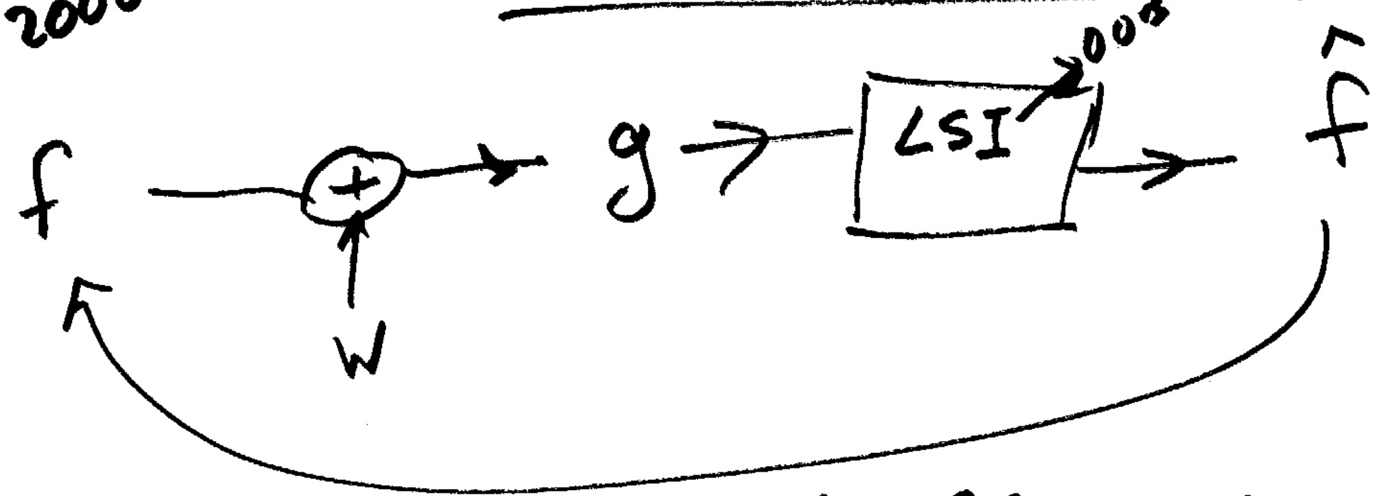
April 7, 2006

Image Restoration



minimize Expected square error between f and $\hat{f}$

$$E\left[(f - \hat{f})^2\right]$$

→ show LSI filter Weiner $= \dfrac{P_f}{P_f + P_w}$

$H(\omega_1, \omega_2) =$

→ Adaptive Weiner filter minimizes excessive blurring.

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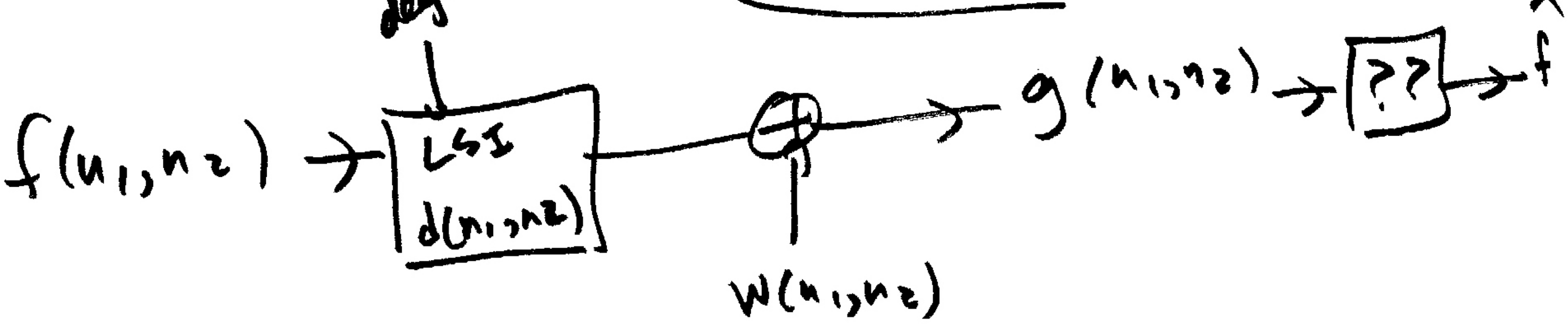
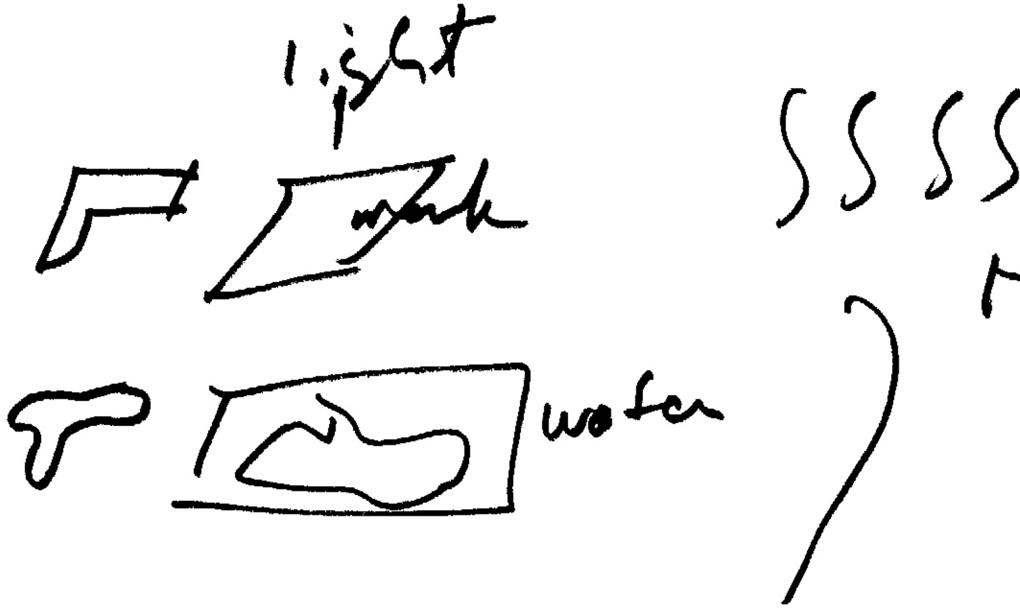
New Problem

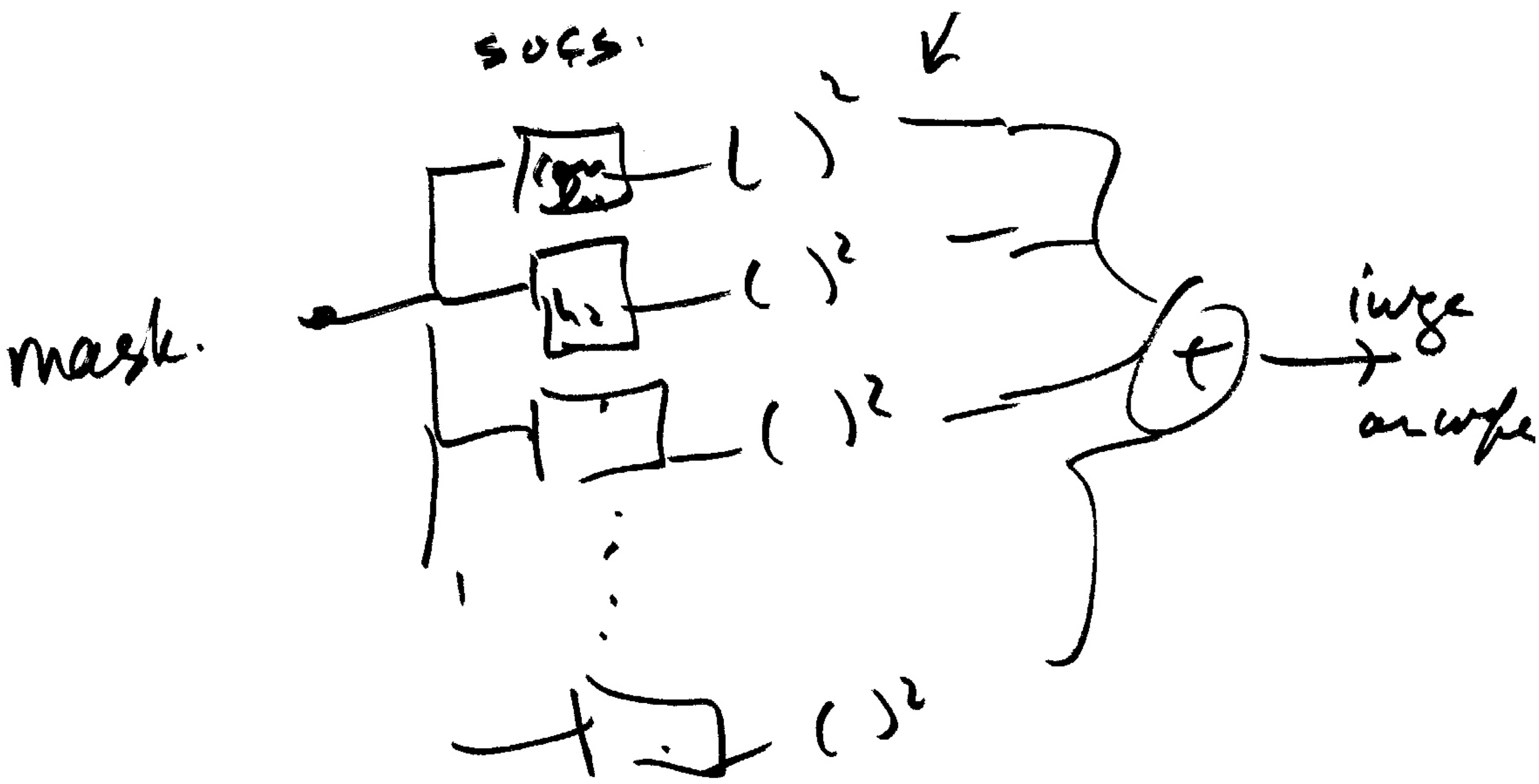


LSI modeling doesn't always present reality.

optics in lithography → Partially coherent projection



Hopkins integral



Optical Proximity Correction OPC.

Blur → motion Blur
→ out of focus Blur
→ Atmospheric Turbulence.

Motion Blur . Assume. scene translates @ constant velocity $\nu_{relative}$ under an angle ϕ radian w.r.t. horizontal axis

- Exposure Time $[0, t_{exposure}]$
- Length of motion $= L = \nu_{relative} \cdot t_{exposure}$.

PSF
$$d_{L,\phi}(x,y) = \begin{cases} \frac{1}{L} & \text{if } \sqrt{x^2+y^2} \leq \frac{L}{2}, \quad \frac{x}{y} = -\tan\phi \\ 0 & \text{otherwise.} \end{cases}$$

Out of focus Blur

$$d_R(x,y) = \begin{cases} \frac{1}{\pi R^2} & \text{if } \sqrt{x^2+y^2} \leq R^2 \\ 0 & \text{otherwise.} \end{cases}$$

R = parameter for how much out of focus.

$$d_R(n_1,n_2) = \begin{cases} \frac{1}{C} & \text{if } \sqrt{n_1^2+n_1^2} \leq R^2 \\ 0 & \text{otherwise} \end{cases}$$

Atmospheric Blur

$$d_{\sigma_G}(x,y) = c \exp\left\{- \frac{x^2+y^2}{2\sigma_G^2}\right\}$$

Techniques. for deconvolution

1. Blur fn is known ←
2. is unknown ← Blind deconvolution.

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3 classes of Restoration / Deconvolution Algs:

1. inverse filtering.
2. Least square filters → weiner, → CLS.
3. Iterative filters.

Inverse Filtering

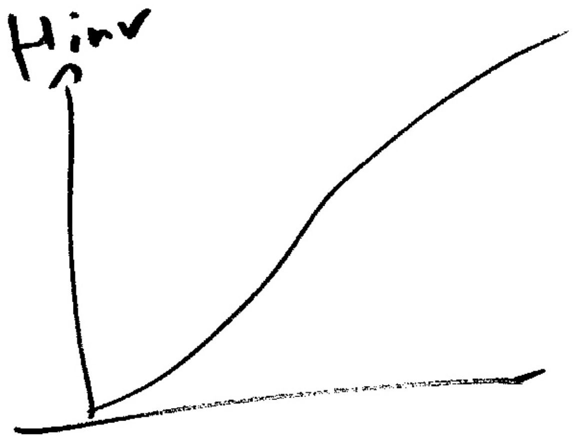
$$h_{inv} * d = \delta(n_1, n_2)$$

$$H_{inv}(w_1, w_2) = \frac{1}{D(w_1, w_2)}$$

$H_{inv}(\omega_1,\omega_2)\, D(\omega_1,\omega_2) = 1.$

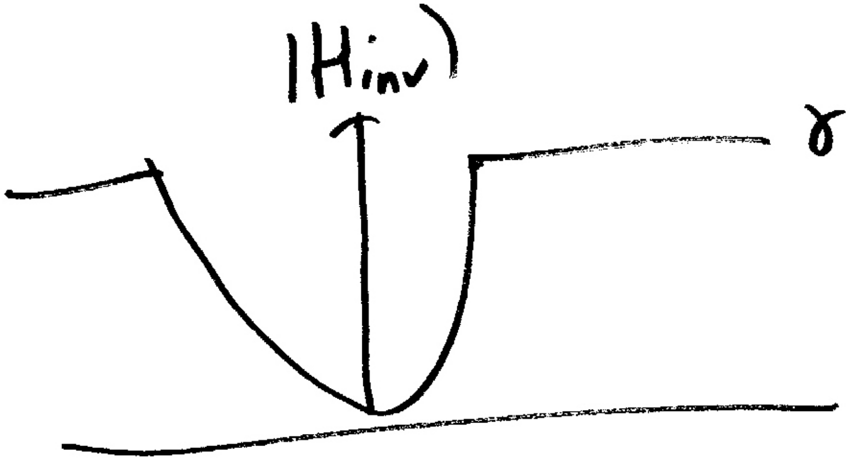
2 problem $\longrightarrow$ ① noise : H_{inv} is High pass filter.

$\Longrightarrow$ accentuates noise.



② $D(\omega_1,\omega_2) \to 0$ everything goes bad

$$H_{inv}(\omega_1,\omega_2) = \begin{cases} \dfrac{1}{D(\omega_1,\omega_2)} & \text{only if } \dfrac{1}{D(\omega_1,\omega_2)} < \gamma \\ \gamma & \text{otherwise} \end{cases}$$



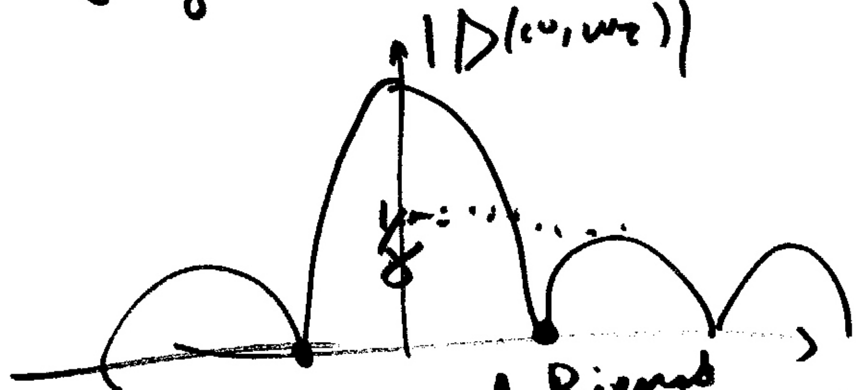


Fig 5 of Biemond
fig 5.29 of G&W 7

Least Squares Filters

$$f \rightarrow \boxed{d} \rightarrow \underset{\substack{\uparrow \\ w}}{\oplus} \rightarrow g \rightarrow \boxed{LSI} \rightarrow \hat{f}$$

$$E\left[(f-\hat{f})^2\right]$$

$$H_{weiner}(\omega_1,\omega_2) = \frac{D^*(\omega_1,\omega_2)}{D^*(\omega_1,\omega_2)D(\omega_1,\omega_2) + \left[\frac{P_W(\omega_1,\omega_2)}{P_f(\omega_1,\omega_2)}\right]}$$

- obs. 1: If $D(\omega_1,\omega_2) = 1 \Rightarrow$ no bluring.

$$H_{weiner} = \frac{P_f}{P_f + P_w}$$

- obs 2: noise $\rightarrow 0$

$$H_{inverse} = \begin{cases} \frac{1}{D(\omega_1,\omega_2)} & \text{for } D(\omega_1,\omega_2) \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

inverse filte.

- obs 3 : For (ω_1, ω_2)

$$P_W(\omega_1, \omega_2) << P_f(\omega_1, \omega_2)$$

(noise is much smaller than signal)

Then weine filter approaches Inverse filter.

For (ω_1, ω_2) $P_W(\omega_1, \omega_2) >> P_f(\omega_1, \omega_2)$

$H_{weine} \to 0$ frequency rejection filter

How to estimate P_f

① $P_f = P_g - P_W$

Model W as a white noise with variance σ_W^2

$$P_f(\omega_1, \omega_2) = P_g(\omega_1, \omega_2) - \sigma_W^2$$

$$= \frac{1}{N_1 N_2} \underline{G(\omega_1, \omega_2) G^*(\omega_1, \omega_2)} - \sigma_W^2$$

Periodogram

2. set of representative image

3. model Based: 2D causal autoregressive model.

$$f(n_1, n_2) = a_{01} f(n_1, n_2 - 1) + a_{11} f(n_1 - 1, n_2 - 1) + a_{10} f(n_1 - 1, n_2) + v(n_1, n_2)$$

white noise with some sort of variance.

Cameraman

$a_{01} = .709$

$a_{11} = -0.467$

$a_{10} = .739$

$\sigma_v^2 = 231$

Fig 6 in Biemond/Langendijk paper.

$$SNR_g = 10 \log_{10} \left(\frac{\text{Variance } f}{\text{Variance } (g - f)} \right) \text{ dB}$$

$$SNR_{\hat{f}} = 10 \log_{10} \left(\frac{\text{Variance } \hat{f}}{\text{Variance } (\hat{f} - f)} \right) \text{ dB}$$

$$\Delta SNR = SNR_{\hat{f}} - SNR_g$$

$$\Delta SNR = 10 \log 10 \left(\frac{\text{Variance of } (g - f)}{\text{Variance of } (\hat{f} - f)} \right) \text{dB.}$$

Fig 6:

$$\hookrightarrow P_f(\omega_1,\omega_2) = \frac{\sigma_v^2}{\left| 1 - a_{01} e^{-j\omega_1} - a_{11} e^{-j\omega_1 - j\omega_2} - a_{10} e^{-j\omega_2} \right|^2}$$

Fig 5.29 G/W.

Alternative To Weiner

$$f \rightarrow \boxed{d} \rightarrow \bigoplus \rightarrow g \rightarrow \boxed{???} \rightarrow \hat{f}$$

$$\uparrow w$$

$$(*) \quad \| d * \hat{f} - g \|^2 \approx \sigma_w^2 \longleftarrow \text{ideally}$$

__Approach__ find a "smooth" signal such that (*) is satisfied.

Define $c(n_1, n_2)$ as high pass filter.

minimize $\| c * \hat{f} \|^2$

subject to (*)

→ Constrained Least Squares.

$$H_{res}(\omega_1, \omega_2) = \frac{D^*(\omega_1, \omega_2)}{\left(D^*(\omega_1,\omega_2)\, D(\omega_1,\omega_2) + \alpha\, C^*(\omega_1,\omega_2)\, C(\omega_1,\omega_2)\right)}$$

α = ideally ~~B~~ ~~so~~ chosen to satisfy *

→ regularization parameter.

Example of c → 2D laplacian operator

$$\begin{array}{ccc} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{array}$$

Fig 8 → Biemond

Fig 5.30 G/W.

Iterative Filters

motivation.

1. Actively control Trade off between ringing + blurring.
2. Can incorporate a priori constraint about your signal.

$$\hat{f}_{i+1}(n_1,n_2) \leftarrow \hat{f}_i(n_1,n_2) + \beta\left[g(n_1,n_2) - d * f_i \right]$$

Show converge if $|1-\beta D(\omega_1,\omega_2)| < 1 \quad \forall (\omega_1,\omega_2)$

Assuming $|D(\omega_1, \omega_2)| \leq 1$

Then convergence

$$0 < \beta < 2 \quad \text{provided } D(\omega_1, \omega_2) > 0$$

$$\lim_{i \to \infty} \hat{f}_i = h_{inv} * g$$

Fig 9 Lagendijk / Biemond.

__Apriori knowledge__: We know img. is positive. $\longrightarrow$ Apply a projection operator:

$$P\left[\hat{f}(n_1,n_2)\right] = \begin{cases} \hat{f}(n_1,n_2) & \text{if } \hat{f}(n_1,n_2) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{f}_{i+1}(n_1,n_2) \leftarrow P\left[\hat{f}_i + \beta(g - d * \hat{f}_i)\right]$$

$$\hat{f}_{i+1} \leftarrow P_1 P_2 P_3 \left[\cdots \cdots \right]$$

POCS $\doteq$ Projection Onto Convex Sets.

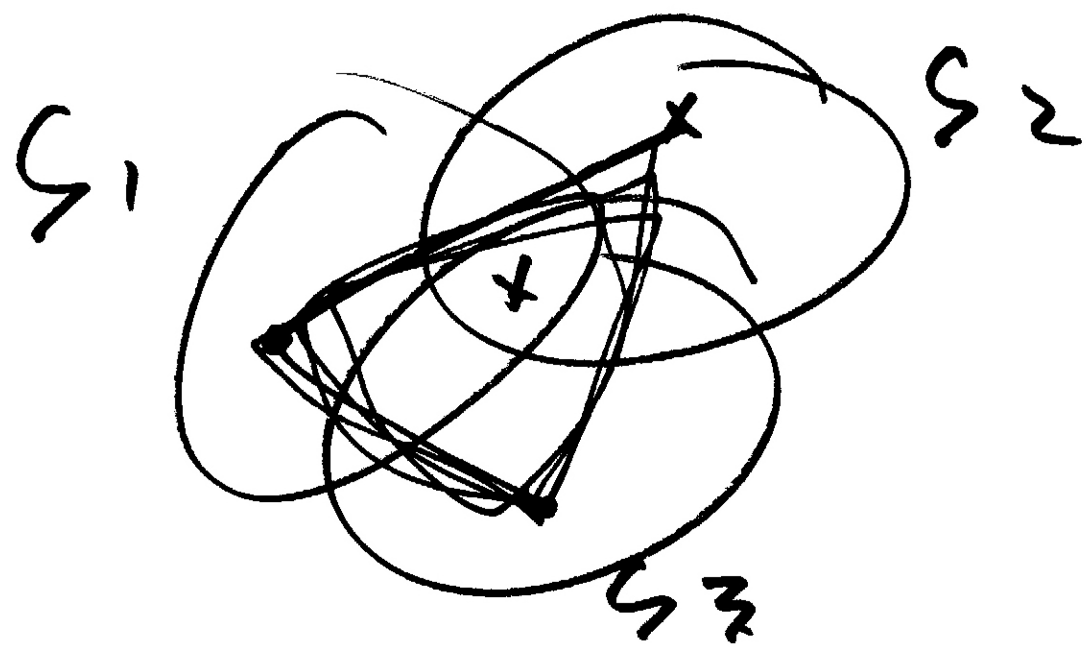
Convex Set $\doteq S$

$a \in S \qquad b \in S$.

$\forall \epsilon \qquad a\epsilon + (1-\epsilon)b$ is also $\in S$

Example: set of Positive image is convex.

$S_1, S_2, S_3 \longrightarrow$ convex



Want to find <u>any</u> point in the intersection

Start at an arbitrary ~~and~~ point, keep projecting $\longrightarrow$ eventually converge

To the intersection.