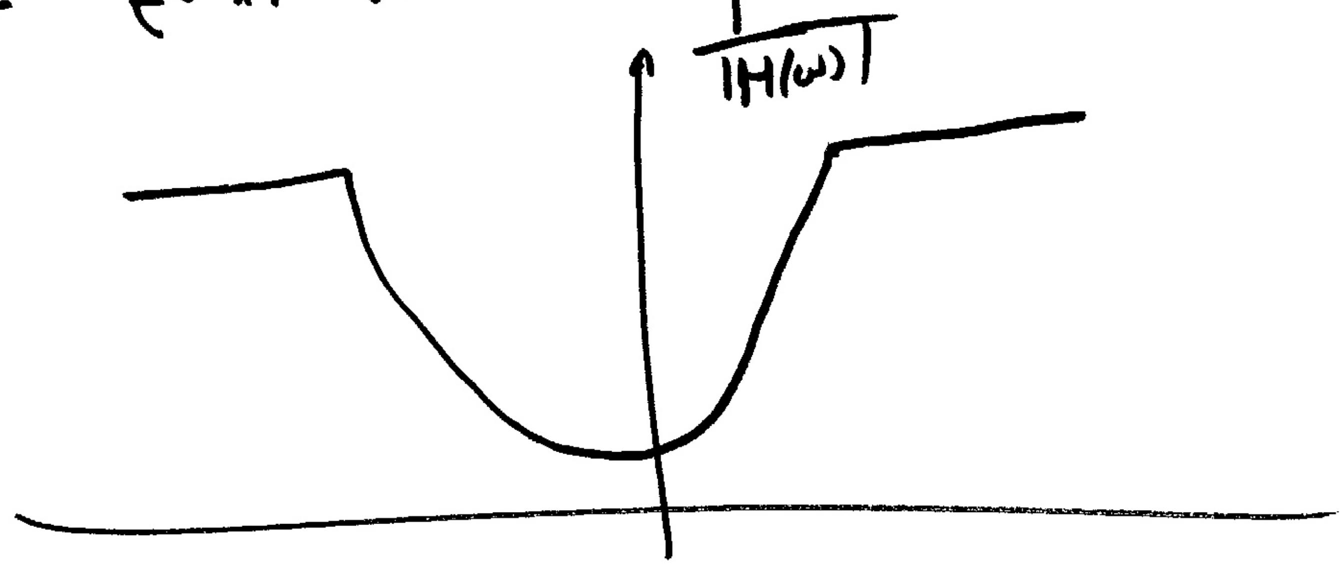
$$H(\omega_x, \omega_y) = \frac{T}{\pi \omega_x a} \sin(\pi \omega_x a) \, e^{-j\pi\omega_x a}$$

$$|H(\omega_x, \omega_y)|$$

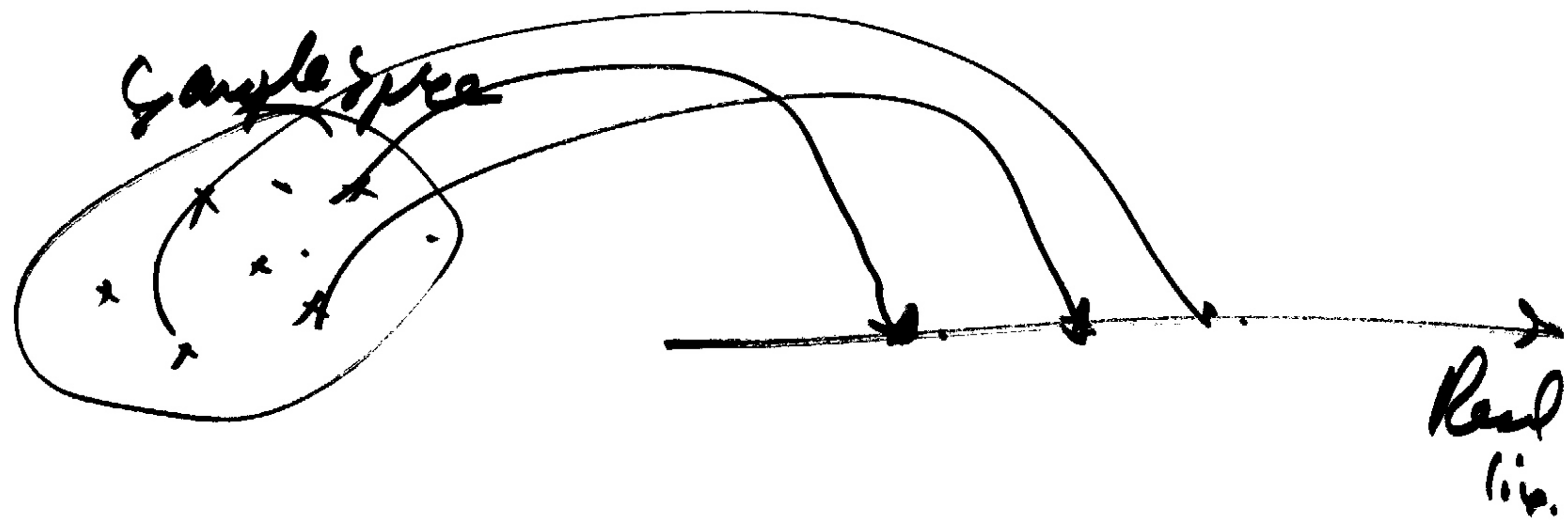




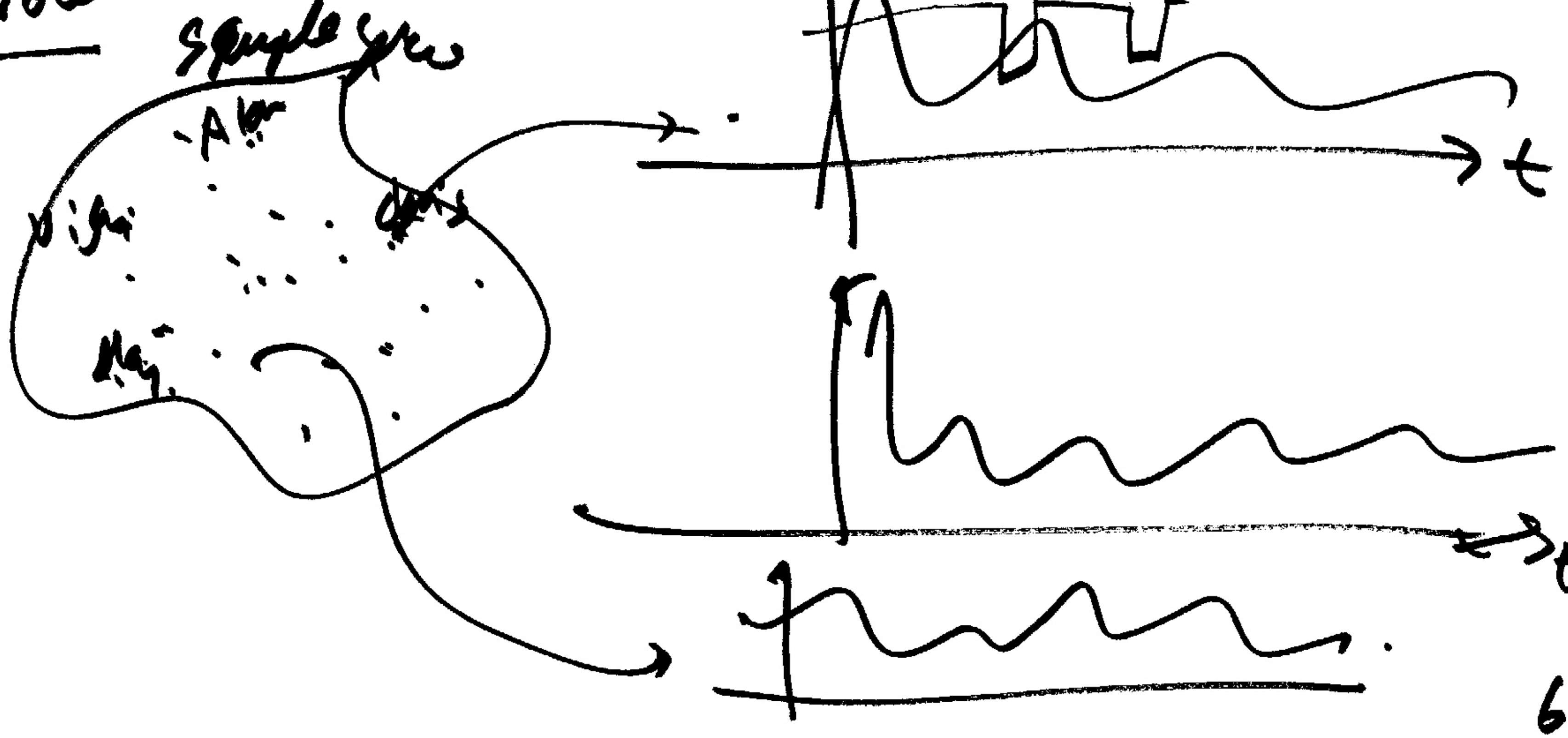
$$G(\omega_x, \omega_y) = F(\omega_x, \omega_y)\, H(\omega_x, \omega_y) + \underline{\underline{\text{Noise}}}$$



R.V



R. Process



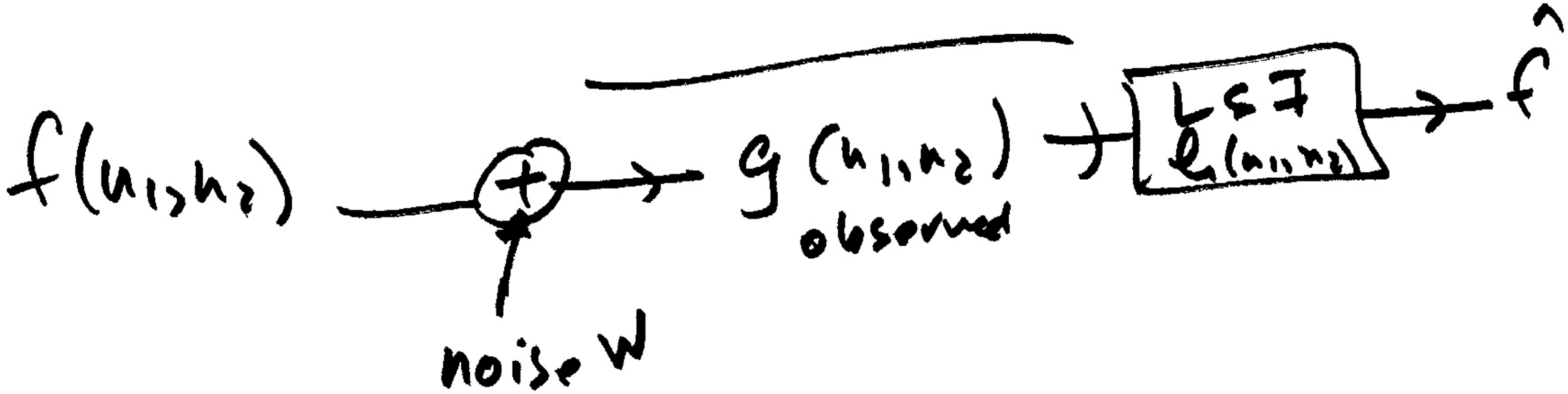
Stationarity

$$P_{x(t_1)\,x(t_2)\ldots\ldots\,x(t_N)}\left(X_1, X_2, X_3, \ldots, X_N\right)$$

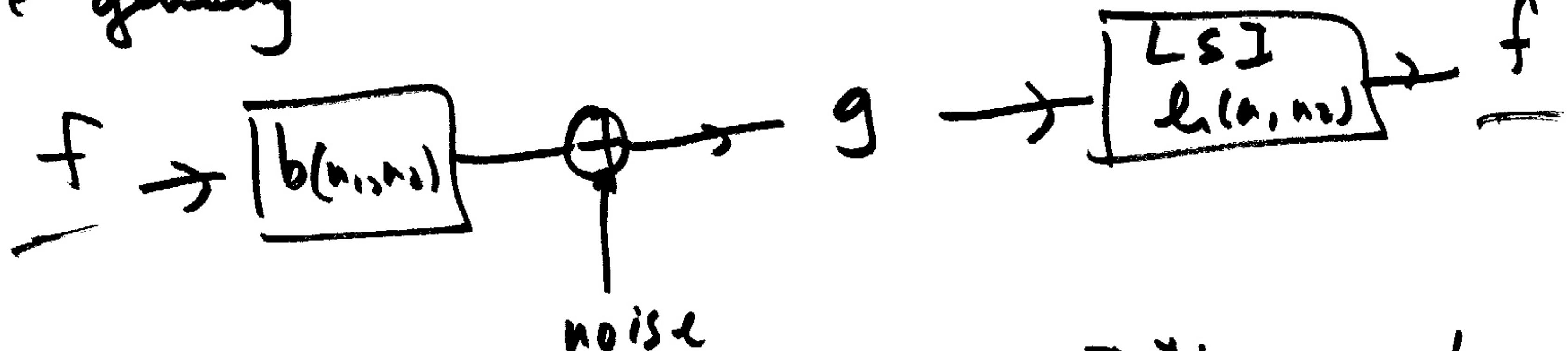
$$=$$

$$P_{x(0),\, x(t_2-t_1),\, \ldots\, x(t_N-t_1)}\left(X_1, X_2, \ldots\ldots X_N\right)$$

Weiner filtering



more generally



f sample of a zero mean stationary random process.

w " " " " " " " "

f, w are independent of each other

8

Define metric : $\hat{f}$ as close as possible to f.

$$E\left[(f-\hat{f})^2\right] \longrightarrow \text{linear least square error.}$$

$\perp$

<u>orthogonality principle</u> :

least square error is achieved when
" error orthogonal To observation"

$\perp$

$$e = f - \hat{f} \perp g \Rightarrow$$

$f - \hat{f}$ must be uncorrelation with g.

$$f \longrightarrow \oplus \longrightarrow g \longrightarrow \boxed{h} \longrightarrow \hat{f}$$

w

Goal: Design h so that $f - \hat{f} \perp g$.

$$E\left[\left(f(n_1,n_2) - \hat{f}(n_1,n_2)\right) \cdot g(m_1,m_2)\right] = 0$$

$$\forall (n_1,n_2), (m_1,m_2).$$

$$\Longrightarrow E\left[f(n_1,n_2)\, g(m_1,m_2)\right] = E\left[\hat{f}(n_1,n_2)\, g(m_1,m_2)\right]$$

find h.

$$g * h = \hat{f}$$

$$E\left[f(n_1,n_2)\,g(m_1,m_2)\right] =$$

$$E\left[\left(\sum_{k_1}\sum_{k_2} h(k_1,k_2)\,g(n_1-k_1,\, n_2-k_2)\right) g(m_1,m_2)\right]$$

$\Longrightarrow$

cross Correlation = R.

cross correlation f,g. $R_{fg}(n_1-m_1,\, n_2-m_2) =$

$$\sum_{k_1}\sum_{k_2} h(k_1,k_2)\, R_g(n_1-k_1-m_1,\, n_2-k_2-m_2)$$

auto correlation of g with itself.

chg of variable $\Longrightarrow$

$$R_{fg}(n_1,n_2) = \sum_{k_1}\sum_{k_2} h(k_1,k_2)\, R_g(n_1-k_1,\, n_2-k_2)$$

$$R_{fg}(n_1,n_2) = h(n_1,n_2) * R_g(n_1,n_2)$$

$\downarrow$ F.T.

$$P_{fg}(\omega_1,\omega_2) = H(\omega_1,\omega_2)\, P_g(\omega_1,\omega_2)$$

Wiener filter

$$\boxed{H(\omega_1,\omega_2) = \frac{P_{fg}(\omega_1,\omega_2)}{P_g(\omega_1,\omega_2)}}$$

$$R_{fg}(n_1,n_2) \triangleq E\left[f(k_1,k_2)\; g(k_1-n_1,k_2-n_2)\right]$$

$$g = f + w.$$

$$R_{fg}(n_1,n_2) = E\left[f(k_1,k_2)\left(f(k_1-n_1,k_2-n_2) + w(k_1-n_1,k_2-n_2)\right)\right]$$

$$R_{fg}(n_1,n_2) = E\left[f(k_1,k_2)\, f(k_1-n_1, k_2-n_2)\right] + E\left[f(k_1,k_2)\, w(k_1-n_1, k_2-n_2)\right]$$

$$\xrightarrow{\text{f, w are indp.}} 0$$

$$R_{fg}(n_1,n_2) = R_f(n_1,n_2)$$

$$R_f(n_1,n_2) = h(n_1,n_2) * R_g(n_1,n_2)$$

F.T.

Weiner

$$\boxed{H(\omega_1,\omega_2) = \frac{P_f(\omega_1,\omega_2)}{P_g(\omega_1,\omega_2)}}$$

$$R_g(n_1,n_2) = E\left[g(k_1,k_2)\,g(k_1-n_1,\,k_2-n_2)\right]$$

$$= E\left[\left(f(k_1,k_2)+w(k_1,k_2)\right)\left(f(k_1-n_1,k_2-n_2)+w(k_1-n_1,k_2-n_2)\right)\right]$$

$$= E\left[f(k_1,k_2)\,f(k_1-n_1,\,k_2-n_2)\right] + E\left[f(k_1,k_2)\,w(k_1-n_1,\,k_2-n_2)\right]^{\to 0} + E\left[w(k_1,k_2)\,f(k_1-n_1,\,k_2-n_2)\right]^{\to 0} + E\left[w(k_1,k_2)\,w(k_1-n_1,\,k_2-n_2)\right]$$

f, w indepd.

$$R_g = R_f(n_1,n_2) + R_w(n_1,n_2)$$

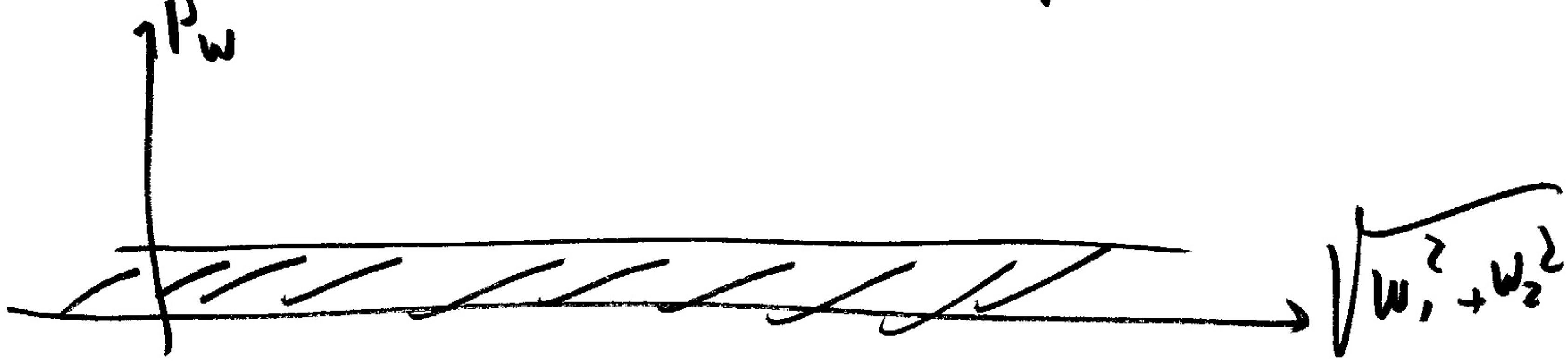
$\downarrow$ F.T.

$$P_g(\omega_1,\omega_2) = P_f(\omega_1,\omega_2) + P_w(\omega_1,\omega_2)$$

$$H(\omega_1, \omega_2) = \frac{P_f(\omega_1, \omega_2)}{P_f(\omega_1, \omega_2) + P_W(\omega_1, \omega_2)}$$

Weiner filter.

most natural image.



Consider 2 cases:

(1) $P_f \gg P_w \Longrightarrow$

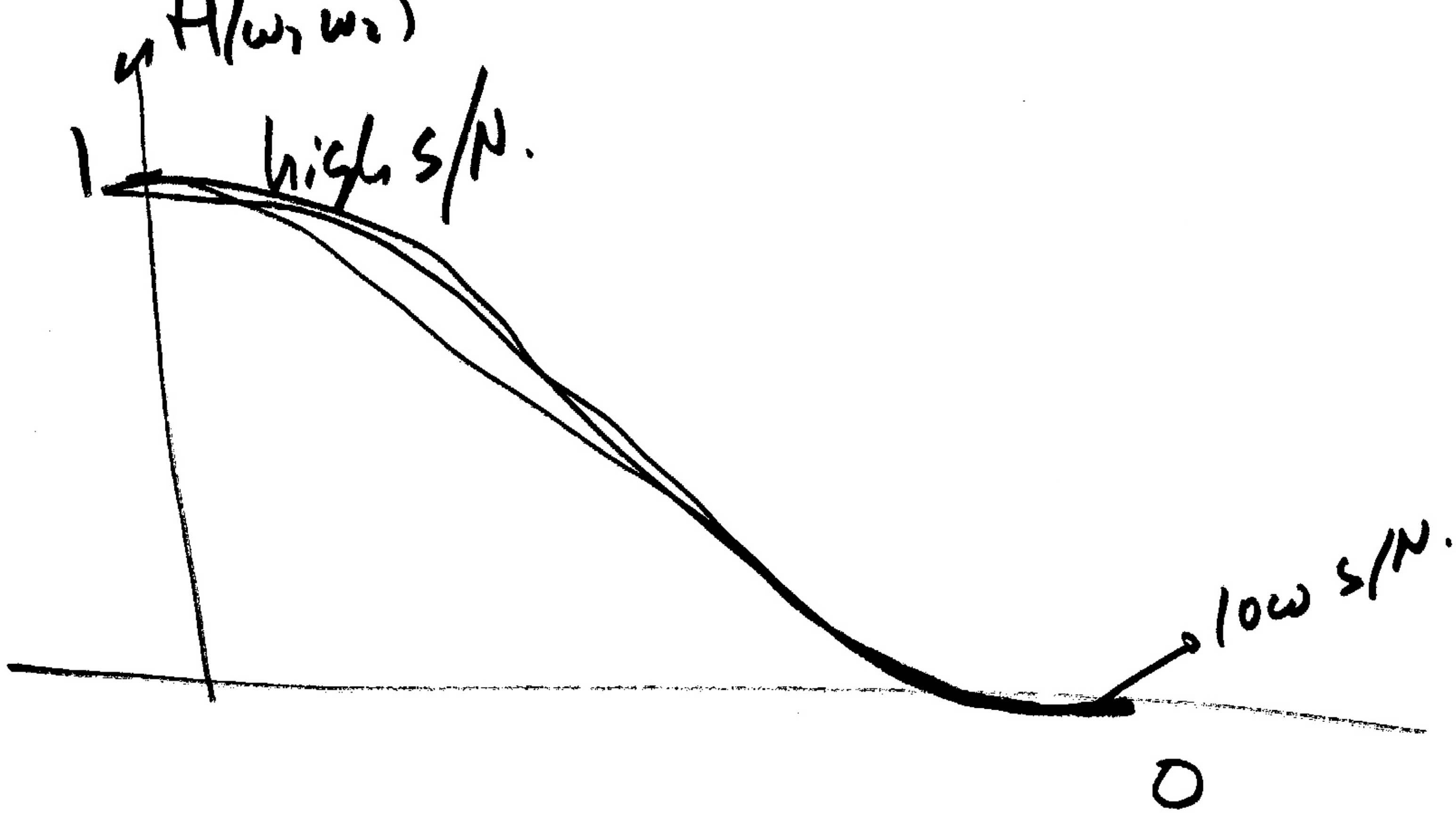
denominator $\simeq P_f \Rightarrow H(\omega_1, \omega_2) \simeq 1$

$\Longrightarrow$ signal gets thru.

(2) $P_f \ll P_w \Longrightarrow$

$H(\omega_1, \omega_2) \simeq \dfrac{\cancel{P_f}\, P_f}{P_w} \simeq 0$

$\Longrightarrow$ Nothing gets thru.



H(ω1, ω2)
1
high S/N.
low S/N.
0

<u>Problem</u> How to find P_f, P_w?

① f is just a sample of R.P.

Average $|F_i(\omega_1, \omega_2)|$ over a lot of natural imagery.

P_f

② Assume model P_f. estimate parameter of P_f by observing g.

$\Rightarrow$ Another problem: Images are not really globally stationary, locally stationary.